Transmission Phase Lapses through a Quantum Dot in a Strong Magnetic Field

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The phase of the transmission amplitude through a mesoscopic system contains information about the system's quantum mechanical state and excitations thereof. In the absence of an external magnetic field, abrupt phase lapses occur between transmission resonances of quantum dots and can be related to the signs of tunneling matrix elements. They are smeared at finite temperatures. By contrast, we show here that in the presence of a strong magnetic field, phase lapses represent a genuine interaction effect and may occur also on resonance. We identify a relevant physical regime where these phase lapses are robust against finite temperature broadening.

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Probability amplitudes, which are the building blocks of quantum mechanics, are complex entities. This simple fact underlines many quantum phenomena in nature. The need to measure such complex entities is thus of substantial importance. Such measurements, performed on the transmission amplitude of electrons passing through a quantum dot (QD), revealed an intriguing phenomenon: contrary to naive expectations, based on the analogy between a double barrier and a QD, the transmission phase through a QD was found to jump abruptly between transmission peaks. The origin of these so-called phase lapses has been heatedly debated [1-17]. The explanation relies on the presence of intradot interactions, which lead to *population switching*: an abrupt "swap" of two level occupations as the gate voltage is varied [13–18]. We also note explanations that invoke quantum chaos correlations in the OD [19,20].

In the presence of a strong magnetic field [21], specifically in the integer quantum Hall (QH) regime, the aforementioned picture is likely to change. This has to do with the chiral motion of electrons along equipotential contours inside the QD, forming one dimensional edge states [22]. In this regime electrons cannot backscatter off impurities (unless a counterpropagating edge is nearby). Moreover, the magnetic-field-acquired phase of the wave functions cannot be gauged out, rendering the tunneling matrix elements complex. Do phase lapses occur under such circumstances too?

We present here a study of a QD operating in the QH regime with filling factor $\nu = 2$, where the Hall bar supports two copropagating edge channels [22,23]. One outer channel (1*R*, cf. Fig. 1) is set to be part of the arm of a Mach-Zehnder interferometer (MZI). This facilitates the measurement of the complex transmission amplitude through the QD [24]. This is a prototypical system and a minimal model in which interedge interactions exist; as

far as we know, it has not been investigated before in the current context.

Here we find the following. (i) Phase lapses may occur also in this regime of a strong magnetic field, but that the underlying physics is utterly different from the zero field case. Importantly, these phase lapses represent a genuine



FIG. 1 (color online). Example of a stability diagram describing the charge distribution in the quantum dot (QD) in the strong coupling regime. The pairs of numbers denote the charge in the outer and inner parts. Inset: A gate-defined QD operating in the quantum Hall regime (filling factor $\nu = 2$). The QD consists of two parts. The outer channels (denoted by 1*R* and 1*L*, blue solid lines) form a ring shaped region. Tunneling between this region and the associated channels outside the QD is denoted by dotted blue lines. The inner channels (denoted by 2*R* and 2*L*, red dashed lines) define an isolated puddle (or state) tunnel coupled (not shown) to leads. The two parts of the QD interact electrostatically (wiggly lines). The transmission amplitude of electrons traveling along channel 1*R* towards the dot is measured by embedding it in one arm of a Mach-Zehnder interferometer (MZI, not shown).

many-body effect, resulting from the interaction between the inner and outer edge channels (the inner edge channel may also be represented by an orbital level or a compressible puddle). (ii) In the standard case, zero transmission and phase lapses are due to the coherent addition of two or more transmission amplitudes through the quantum dot. In contradistinction, in the strong magnetic field case phase lapses are due to true dephasing as an internal degree of freedom fluctuates inside the quantum dot. (iii) For zero magnetic field phase lapses acquire a width $\sim T^2$ at a finite temperature T [7]. By contrast, we find that for a realistic, experimentally relevant physical regime, strong magnetic field phase lapses are robust against broadening at finite temperatures.

