Higgs Inflation is Still Alive after the Results from BICEP2

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The observed value of the Higgs boson mass indicates that the Higgs potential becomes small and flat at the scale around 10^{17} GeV. Having this fact in mind, we reconsider the Higgs inflation scenario proposed by Bezrukov and Shaposhnikov. It turns out that the nonminimal coupling ξ of the Higgs squared to the Ricci scalar can be smaller than 10. For example, $\xi = 7$ corresponds to the tensor-to-scalar ratio $r \simeq 0.2$, which is consistent with the recent observation by BICEP2.

The observed value of the Higgs boson mass 125.9 ± 0.4 GeV [1] indicates that the standard model (SM) Higgs potential becomes small and flat at the scale around 10^{17} GeV; see, e.g., [2–9] for latest analyses [10]. See Fig. 1 for the Higgs potential around that scale for various values of the top quark mass [11]. We see that by tuning the top quark mass, we can make the first derivative at the inflection point arbitrarily small as shown by the blue (center) line. Note that the required tuning of the top quark mass is rather strict. The values of M_t are given to show the amount of tuning and should not be taken literally [17]. There are several arguments that this tuning is required by a principle such as the multiple point principle [20–22], the maximum entropy principle [23,24], the classical conformality [25–32], and the asymptotic safety [33].

It is known that this inflection point cannot be used to achieve a successful inflation [34–36]. Slow-roll condition $|\eta_V| \lesssim 1$ restricts the field value to be very close to the inflection point. To earn a sufficient e folding $N_* \simeq 60$ within this range of φ_*, the first derivative at the inflection point must be very small, and hence cannot yield the right amount of the amplitude $A_s \propto V_*/\epsilon_V$ at φ_*.

In Ref. [35], we have discussed a possibility that a new physics, such as string theory, modifies the Higgs potential above the scale $\Lambda \sim 10^{17}$ GeV. In this Letter, we pursue another possibility that the nonminimal coupling of the Higgs squared to the Ricci scalar, $\xi\varphi^2\mathcal{R}$, leads to a successful inflection point inflation.

The main differences from the ordinary Higgs inflation scenario [39–43] are the following two points [44]: (i) The e folding is earned in passing the inflection point, and hence the relation $\epsilon_V \sim 1/N_*^2$ no longer holds. Therefore, the scalar-to-tensor ratio $r = 16\epsilon_V$ can be sizable to match the recent BICEP2 result [49]:

$$r = 0.2^{+0.07}_{-0.05} \tag{1}$$

at the 68% C.L. (ii) ξ can be smaller than 10, since the Higgs quartic coupling λ is small at φ_*, due to the tuning mentioned above.

We start from the same Lagrangian as the ordinary Higgs inflation [39,41,42,50]. The potential in the Einstein frame can be obtained from the effective potential

$$V(\varphi) = \frac{\lambda(\varphi)}{4}\varphi^4 \tag{2}$$

in the flat space, by setting $\varphi = \varphi_h$ with

$$\varphi_h := \frac{h}{\sqrt{1 + \xi h^2/M_P^2}}, \tag{3}$$

where h is the Higgs field in the Jordan frame [53].

The running coupling $\lambda(\mu)$ has a minimum at $\mu_{\min} \sim 10^{17–18}$ GeV, depending on the Higgs boson mass [2–9,54]. Around the minimum, $\lambda(\mu)$ can be expanded as

$$\lambda(\mu) = \lambda_{\min} + \frac{\beta_2}{(16\pi^2)^2}\left(\ln\frac{\mu}{\mu_{\min}}\right)^2$$
$$+ \frac{\beta_3}{(16\pi^2)^3}\left(\ln\frac{\mu}{\mu_{\min}}\right)^3 + \cdots, \tag{4}$$
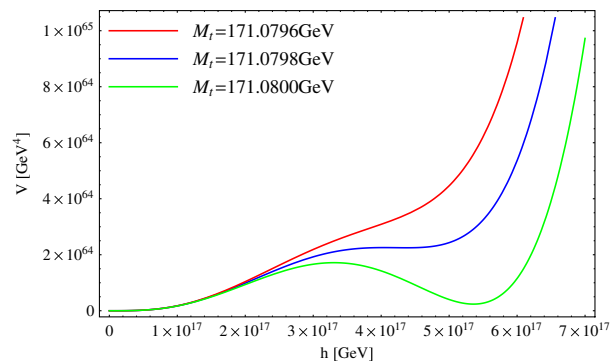

