# Adaptive Hybrid Optimal Quantum Control for Imprecisely Characterized Systems

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Optimal quantum control theory carries a huge promise for quantum technology. Its experimental application, however, is often hindered by imprecise knowledge of the input variables, the quantum system's parameters. We show how to overcome this by adaptive hybrid optimal control, using a protocol named Ad-HOC. This protocol combines open- and closed-loop optimal control by first performing a gradient search towards a near-optimal control pulse and then an experimental fidelity estimation with a gradient-free method. For typical settings in solid-state quantum information processing, adaptive hybrid optimal control enhances gate fidelities by an order of magnitude, making optimal control theory applicable and useful.

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The last decades have seen the transformation of quantum theory from a mere description of nature to a tool in research and applications, prominently in quantum information processing [1], spectroscopy, sensing, and metrology [2]. Quantum control describes the science of shaping the time evolution of quantum systems in a potentially useful way [3,4]. Control parameters typically are parameters of an external field parametrized in a technologically appropriate way, e.g., into a quantum logic gate [5], into a higher coherence in NMR [6–8], or into states important for sensing [9]. While analytically accessible only in highly specialized cases, these pulse shapes can in many cases be found using the powerful mathematical technique of optimal control theory (OCT); by solving a Schrödinger or master equation iteratively, a pulse shape producing the desired time evolution can be found [6]. This results in complex pulses that are used in a wide variety of cases such as controlling the cooperative effects of driving and dissipation [10], to control nonintegrable quantum many-body [11] and many electron [12] systems, generating matter-wave entanglement [13,14] and quantum information devices [15–17]. These pulses are designed based on the best available knowledge of the system. This can be insufficient for two reasons: (i) in many cases, the underlying model cannot be solved with sufficient precision as in the case of many-body systems [12]; (ii) in quantum systems that are engineered or when a human-made apparatus is a key part of the setup, parameters need to be measured with precision compatible with the control task at hand [18], which is often not possible. This necessity to precisely know the underlying model strongly limits harvesting the benefits of optimal control in complex quantum systems.

In this Letter we solve this problem with a hybrid open- or closed-loop optimal control method called adaptation by hybrid optimal control (Ad-HOC). It combines a model based gradient search and the model free Nelder-Mead (NM) algorithm [19]. Ad-HOC is designed to overcome shortcomings of the assumed physical model [20], errors on the controls and inaccurate knowledge of the parameters. We demonstrate this approach along two tasks: We first show that pulses can be optimized using only feedback from the experiment. We then show the efficiency of the hybrid method for the example of two superconducting qubits [21].

Model-free calibration was pioneered for state transfers in chemical reactions [22] using genetic algorithms and was implemented for state transfer in optical lattices in [23]. The many successes of this method as well as improvements can be found in [24]. We in turn optimize gates, i.e., transfers of a full basis of Hilbert space over a short distance in the control landscape, a task for which we found NM to be 1.5 orders of magnitude faster. The NM algorithm has been used in tuning dynamical decoupling sequences in [25] and is part of the chopped random basis (CRAB) optimal control scheme [11] without initial gradient search. The closed-loop part of Ad-HOC has been experimentally demonstrated on a CZ gate between two coupled superconducting qubits [26] and enabled the high gate fidelities in [27].

*Problem setting.*—Delicate engineering of controlled quantum systems, in particular the need to isolate quantum systems from their environments, makes quantum control setups very complex. Such an experiment, sketched in Fig. 1 is made of the system to be controlled and the unit (the AWG) producing the control pulses. The pulses are brought from the latter to the former by a chain of electronic or optical components referred to as *the "control transfer chain*." We assume that this chain and the AWG have a sufficiently large bandwidth to manipulate the system in the required way. In this setup, four different mechanisms will degrade the fidelity of an OCT designed pulse: (i) Parameter estimation: The quantum system to be controlled is modeled by a drift and control Hamiltonians $\hat{H}=\hat{H}_\mathrm{d}+\sum_i u_i(t)\hat{H}_{\mathrm{c},i}$ with $\boldsymbol{u}(t)$ the control fields to be shaped. Imprecise characterization of parameters entering the drift $\hat{H}_\mathrm{d}$ and controls

 

