Quench Dynamics of One-Dimensional Interacting Bosons in a Disordered Potential: Elastic Dephasing and Critical Speeding-Up of Thermalization

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The dynamics of interacting bosons in one dimension following the sudden switching on of a weak disordered potential is investigated. On time scales before quasiparticles scatter (prethermalized regime), the dephasing from random elastic forward scattering causes all correlations to decay exponentially fast, but the system remains far from thermal equilibrium. For longer times, the combined effect of disorder and interactions gives rise to inelastic scattering and to thermalization. A novel quantum kinetic equation accounting for both disorder and interactions is employed to study the dynamics. Thermalization turns out to be most effective close to the superfluid-Bose-glass critical point where nonlinearities become more and more important. The numerically obtained thermalization times are found to agree well with analytic estimates.

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One of the most challenging questions in strongly correlated systems is understanding the combined effect of disorder and interactions. This old problem has recently received some fresh input both in the form of experiments where ultracold gases with tunable interactions and tunable disordered potentials have been realized [1–3], and in the form of theory where phenomena such as many-body localization have been proposed [4–6]. These studies indicate that the combined effect of disorder and interactions is most dramatic in the nonequilibrium regime. While even for clean interacting systems, quantum dynamics is poorly understood, disorder adds yet another layer of complexity to the problem.

In this Letter we study quench dynamics of a one-dimensional (1D) interacting Bose gas in a disordered potential. The quench involves a sudden switching on of the disordered potential. Past studies of such quenches have primarily focused on the limit of strong disorder and weak interactions where many-body localization may lead to a breakdown of equilibration [7–9]. We focus on the complementary regime of strong interactions and weak disorder. More precisely, we investigate a regime where disorder is nominally irrelevant by studying the superfluid side of the superfluid-Bose-glass quantum critical point.

A quantum quench drives a system out of equilibrium, and the key question is how the system relaxes. We show that the nonequilibrium bosons generated by the quench can relax by means of two different kinds of scattering processes in the presence of disorder. One is a random elastic forward scattering which leads to dephasing. The second is inelastic scattering arising due to the interplay of disorder and interactions which eventually thermalizes the system. We use a novel quantum kinetic equation that accounts for both disorder and interactions to numerically investigate how the system thermalizes. We also present analytic estimates for the thermalization time. However, we do not investigate the role of hydrodynamic long time tails which ultimately dominate equilibration at the longest time scales [10].

Upon approaching a classical or quantum critical point, two competing phenomena can occur: "critical slowing down" arises when the relaxation becomes slower and slower due to the dynamics of larger and larger domains. But also the opposite, "critical speeding up," can occur: because of the abundance of critical fluctuations and the importance of nonlinearities, thermalization can become more efficient close to criticality. Both effects can even occur simultaneously. For magnetic quantum-critical points in 3D metals, for example, electron relaxation becomes more efficient close to the transition, while the order parameter relaxes more slowly [11]. A dramatic critical speeding up has, for example, recently been observed close to the liquid-gas transition of monopoles in spin ice [12]. In addition, experimental, numerical, and analytic results on the short-time [13–15] and long-time dynamics [16] of the superfluid-Mott transition suggest that the dynamics becomes faster upon approaching the transition. In this case, however, the proximity to integrable points makes the theoretical analysis of equilibration more challenging, a complication absent in our study. We find that the enhanced role of backscattering close to the critical point does give rise to a striking enhancement of equilibration upon approaching the critical point.

The equilibrium phase diagram of 1D interacting bosons in the limit of weak disorder was studied in Refs. [17,18], where a Berezenskii-Kosterlitz-Thouless transition from the superfluid phase to a Bose-glass phase was identified (for strong disorder see, e.g., Refs. [19–21], and for quasiperiodic lattices see Refs. [22,23]). We study quench dynamics in the regime of weak disorder when bosons are delocalized in the ground state. However, we show that, out of equilibrium, even very weak disorder can be quite potent, causing elastic dephasing and inelastic scattering. These effects will be identified by studying the time evolution of some key correlation functions and the boson distribution function.