FIG. 1 (color online).   Standard model Higgs potential for the Higgs boson mass 125.6 GeV.
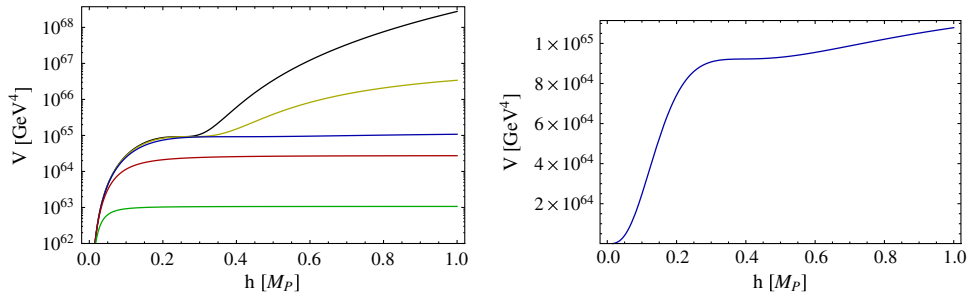
FIG. 2 (color online).   Left: Inflaton potential for $\xi = 0, 3, 10, 100$, and 1000 from above to below in a log-linear plot. Right: the same for $\xi = 10$ in a linear-linear plot.

where $\beta_2 \simeq 0.6$ in the SM [35]. The terms proportional to β_3 and higher are small in the region of our interest, and we will neglect them hereafter. The value of $\lambda_{\min}$ depends on the top quark mass, and we can set it arbitrarily small by tuning the top quark mass within the current experimental bound.

For the potential $V(\varphi)$ to be monotonically increasing around the inflection point, it is necessary and sufficient that

$$\lambda_{\min} \geq \lambda_c := \frac{\beta_2}{(64\pi^2)^2} \sim 10^{-6}. \quad (5)$$

The equality holds when the potential has a plateau. That is, when we put $\lambda_{\min} = \lambda_c$, the point $\varphi_{\text{inflection}} = e^{-1/4}\mu_{\min} \simeq 0.8\mu_{\min}$ becomes a saddle point with vanishing first and second derivatives [55].

We set the value of $\lambda_{\min}$ slightly larger than λ_c to realize an inflection point inflation, while keeping the potential above $\varphi_{\text{inflection}}$ sufficiently small by the introduction of ξ in order to evade the problem described above. The three cases $\lambda > \lambda_c$, $\lambda = \lambda_c$, and $\lambda < \lambda_c$ correspond to the red (upper), blue (middle), and green (lower) curves in Fig. 1, respectively. An important point here is that the value of φ_h in Eq. (3) is saturated to $M_P/\sqrt{\xi}$ for large values of h ($\gg M_P/\sqrt{\xi}$), and therefore the potential does not grow rapidly. In order for this saturation to work to avoid too large η_V, we need $\varphi_{\text{inflection}} \sim M_P/\sqrt{\xi}$, that is, $\xi \sim M_P^2/\mu_{\min}^2$.

As concrete examples, we show our results for several benchmark points with the parameter choice $\xi = 0, 3, 10, 100$, and 1000 with $\lambda_{\min} = 1.01\lambda_c$, $\beta_2 = 0.6$, and $\mu_{\min} = M_P/\sqrt{10}$ in the left panel in Fig. 2; the same figure is drawn in a linear plot for $\xi = 10$ in the right panel.