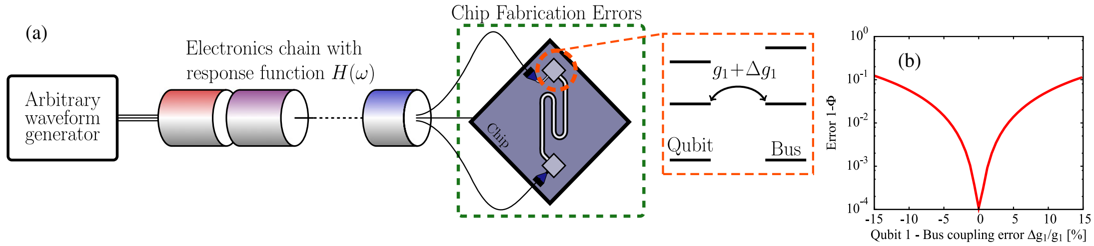


FIG. 1 (color online). (a) Sketch of quantum control experiment. The unit generating the control pulses, typically an arbitrary waveform generator (AWG), at room temperature generates the control pulses that are sent through the control transfer chain (sketched as the chain of cylinders) to finally reach the quantum system, often cooled to less than a Kelvin. Error sources are in the parameters modeling the "chip," the electronics, and the calibration of the control signals. (b) Rapid degradation of a 99.99% fidelity CZ gate between two qubits coupled via a bus assuming only an error on the $g_1$.

$\hat{H}_{c,i}$ will degrade fidelity. (ii) Improper characterization of the control transfer chain's distortion of the pulses [14,28]. (iii) Signal calibration: In practice the control unit generates an electrical signal or laser impulse which is related to $\boldsymbol{u}(t)$. Imprecisions in this relation, e.g., a constant offset, generate errors on the controls. (iv) Effects that are not taken into account in $\hat{H}$. Among many examples are other idling components of a complex quantum system such as a quantum processor, spurious two level systems (TLS) in Josephson Junctions, as well as slow non-Markovian noise. Errors in parameters and controls could be addressed in viewing the experiment as part of an ensemble and then using broadband control [29,30]. This typically leads to cumbersome pulses since high-order Lie brackets have to be generated by the compensating pulse [31]. Instead with Ad-HOC the pulses are suited to the *single* yet *uncertain* physical system at hand, thus avoiding complexity based on a simpler task.

*Proposed method.*—In order to address imperfections of the model, the control loop can be closed by using the experiment as feed-back to calibrate the control pulses. An initial gradient search [32], e.g., done with the gradient ascent pulse engineering (GRAPE) algorithm [6], of the optimal pulse, taking into account constraints on the controls as well as robustness is performed with the best reasonably achievable (to be quantified) model of the system. This gives control pulses that yield high fidelity on the model but perform sub-optimally in the real system. As long as the model is a reasonably good approximation of the physical system, these pulses will still lie close to the optimal point in the control landscape. A set of similar pulses (with model parameters drawn from the error bars of the initial characterization of the system) are sent to the experiment and their performance measured. The pulses are then updated and the procedure is iterated until either a target performance is reached or convergence halts. Measuring pulse performance is time consuming; thus, we chose the NM algorithm [19]. It is robust and typically only evaluates 1–2 pulses per iteration. Once the calibration is done, the pulses can be used. At a latter time a few pulse calibration iterations correct for drifts in parameter values and experiments can resume. The Ad-HOC protocol is illustrated in Fig. 2. Note that the precise experimental parameters are never identified. Ad-HOC hinges on an efficient method to experimentally estimate the performance index. Here, the performance index is the process fidelity which can be estimated using randomized benchmarking (RB) [33–35]. Other than standard process tomography, it is significantly faster to obtain and minimizes the impact of state preparation and measurement errors. RB yields the average fidelity

$$\bar{\mathcal{F}} = \int d\hat{U}\langle\psi|\hat{U}^\dagger\hat{U}_t^\dagger\Lambda(\hat{U}|\psi\rangle\langle\psi\hat{U}^\dagger)\hat{U}_t\hat{U}|\psi\rangle, \qquad (1)$$

estimating how well the channel $\Lambda$ implements the target Clifford gate $\hat{U}_t$. As shown in [26], RB is well adapted to fast experimentation and catches a variety of practical errors of different scales.