Two gate controlled constrictions in the Hall bar form a QD (cf. Fig. 1). In the QH regime with $\nu = 2$ the electrons move inside the QD along two chiral edge modes. We focus on the transmission of the outer channel. Assuming that the magnitude of charge fluctuations on the inner mode do not exceed an electron charge, we treat it as a localized level which may be either occupied or empty. The spatial structure of the outer edge channel of the QD is important, and in what follows will be taken into account. Tunneling between the two edge channels is suppressed as these correspond to oppositely spin polarized modes. The respective couplings of the outer channel and the inner puddle to the external edge modes define two time scales, namely, the typical times for charge fluctuations in the corresponding region. It will be assumed that during the passage of one electron through the outer region, the localized level's occupation remains unchanged; i.e., each passing electron through the outer region senses the localized level as a nondynamical environment [25]. In addition, we use the inert band approximation and do not consider the renormalization of the width Γ of the localized level due to the Coulomb interaction with the outer channel.

Our aim is to calculate the transmission amplitude through the QD. We first consider the zero temperature quantum regime, and later will generalize our discussion to finite temperatures. The effect of the localized level is to provide an electron passing through the outer region of the QD with an extra phase, if this level is occupied [26]. Specifically, an electron occupying the localized level induces a change in the density of electrons at channels 1R and 1L. Employing the Thomas-Fermi approximation, this change is $\delta \rho_{R/L}(x) =$ $-eV_{R/L}(x)/2\pi\hbar v$, where $V_{R/L}(x)$ is the potential induced in channel 1R (1L) by the electron occupying the localized level, whose charge is e < 0; x and v are the spatial coordinate and velocity of electrons along the channel. When the localized level is empty, an electron at the Fermi level ε_F acquires a phase $\varepsilon_F \Delta x / \hbar v$ while traversing a distance Δx . In the presence of the potential $V_{R/L}(x)$, the chemical potential changes locally by $-eV_{R/L}(x)$. This, in turn, induces an extra phase equal to $-e \int_{0}^{\Delta x} dx V_{R/L}/\hbar v =$ $2\pi \int_0^{\Delta x} dx \delta \rho_{R/L}(x) \equiv \theta_R(\theta_L)$, where $\theta_R + \theta_L = 2\pi$. The last equality reflects the fact that the total screening charge is e. For symmetric screening between channels 1R and 1L, $\theta_R = \theta_L = \pi$. Similarly, we define the screening phase θ , which denotes the extra phase accumulated by an electron while winding once along channels 1*R* and 1*L* inside the QD. The results of our calculation can be formulated using only the screening phases θ and θ_R .

The spatial dependence of $\delta \rho_{R/L}(x)$ and the ensuing screening phase is important for the analysis of the transmission amplitude. Part or all of the screening takes place inside the QD. Then, multiple winding trajectories imply multiple accumulation of the screening phase θ . Clearly, $0 \le \theta \le 2\pi$ ($0 \le \theta_R \le 2\pi$), where $\theta/2\pi$ ($\theta_R/2\pi$) is the fraction of the electron charge screened inside the QD (along channel 1*R*). Below, we assume that screening does not take place along channels 2*R* and 2*L* (generalization beyond this assumption is straightforward).

In order to measure the transmission amplitude through the QD, the latter is embedded in one arm of a MZI ("upper"). The wave packet of an electron injected into the MZI is split into two upon arriving at its first junction. The lower partial wave, $|d\rangle$, goes directly towards the second junction and interferes with the part of the upper partial wave that is transmitted through the QD, $|u\rangle$. The current through the MZI as measured at one of its drains is proportional to the probability of an electron to arrive at that drain. Thus, the current is a function of the transmission phase through the QD.

The scattering matrix of the QD depends on the initial state of the isolated subsystem consisting of the localized level and the tunnel-coupled lead(s). Formally, this state is a Slater determinant built of the eigenstates of that subsystem. Here, we do not include the interaction between the localized state and the outer edge mode of the QD since it does not change our picture in a qualitative manner. However, it is possible to show [27] that this subsystem can be treated as a two-state system, whose wave function is $\sqrt{1-n|0\rangle} + \sqrt{n}|1\rangle$. Here, $|\sigma\rangle$ is a basis state vector corresponding to an empty ($\sigma = 0$) or occupied ($\sigma = 1$) localized level; an unimportant relative phase factor is omitted. Because of the fermionic statistics of the electrons, the probability of the localized level to be occupied, n, equals its *mean occupation*. The calculation of n is elementary [28]. The result is

$$n = \int_{-\infty}^{\infty} \frac{dE}{2\pi} \frac{1}{e^{(E-\mu)/T} + 1} \frac{2\Gamma}{(E-\epsilon_0)^2 + \Gamma^2},$$
 (1)

where μ is the chemical potential of the system, ϵ_0 the eigenenergy of the localized state, and Γ its width due to the tunnel-coupled leads. At T = 0 one has $n = \{\arctan[(\mu - \epsilon_0)/\Gamma] + \pi/2\}/\pi$.