Our quench protocol is as follows. First, the bosons are prepared in the ground state of a Hamiltonian H_i characterized by an interaction parameter K and sound velocity u, $H_i = (u/2\pi)\int dx[K(\pi\Pi(x))^2 + (1/K)(\partial_x\phi(x))^2] = \sum_{p\neq 0} u|p|a_p^\dagger a_p$. $\Pi = \partial_x\theta/\pi$ is canonically conjugate to the field ϕ, $-\partial_x\phi/\pi$ is the smooth part of the boson density, and the theory is diagonal in terms of $a_p^\dagger, a_p$, the creation and annihilation operators for the sound modes [24,25]. At $t = 0$, a disordered potential is suddenly switched on so that the time evolution from $t > 0$ is governed by the final Hamiltonian $H_f = H_i + V_{\text{dis}}$, where

$$V_{\text{dis}} = \int dx\left[-\frac{1}{\pi}\eta(x)\partial_x\phi + (\xi^* e^{2i\phi} + \xi e^{-2i\phi})\right]. \quad (1)$$

η and ξ are the strength of the forward- and backward-scattering disorder, respectively [24]. These are assumed to be time independent and Gaussian distributed so that disorder averaging (represented by $\overline{\dots}$) gives $\overline{\eta(x)\eta(x')} = D_f\delta(x-x')$, $\overline{\xi(x)\xi^*(x')} = D_b\delta(x-x')$. We find it convenient to define $\mathcal{D}_b = 2\pi D_b u/\Lambda^3$ and $\mathcal{D}_f = D_f(\alpha/u^2)$ as the dimensionless strength of the forward- and backward-scattering disorder, respectively, where $\Lambda = u/\alpha$ is a UV cutoff. Note that $K\to\infty$ is the limit of noninteracting bosons, while $K = 1$ corresponds to hard-core bosons (free fermions), with the superfluid-Bose-glass critical point located near $K = 3/2$ [24].

We will study the time evolution after the quench of the boson density-density correlation function $R_{\phi\phi}$ and the single-particle correlation function $R_{\theta\theta}$, the latter being a measure of the superfluidity in the system. These quantities in the language of bosonization are

$$R_{\phi\phi}(r,t) = \langle\psi_i|e^{iH_f t}e^{2i\phi(r)}e^{-2i\phi(0)}e^{-iH_f t}|\psi_i\rangle, \quad (2)$$

$$R_{\theta\theta}(r,t) = \langle\psi_i|e^{iH_f t}e^{i\theta(r)}e^{-i\theta(0)}e^{-iH_f t}|\psi_i\rangle, \quad (3)$$

where $|\psi_i\rangle$ is the state before the quench (the ground state of H_i). Note that $R_{\phi\phi}$ is the correlator for the component of the density that oscillates at $2\pi\rho_0$ (where ρ_0 is the average boson density). We choose to study this because in the vicinity of the superfluid-Bose-glass critical point, charge density wave fluctuations dominate.

We employ a Keldysh path-integral formalism wherein the expectation value of the observable R_{aa} (where $a = \theta/\phi$) is given by

$$\begin{aligned}\langle\psi_i|R_{aa}(t)|\psi_i\rangle &= \text{Tr}[e^{-iH_f t}|\psi_i\rangle\langle\psi_i|e^{iH_f t}R_{aa}] \\ &= \int\mathcal{D}[\phi_{cl},\phi_q]e^{i(S_0+S_{\text{dis}})} \\ &\quad\times R_{aa}[\phi_{cl/q}(t),\theta_{cl/q}(t)], \quad (4)\end{aligned}$$

where $\phi_{cl,q}, \theta_{cl,q}$ are linear combinations of the fields $\phi_\pm, \theta_\pm$ in the two-time Keldysh formalism [26]. Above, S_0 captures the correlators of the clean interacting Bose gas after the quench, exactly known within our Luttinger liquid approximation [27]. S_{dis} contains the forward- and backward-scattering disorder. While the forward-scattering disorder may be treated exactly, we will treat the backward-scattering disorder perturbatively. Within the Keldysh formalism, disorder averaging may be carried out without the complication of introducing replicas

$$\begin{aligned}\overline{\langle\psi_i|R_{aa}(t)|\psi_i\rangle} &= \int\mathcal{D}[\eta,\xi,\xi^*]e^{-(\eta^2(x)/2D_f)}e^{-(\xi(x)\xi^*(x)/D_b)} \\ &\quad\times\langle\psi_i|R_{aa}(t)|\psi_i\rangle. \quad (5)\end{aligned}$$