In summary, the gradient search approaches a favorable control over a large distance based on theory and simulation whilst the closed-loop design, done on the experiment, takes into account all experimental details [22].

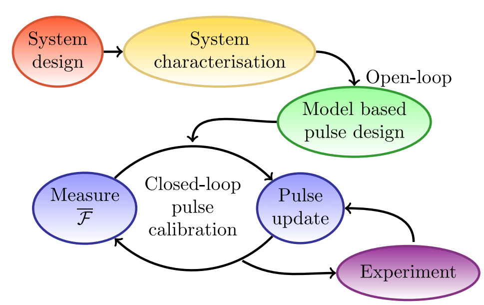


FIG. 2 (color online). Ad-HOC protocol: The physical system and surrounding control and measurement apparatus are designed taking control problems into consideration. For instance the AWG has to have a sufficient bandwidth for the desired control task. The system is then characterized with the best possible precision. Using the resulting parameters the control pulses are created. These are then fine tuned to the system using closed-loop OCT. The pulses are then ready to be used in the experiment and can be recalibrated at a later time to account for drift.

*Closed loop optimization.*—To show that a pulse can be optimized based only on its performance index we consider random gate synthesis. Inside a black box is a TLS in which the drift and control Hamiltonians are both random Hermitian matrices. The black box input is a pulse and the output its fidelity. The target is a random unitary matrix. Figure 3 shows the mean and median error as function of iteration for 100 different realizations of the TLS (see Supplemental Material for details [36]). The convergence is consistent with an exponential decrease of the error as a function of the number of steps. It is important to recognize in Fig. 3 that while demonstrating the power of the closed-loop part of Ad-HOC it also highlights that closed-loop control alone needs a large number of steps for a rather elementary control task. Going down this convergence curve with gradient search drastically reduces the number of steps to about 50 per order of magnitude error reduction.

*Numerical demonstration for a realistic setting.*—To demonstrate hybrid optimal control in a more complicated yet realistic and genuine system, we choose to create a CZ gate between two superconducting qubits in the qubit-bus-qubit system [39,40]. These systems are well described by the typical setup of Fig. 1. The qubit-bus-qubit Hamiltonian is modeled by

$$\hat{H} = \sum_i \delta_i(t)\hat{\sigma}_i^+\hat{\sigma}_i^- + \Delta_i|2\rangle_{ii}\langle 2| + \frac{g_i}{2}(\hat{\sigma}_i^+\hat{a} + \hat{\sigma}_i^-\hat{a}^\dagger).$$

The control $\delta_i(t)$ is the $i$th qubit-bus detuning. Their coupling strength is $g_i$. $\Delta_i$ is the qubit's nonlinearity. $\hat{\sigma}_i^+$ and $\hat{a}^\dagger$ respectively create an excitation in qubit $i$ and the bus. This system is particularly vulnerable to errors on the controls and parameters [18]. For instance Fig. 1(b) shows the fidelity loss due to a small error on $g_1$. 5% imprecision increases the error by two orders of magnitude. In fact, albeit the initial numerical optimization leading to a pulse that is first-order insensitive to errors, the second derivative is large, making this an example that is specifically unforgiving to model uncertainty and the ideal case for showing Ad-HOC's performance.

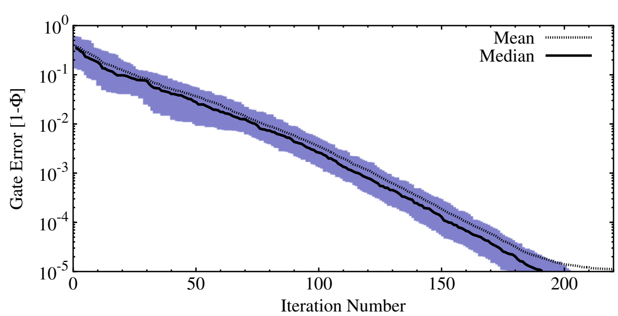


FIG. 3 (color online). Convergence during optimization of random gates. 100 pulses were optimized each for a different realization of the random TLS. The target fidelity of $1–10^{-5}$ is reached rapidly as indicated by the median. The shaded area includes 68% of all runs centered around the median.