To fit the cosmological data, we can, e.g., take $h_* = 0.896M_P$, $\lambda_{\min} = 1.01\lambda_c$, $\mu_{\min} = 0.37M_P$, $\xi = 7$ to get $r = 16\epsilon_V(h_*) = 0.19$, $N_* = 58$, $V(\varphi_{h_*})/\epsilon_V(h_*) = 5.0 \times 10^{-7}$ and $n_s(h_*) = 0.955$, where

$$\epsilon_V = \frac{M_P^2}{2V(\varphi_h)^2}\left(\frac{dh}{d\chi}\frac{dV(\varphi_h)}{dh}\right)^2,$$
$$\eta_V = \frac{M_P^2}{V(\varphi_h)}\frac{dh}{d\chi}\frac{d}{dh}\left(\frac{dh}{d\chi}\frac{dV(\varphi_h)}{dh}\right), \quad (6)$$

with

$$\frac{d\chi}{dh} = \frac{\sqrt{1 + \xi(1 + 6\xi)h^2/M_P^2}}{1 + \xi h^2/M_P^2}. \quad (7)$$

For the same parameters, the Einstein-frame time evolution of the Higgs field h is plotted in Fig. 3. We see that substantial time is spent around the inflection point.

Once the tensor-to-scalar ratio is fixed to be $r \simeq 0.2$, the slow-roll parameter becomes $\epsilon_V(h_*) \simeq 0.013$, and the amplitude $A_s \propto V(\varphi_{h_*})/\epsilon_V(h_*)$ fixes the potential height $V(\varphi_{h_*})^{1/4} \simeq 2 \times 10^{16}$ GeV. The potential height is determined in our case to be $V(\varphi_{h_*}) \simeq \lambda(\varphi_{h_*})M_P^4/\xi^2$, which is the same as the Higgs inflation. The difference is the value of $\lambda(\varphi_{h_*}) \simeq \lambda_{\min} \simeq \lambda_c \sim 10^{-6}$ that allows us to take $\xi \lesssim 10$.

In this Letter, we have matched the renormalization scale in the Einstein frame, as in Eq. (3). If we instead match it in the Jordan frame [56], i.e., if we set $\varphi = h$ in Eq. (2), we obtain the chaotic inflation at $h \gg M_P/\sqrt{\xi}$. In this region, the canonically normalized field is $\hat{\chi} \simeq \sqrt{6}M_P \ln(hM_P/\sqrt{\xi}\mu_{\min}^2)$ in the Einstein frame. The potential for $\hat{\chi}$ becomes quadratic:

$$V \simeq \frac{\lambda_{\min}M_P^4}{4\xi^2} + \frac{1}{2}\frac{\beta_2 M_P^2}{48\xi^2(16\pi^2)^2}\hat{\chi}^2. \quad (8)$$

We see that by taking $\xi \sim 100$, we get the right amount of the inflaton mass $\sim 10^{13}$ GeV.

Finally, we comment on the unitarity issue in the Higgs inflation due to the large nonminimal coupling, which
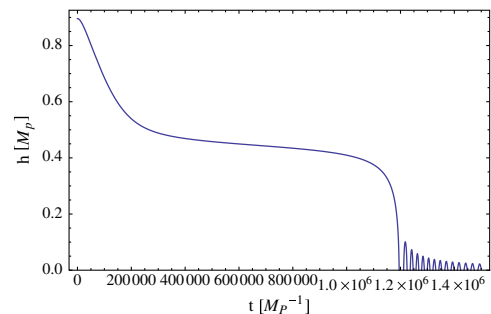

FIG. 3 (color online). h vs t in the Einstein frame in Planck units.

requires a new physics above the scale $\Lambda \sim M_P/\xi$ in order to cure the scattering being strongly coupled on the electroweak vacuum $(\varphi \ll M_P/\sqrt{\xi})$ [59–64]. It is an implicit assumption of the Higgs inflation that such an extension does not affect the result qualitatively, that is, the Wilson coefficients of the higher order terms are sufficiently smaller than $1/\xi$. Note that our ξ is greatly reduced from the value $\xi \sim 10^5$ in the ordinary scenario.

Note added.—After submission of this Letter, there appeared an article treating a similar subject [65], which is consistent with our result [66].

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[55] There appears another inflection point at $e^{-11/12}\mu_{\min} \simeq 0.4\mu_{\min}$ too.

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