Writing $R_{aa} = R_{aa}^{(0)} + R_{aa}^{(1)} + \dots$, where $R^{(i)}$ is $\mathcal{O}(D_b^i)$, to leading order, only the forward-scattering disorder affects the correlators, but already at this order elastic dephasing effects will be apparent. To see this, note that when $D_b = 0$, H_f may be diagonalized, $H_f(D_b = 0) = \sum_p u|p|\Gamma_p^\dagger\Gamma_p$, where $\Gamma_p = a_p + (\tilde\eta_p/u|p|)$, and $\tilde\eta_p = (\sqrt{K/L})\sqrt{L|p|/2\pi}e^{-\alpha|p|/2}\int dx\eta(x)e^{-ipx}$, L being the system size. The quench creates a highly nonequilibrium distribution of the Γ_p quasiparticles so that, before disorder averaging, the leading order correlators at a time t after the disorder quench are [28]

$$\begin{aligned}R_{\phi\phi}^{(0)}(r,t) &= \langle\psi_i|e^{2i\phi(r,t)}e^{-i2\phi(0,t)}|\psi_i\rangle_{D_f=0} \\ &\quad\times e^{-(iK/u)\sum_{\epsilon=\pm}\left[\int_r^{r+\epsilon ut}dy\eta(y)-\int_0^{\epsilon ut}dy\eta(y)\right]}, \quad (6)\end{aligned}$$

$$\begin{aligned}R_{\theta\theta}^{(0)}(r,t) &= \langle\psi_i|e^{i\theta(r,t)}e^{-i\theta(0,t)}|\psi_i\rangle_{D_f=0} \\ &\quad\times e^{-(i/2u)\left[\int_{r-ut}^{r+ut}dy\eta(y)-\int_{-ut}^{ut}dy\eta(y)\right]}. \quad (7)\end{aligned}$$

The correlators are what they would have been in the absence of the forward-scattering disorder ($D_f = 0$) but multiplied by random phases. These phases arise because the quench creates excited left- and right-moving quasiparticles which, as they travel along the chain, pick up random phases due to the forward-scattering disorder. Thus, the operator at position r will be affected by phases picked up in the region $[r - ut, r]$ by the right movers and phases picked up in the region $[r + ut, r]$ by the left movers.

Because of these random phases, disorder averaging leads to dephasing that causes the correlators to decay exponentially in time or position,

$$\bar{R}^{(0)}_{\phi\phi}(r,t) = \left[\frac{1}{\sqrt{1+r^2\Lambda^2}}\right]^{2K} \exp\left\{-\frac{K^2 D_f}{u}[2t\Theta(|r|/u-2t) + (4t-|r|/u)\Theta(2t-|r|/u)\Theta(|r|/u-t) + 3|r|\Theta(t-|r|/u)]\right\}$$

$$\bar{R}^{(0)}_{\theta\theta}(r,t) = \left[\frac{1}{\sqrt{1+r^2\Lambda^2}}\right]^{1/(2K)} \exp\left\{-\frac{D_f}{4u}[2t-(2t-|r|/u)\Theta(2t-|r|/u)]\right\}. \quad (8)$$

Here, Θ is the Heaviside function. Thus, the disorder-averaged correlators are found to decay exponentially with time for short times $ut < r/2$, with a crossover to a steady-state behavior with an exponential decay in position at long times ($ut > r/2$ for $R_{\theta\theta}$ and $ut > r$ for $R_{\phi\phi}$). It is interesting to contrast this behavior with the situation in equilibrium. There, the forward-scattering disorder also imposes an exponential decay in the position of the density correlator $R^{\rm eq}_{\phi\phi} \sim (1/r^{2K})e^{-(2K^2 D_f|r|/u^2)}$, but it does *not* affect the single-particle propagator at all $R^{\rm eq}_{\theta\theta} \sim 1/r^{1/(2K)}$, implying that it cannot suppress superfluidity. Only backward-scattering disorder suppresses superfluidity in equilibrium, eventually causing a transition to the Bose-glass phase [17]. In contrast, our leading order result shows that when the system is quenched, even *forward* scattering strongly affects superfluidity due to random dephasing caused by the emitted nonequilibrium quasiparticles.