We calculate the transmission amplitude through the QD employing scattering matrices and taking into account properly the extra phases θ and θ_R . If the localized level is occupied, the transmission amplitude through the QD for an electron traveling along the channel 1*R* is

$$t_{\rm QD}(\epsilon,\theta,\theta_R) = \frac{\gamma e^{i(\epsilon\ell_d/\ell+\theta_R)}}{1 - e^{i(\epsilon-\theta)} + \gamma}.$$
 (2)

Here, $\ell = \ell_d + \ell_u$ is the circumference of the outer channel inside the QD, consisting of the lower and upper lengths. The dimensionless parameter $\epsilon \equiv 2\pi \alpha V_g/\Delta$, shifting the outer region energy levels, is proportional to the gate voltage V_g with lever arm $\alpha > 0$. $\Delta = 2\pi \hbar v/\ell$ is the level spacing in the *bare* outer region, namely, in the absence of the inner puddle. The dimensionless parameter γ reflects the corresponding levels width. The phase θ_R accounts for the fact that part of the screening takes place on channel 1*R* outside the QD. In a generic case (besides the cases $\ell_d/\ell = 0, 1$ which are unfeasible), and in a situation where the phases θ and θ_R do not vary with energy, the transmission amplitude described by Eq. (2) does not have phase lapses.

The transmission probability through the MZI is obtained by employing a pure state density matrix. This yields

$$T = \operatorname{Tr}(\hat{\rho}\,\hat{D}) = \frac{1}{4} + \frac{1}{4}[(1-n)|t_{\text{QD}}(0)|^2 + n|t_{\text{QD}}(1)|^2] + \frac{1}{2}\Re\{e^{-i\phi}[(1-n)t_{\text{QD}}(0) + nt_{\text{QD}}(1)]\} = \frac{1}{4} + \frac{1}{4}\langle|t_{\text{QD}}|^2\rangle + \frac{1}{2}\Re[e^{-i\phi}\langle t_{\text{QD}}\rangle].$$
(3)

Here $\hat{\rho}$ is a density matrix constructed from the wave function of the whole system. It corresponds to the interfering electron being either scattered by the QD or transmitted through the lower MZI arm. The operator \hat{D} is defined by $(\langle \sigma | \otimes \langle s |) \hat{D}(|s' \rangle \otimes |\sigma' \rangle) = \delta_{\sigma\sigma'}/2$ for all combinations of s, s' = u, d. The operator \hat{D} has two functionalities, namely, it selects only the part of the wave function that arrives at the measured drain, and taking the trace over \hat{D} integrates out the environmental degrees of freedom. The phase $\phi = 2\pi \Phi / \Phi_0$, where Φ is the magnetic flux enclosed by the MZI arms and Φ_0 the magnetic flux quantum. The transmission amplitudes $t_{OD}(1)$ and $t_{OD}(0)$ are abbreviations for $t_{\text{QD}}(\epsilon, \theta, \theta_R)$ and $t_{\text{QD}}(\epsilon, 0, 0)$, respectively [cf. Eq. (2)]. It should be emphasized that our calculation is valid in the regime where the time interval between two consecutive transmitted electrons is sufficient for the inner puddle to relax to its ground state. In the third line of Eq. (3)and henceforth, angular brackets $\langle ... \rangle$ denote the average value of the quantity inside the brackets, calculated with respect to the probability distribution function

$$P(\tilde{\theta}, \tilde{\theta}_R) = \begin{cases} n & \text{for the phases to be } (\theta, \theta_R), \\ 1 - n & \text{for the phases to be } (0, 0). \end{cases}$$
(4)

The parameters θ and θ_R are defined above, and corresponding random variables are denoted by $\tilde{\theta}$ and $\tilde{\theta}_R$. Thus, the last equality in Eq. (3) shows that the presence of the

localized state turns the transmission amplitude of the QD into a random quantity, whose probability distribution function is determined by n (cf. Ref. [25]).