Thus, even though the disorder is weak, and even though we are in the short-time or intermediate-time regime where the full effect of the disorder has not yet set in, disorder is very effective in destroying the superfluidity due to random dephasing. Moreover, in stark contrast to equilibrium, it is the forward-scattering disorder which is the most potent in this prethermalized regime, as random dephasing caused by it also makes the backward-scattering disorder more "irrelevant" than in equilibrium. Thus, while superfluidity is destroyed, the phase that replaces it is not a backward-scattering disorder induced localized phase either. In fact, as we discuss in detail below, the role of backward-scattering disorder is to facilitate inelastic scattering, causing the system to thermalize into a delocalized high temperature phase.

We now discuss the long-time regime where inelastic effects are important. Even in clean interacting systems, inelastic effects set in after a quench; however, for the Luttinger model, where only forward-scattering interactions are retained, the clean system is incapable of thermalizing. In contrast, once disorder is present, the combined effect of disorder and interactions can cause inelastic scattering, leading to thermalization. We now explore this phenomenon. Of course, for free fermions with disorder ($K = 1$), there is again no inelastic scattering; however, our treatment is valid for strong attractive (albeit forward scattering) interactions and weak disorder.

The quantum quench generates nonequilibrium quasiparticles with density $n_p(t) = \langle\psi_i(t)|\Gamma_p^\dagger\Gamma_p|\psi_i(t)\rangle$. At short times $\gamma_0 t < 1$ (below we give an estimate for γ_0), these may be considered to be almost free; this is the so-called prethermalized regime [29–33] discussed above. In contrast, at longer times, these quasiparticles eventually scatter among each other, with the distribution function evolving according to the quantum kinetic equation [28]

$$\frac{u|p|}{\Lambda^2}\frac{\partial}{\partial t}n_p(t) = -\frac{i\pi K}{2}\left\{n_p(t)[\Sigma^R - \Sigma^A](p,t) - \frac{1}{2}[\Sigma^K(p,t) - (\Sigma^R - \Sigma^A)(p,t)]\right\}. \quad (9)$$

$\Sigma^{R,A,K}$ are the self-energies to $\mathcal{O}(\mathcal{D}_b)$, and they depend on the nonequilibrium population $n_p(t)$. A kinetic equation similar to the one above was derived for a commensurate periodic potential [16]. For the disordered problem, the derivation follows analogously. Because the interaction vertex is of the form $e^{2i\phi}$, a key feature of the kinetic equation is that it allows for multiparticle scattering between bosons. Besides this, it has all the usual properties of a kinetic equation in that it conserves energy, and the right-hand side vanishes when n_p is the Bose distribution function. We solve the kinetic equation numerically, where the initial condition entering the kinetic equation is the nonequilibrium quasiparticle density n_p generated by the quench. Note that the kinetic equation has been obtained after a leading order gradient expansion and, in doing so, has lost some of the initial memory effects and is therefore not valid at very short times after the quench. We smoothly connect between the short-time dynamics and the long-time dynamics of the kinetic equation by perturbatively evolving $n_p(t)$ forward in time at short times, and we use this distribution as the initial condition for the kinetic equation. For $u|p| \ll \Lambda$, such a perturbative short-time evolution gives [28]

$$n_p(t \simeq 0) = \langle\psi_i|\Gamma_p^\dagger\Gamma_p|\psi_i\rangle = \frac{T^f_{\rm eff} + T^b_{\rm eff}}{u|p|} = \frac{T_{\rm eff,0}}{u|p|}, \quad (10)$$

where $T_{\rm eff,0} = T^f_{\rm eff} + T^b_{\rm eff}$ with $T^f_{\rm eff} = K\mathcal{D}_f\Lambda/2\pi$, $T^b_{\rm eff} = \Lambda 8\pi K\mathcal{D}_b[\Gamma(2K-2)/\Gamma(2K)]$. Thus, the density is a sum of two terms, one proportional to the strength of the forward-scattering disorder and the second proportional to the strength of the backward-scattering disorder. The symbol $n_p(t \simeq 0)$ is used to imply that this distribution is obtained after an initial time evolution. At long wavelengths, the distribution $n_p(t \simeq 0)$ has the appearance of an effective temperature, however, unlike a true temperature where for $u|p| \geq T_{\mathrm{eff},0}$ the distribution function is exponentially suppressed; for our case, the distribution function maintains a slow power-law decay with momentum up to energy scales of the order of the cutoff Λ. We will use $T_{\mathrm{eff},0}$ as a measure of the quench amplitude, and all energy scales will be measured in units of Λ.

We now present results for the numerical solution of the kinetic equation for a point far away from $(K = 3)$ and at $(K = 3/2)$ the superfluid-Bose-glass critical point. In the main panel of Fig. 1, $qn(q)$ is plotted at different times after the quench, and it is found to reach thermal equilibrium $qn_{\mathrm{eq}} = q/(e^{u|q|/T_{\mathrm{eq}}} - 1)$, with T_{eq} being determined from energy conservation. The high-q modes thermalize the fastest; thus, the thermalization time is set by the behavior of the long-wavelength modes, an observation which will allow us to make analytic estimates for the thermalization time. The numerics also show that the relaxation to equilibrium is not determined by a single time scale [34] and therefore not described by a single exponential function. This is most directly seen by studying how $u[qn(q)]_{q=0} = T_{\mathrm{eff}}$ approaches T_{eq} starting from its initial value of $T_{\mathrm{eff},0}$ (see insets of Figs. 1 and 2). The inset of Fig. 1 shows that the system thermalizes much faster at the critical point $K = 3/2$ in comparison to away from it (see also [28]). The inset of Fig. 2 clearly shows that at least two different relaxation rates appear in the dynamics. Below we discuss these rates analytically.

Since the longest-wavelength mode relaxes the slowest, let us consider the outscattering rate in the long-wavelength limit,

$$\gamma(p,t) = \left(\frac{\pi K}{2}\right)\frac{i(\Sigma^R - \Sigma^A)}{u|p|} \xrightarrow{p\to 0}$$
$$= 4K\mathcal{D}_b \int_{-\infty}^{\infty} d(\Lambda\tau) \sin\left[2K\tan^{-1}\Lambda\tau\right](\Lambda\tau)e^{-I(t,\tau)}, \tag{11}$$

where $I(t,\tau) = 2K\int_0^\infty (dq/q)e^{-\alpha q}[1+2n_q(t)][1-\cos(qu\tau)]$. Two time scales may be extracted from Eq. (11). One is γ_0^{-1}, the time scale for leaving the prethermalized regime, and the second is $\gamma_{\mathrm{th}}^{-1}$, the thermalization time when the system is weakly perturbed from thermal equilibrium. To determine the former, we substitute $n_p(t \simeq 0)$ into Eq. (11) to obtain $\gamma_0 \sim \mathcal{D}_b T_{\mathrm{eff},0} \sim \mathcal{D}_b(\mathcal{D}_f + b\mathcal{D}_b)$.

As the system evolves, the distribution function approaches thermal equilibrium. The time scale $\gamma_{\mathrm{th}}^{-1}$ for the final approach to thermal equilibrium may be estimated by substituting $n_p = 1/(e^{u|p|/T_{\mathrm{eq}}} - 1)$ in Eq. (11). This yields a thermalization rate of $\gamma_{\mathrm{th}} \sim \mathcal{D}_b(T_{\mathrm{eq}})^{2K-2}$. Since $T_{\mathrm{eq}} \sim \sqrt{T_{\mathrm{eff},0}}$ for small quench amplitudes [28],

$$\gamma_{\mathrm{th}} \sim \mathcal{D}_b[T_{\mathrm{eff},0}]^{K-1}. \tag{12}$$

Our numerical results show that high-energy modes relax sufficiently fast such that the total time needed for thermalization can be estimated from $t_{\mathrm{th}} \sim \gamma_{\mathrm{th}}^{-1}$. The relaxation rate towards thermal equilibrium obtained from the long time tail of the time evolution is shown in the main panel of Fig. 2, and it agrees well with γ_{th}. Note that the dramatic reduction of thermalization time on approaching the superfluid Bose-glass critical point is due to the backward-scattering disorder becoming more relevant, facilitating thermalization. We emphasize that our results are valid as long as the backscattering disorder is a weak perturbation, which is the case for $K > 3/2$ where $\mathcal{D}_b$ is renormalization group irrelevant. While our expressions remain well