The two quantities of interest are the transmission phase through the QD, $\arg\langle t_{\rm QD}\rangle$, and the magnitude of the coherent oscillations of the current through the MZI, $|\langle t_{\rm QD}\rangle|$. These two quantities are measurable experimentally [1,29]. From Eq. (3) we find

$$\arg\langle t_{\rm QD}\rangle = \arg\left[t_{\rm QD}(\epsilon, 0, 0)\langle\zeta\rangle\right],$$
 (5a)

$$|\langle t_{\rm QD} \rangle| = |t_{\rm QD}(\epsilon, 0, 0)||\langle \zeta \rangle|, \tag{5b}$$

$$\zeta(\epsilon, \tilde{\theta}, \tilde{\theta}_R) = \frac{1 + \gamma - e^{i\epsilon}}{1 + \gamma - e^{i(\epsilon - \tilde{\theta})}} e^{i\tilde{\theta}_R}.$$
 (5c)

Here averages are calculated with respect to the probability distribution (4), e.g., $\langle \zeta \rangle = 1 - n + n\zeta(\epsilon, \theta, \theta_R)$. Clearly, the presence of the inner puddle induces a change in the transmission phase such that $\arg[t_{\rm QD}(\epsilon, 0, 0)] \rightarrow$ $\arg[t_{\rm QD}(\epsilon, 0, 0)\langle \zeta \rangle]$. The resulting phase is the sum of the transmission phase through the "bare" outer region and $\arg(\langle \zeta \rangle)$. Phase lapses can occur only due to the phase evolution of $\langle \zeta \rangle$, since $\arg[t_{\rm QD}(\epsilon, 0, 0)]$ by itself evolves continuously. Moreover, it is evident that any interesting physics that may be hidden in $\langle \zeta \rangle$ will generically be more pronounced if it happens to occur in between resonances of the bare outer region—there the phase of $t_{\rm QD}(\epsilon, 0, 0)$ is practically constant.

A phase lapse occurs if $\langle \zeta \rangle$ vanishes at a certain V_g . Equation (5b) shows that this abrupt jump in the phase is accompanied by complete suppression of the coherent oscillations in the MZI. Solution of the complex equation $\langle \zeta \rangle = 0$ implies fine tuning of the phases θ and θ_R such that arg $[\zeta(\epsilon, \theta, \theta_R)] = \pi$. These phases are determined by the geometry of the setup, which fixes the way screening is divided in the system. The geometry can be controlled by tuning the gates that define the QD.

This very general picture outlined above can be put to work employing parameters that reflect the sample's specific electrostatic features. These parameters determine the effect of the gate voltage on the inner and outer parts of the QD, and, hence, the evolution of the transmission phase. Specifically, we employ a charging energy model, which leads to a stability diagram of the charge distribution between the inner and outer parts of the QD, with charges N_i and N_o , respectively (see Fig. 1).

Figure 2 depicts the emergence of phase lapses in the transmission amplitude through the QD and the accompanying dephasing of the MZI as a function of ϵ . The energy level of the outer region and the occupancy of the inner region [cf. Eq. (1)] are controlled by a common gate voltage V_g with lever arms α and β through the relations $\epsilon = 2\pi \alpha V_g / \Delta$ and $\mu - \epsilon_0 = \beta V_g + c$ (c is a constant),



FIG. 2 (color online). Transmission phase through the QD (red dashed lines, top) and magnitude of coherent oscillations in the MZI (blue solid lines, bottom) in the strong coupling regime for symmetric ($\theta_R = \pi$, left) and slightly asymmetric [$\theta_R = 1.3\pi$ (center) and $\theta_R = 0.7\pi$ (right)] setups at T = 0. The left (center, right) plot depicts a sharp (smeared) phase lapse accompanied by a full (partial) suppression of the coherent oscillations through the MZI. Other parameters are $\ell_d/\ell = 1/2$, $\gamma = 1/4$, (β/α)($\Delta/2\pi\Gamma$) = 20 and $c/\Gamma = 22\pi$. The screening phase $\theta = 2\pi$ (left) or $\theta = 1.7\pi$ (center and right) implies full and almost full screening inside the QD, respectively.

respectively. Three representative cases in the strong coupling regime are shown, in which the occupation numbers of the QD follow the sequence $(N_o, N_i) \rightarrow (N_o + 1, N_i) \rightarrow (N_o, N_i + 1) \rightarrow (N_o + 1, N_i + 1)$. The occurrence of a phase lapse, either sharp or smeared, accompanied by dephasing of the MZI, is a result of screening, i.e., the expulsion of an electron from the outer part of the QD due to population of its inner part. This effectively results in an average over two transmission amplitudes, associated with $N_o + 1$ and N_o electrons in the outer part [cf. Eqs. (5)]. The (smeared) phase lapse and (almost) full dephasing occur at the point where the two amplitudes are (almost) equal in magnitude and have a phase difference of (almost) π .