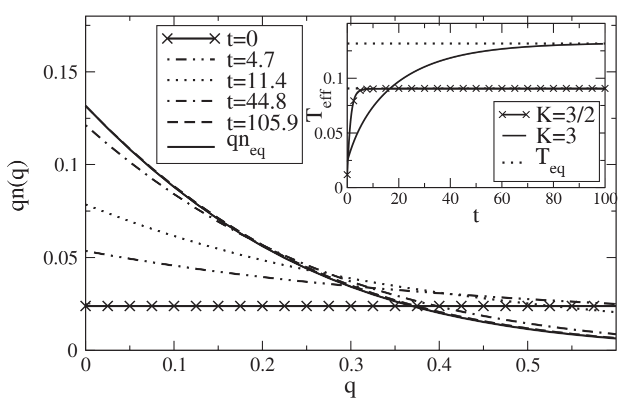

FIG. 1. Main panel: Time evolution of $qn(q)$ for a quench where $K = 3$ and the quench amplitude $T_{\mathrm{eff},0} = 0.024$. The system thermalizes with $n(q)$ approaching $n_{\mathrm{eq}}(q) = 1/(e^{u|q|/T_{\mathrm{eq}}} - 1)$, with T_{eq} determined from energy conservation. Inset: Time evolution of $u[qn(q)]_{q=0} = T_{\mathrm{eff}}$ for $K = 3$ and $3/2$. $T_{\mathrm{eff}}(t = 0) = T_{\mathrm{eff},0}$ and it approaches T_{eq} at long times. $u = 1$, q, t are in units of Λ, $[8\mathcal{D}_b\Lambda/\pi]^{-1}$, respectively.

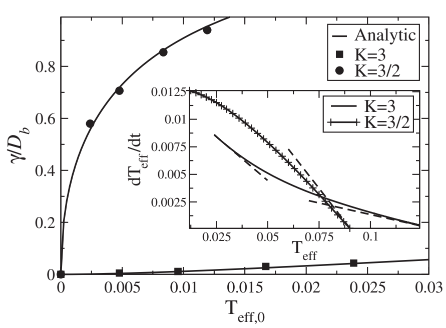

FIG. 2. The thermalization rate γ obtained from the long time tail agrees well with the analytic estimate which for small quench amplitudes is $\gamma_{\mathrm{th}} \sim \mathcal{D}_b(T_{\mathrm{eff},0})^{K-1}$. Inset: The relaxation rates for $K = 3$ and $3/2$ (the latter has been scaled down). For $K = 3$, at short $(T_{\mathrm{eff}} \simeq T_{\mathrm{eff},0})$ and long times $(T_{\mathrm{eff}} \simeq T_{\mathrm{eq}})$ the relaxation rates agree well with γ_0, γ_{th} respectively indicated by the dashed lines. For $K = 3/2$, the relaxation rate at long times agrees well with γ_{th}.

defined for $1 < K < 3/2$, they clearly break down in the hard-core boson (or free-fermion limit), where the perturbative expression for the density (see $T^b_{\rm eff}$) and the zero temperature outscattering rate $\gamma_{\rm th} \sim \mathcal{D}_b \int_1^\infty d\tau 1/\tau^{2K-1}$ acquire infrared divergences [28].

To summarize, we have studied quench dynamics in a system where both interactions and disorder are present. A key effect of the disorder is to give rise to random forward-scattering induced elastic dephasing, important even at short times, which destroys superfluidity. At longer times, the interplay of disorder and interactions leads to thermalization which is strongly enhanced close to the superfluid-Bose-glass transition. Both in the short-time elastic dephasing regime and the long-time thermal regime, correlations decay exponentially; however, one may differentiate between these two regimes by an echo [35] experiment: an echo visible in the short-time dephasing regime will be suppressed exponentially when inelastic scattering dominates. The two regimes may also be identified by the length scale determining the decay of the correlations, which is D_f in the elastic dephasing regime and $T_{\rm eq}$ in the thermal regime. The dephasing dominated regime should also be observable in short-time numerical simulations on disordered lattice systems. An interesting direction is to study quenches on the insulating side of the superfluid-Bose-glass transition where the growth of disorder under renormalization competes with dephasing and decoherence arising from the nonequilibrium population of quasiparticles.

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