In order to extract a physical picture out of this manyparameter problem, we focus on two important limits, namely, that of a strong ("S") and a weak ("W") coupling between the two parts of the QD. We examine each of these limits in view of two interesting scenarios that may occur *vis-à-vis* the change in occupancy of the two parts of the dot as a common gate voltage is varied. These scenarios are (a) $(N_o, N_i) \rightarrow (N_o, N_i + 1)$, and (b) $(N_o, N_i) \rightarrow$ $(N_o - 1, N_i + 1)$.

We begin with the strong interaction case (S), which implies $\theta \simeq 2\pi$. In S(a) the outer part is positioned in a valley between resonances, while the inner part is tuned to be near a resonance peak and eventually crosses this peak as a function of gate voltage. Under these conditions a phase lapse occurs if $\theta_R \simeq 0, 2\pi$. In S(b) there is a population switching (see e.g., [30]); i.e., both parts of the QD change their occupation by ± 1 . If $\theta_R \simeq \pi$, as appears to be achieved quite naturally in experiments [24], then a phase lapse occurs.

We turn now to the weak coupling regime, where $\theta \simeq 0$. Scenario W(a) may occur when the outer channel is not too close to a resonance, so that its occupation is not affected by a change in the occupation of the inner puddle. Then there is no discontinuity (yet possibly a sharp signature) in the transmission phase. Scenario W(b), which implies a population switching, can occur only if the outer channel is close to a resonance. Then (in a generic case) a phase lapse occurs if $\theta_R \simeq 0, 2\pi$, where $\theta_R \simeq 0$ is more likely in a weak coupling scenario.

Finite temperatures.—Our analysis so far pertains to the strictly zero temperature limit. Two modifications need to be introduced at finite temperatures. (i) The initial state of the subsystem composed of the localized level and the tunnelcoupled lead(s) must be described by a mixed density matrix (rather than a wave function). This is easily handled as the operator \hat{D} is diagonal in the localized level coordinate σ . This implies that only the corresponding diagonal elements of $\hat{\rho}$ are of importance for the calculation of the transmission probability through the MZI. Thus, at finite temperatures one should use Eq. (1). (ii) The electronic beam traveling along the arms of the MZI has a finite width in energy; i.e., the entire interference pattern is a juxtaposition of many monochromatic beams. Summing over all contributions will naturally lead to thermal smearing and reduction of the interference signal. This, however, is not our main focus here. We note that in scenario S(b) above, and when the entire screening takes place inside the QD ($\theta = 2\pi$), each such partial interference would be shifted by a phase π due to the entry or exit of an electron to the localized level. When the weights of these two patterns are equal (n = 1/2), the interference signal is fully dephased. Formally, $\langle t_{\rm QD}(\mathsf{T}) \rangle = \langle \zeta \rangle \int_{-\infty}^{\infty} dE(-\partial_E f) t_{\rm QD}(E,0,0)$, where f is the Fermi function. Thus, an abrupt phase lapse accompanied by full dephasing will take place at finite temperature as well. This phase lapse and dephasing will take place on the background of an interference contrast which decreases with temperature [31]. We note that the physics is less simple with the other scenarios outlined above. Charge fluctuations on the localized level will affect electron trajectories with different winding numbers differently. That would imply, in turn, that the efficiency of dephasing will vary with energy, leading to temperature dependent smearing of the phase lapses.

To conclude, we have studied the transmission amplitude through a QD operating in the QH regime and have found that it displays phase lapses due to interactions between different spin populations inside the dot. Specifically, phase lapses occur in the presence of quantum or thermal fluctuations, and are related to full dephasing of the electrons. We have managed to take into account the influence of both types of fluctuations in a unified way, and have identified the experimentally relevant regime of a strongly interacting and spatially symmetric setup, where phase lapses are expected to occur due to population switching in the valley between transmission resonances. These phase lapses are not thermally broadened, in contrast to the zero magnetic field case.

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