First, a gradient search optimizes down to machine precision the error of a CZ gate using the quantum process fidelity $\Phi = |\mathrm{Tr}\{\hat{U}_{\mathrm{CZ}}^\dagger \hat{U}[\delta_1, \delta_2]\}|^2/d^2$. $\Phi$ measures the overlap between the ideal CZ gate $\hat{U}_{\mathrm{CZ}}$ and the gate implemented by the controls $\delta_i$. $d$ is the dimension of the Hilbert space. GRAPE optimizes $\Phi$ by slicing time into intervals across which the controls are constant, i.e., $\delta_i(t) \to \{\delta_{ij}\}$. It then searches in the direction of steepest $\partial\Phi/\partial\delta_{ij}$ which can be computed analytically [41]. Next, the model parameters $g_i$ and $\Delta_i$, as well as the standard deviation of the transfer chain's impulse response are promoted to random variables following Gaussian statistics with variances reflecting the precision of actual parameter estimations [42]. Additionally, random calibration offsets are introduced on the pulses. The difference between the new and old optimal controls is five times smaller than between the initial GRAPE guess and the resulting optimal control (see Supplemental Material [36]). We then compute the average gate fidelity $\bar{\mathcal{F}}$ for many different realizations of the system, see the red histograms in Fig. 4(a). As expected the fidelities are nowhere close to optimal ranging between 99% and 68%, clearly insufficient for quantum computing [43]. Finally each instance is reoptimized using the closed loop part of

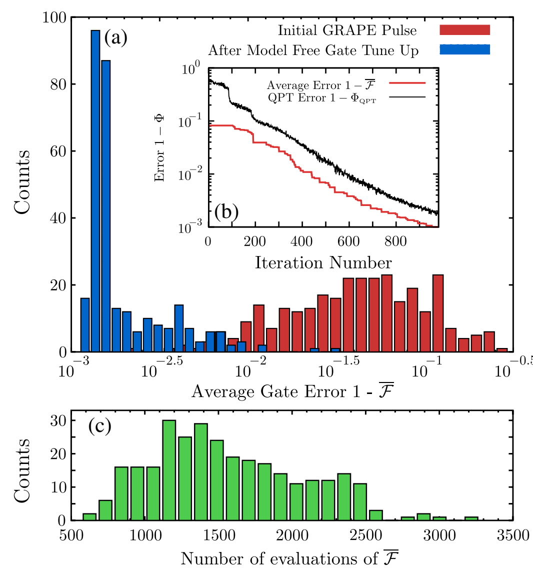


FIG. 4 (color online). Debugging procedure for parameter errors, control transfer chain errors and control dc offset errors. The error of the initial pulse was minimized using a gradient search down to machine precision. The pulses are then calibrated to a specific realization of the system. (a) Histograms for 300 system realizations. The red histograms show the fidelity of the initial uncalibrated numerical pulse. The blue histograms show the improvement in average gate fidelity after running Ad-HOC. (b) Gate errors as function of the calibration algorithms iteration number. (c) Histogram of the number evaluations of $\bar{\mathcal{F}}$ needed to calibrate the pulse, i.e., to take the red histograms to the blue ones.

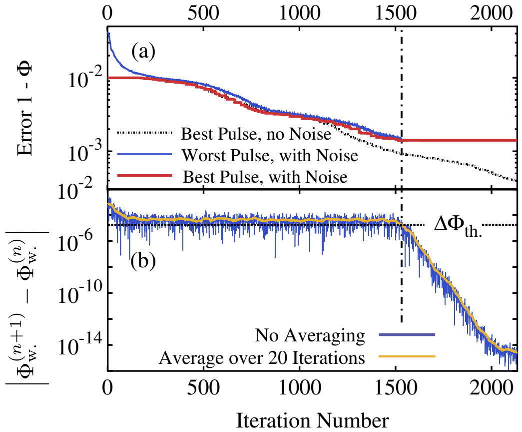


FIG. 5 (color online). (a) Convergence speed of a single optimization comparing the cases when a depolarizing channel adds noise and when the optimization is noiseless. (b) Difference in fidelity of the worst point in the simplex between subsequent iterations in a noisy optimization. As long as, on average, this difference is greater than the noise level, the optimization continues.

Ad-HOC; i.e., a pulse for that specific parameter set is found. For each realization, Ad-HOC increased the fidelity by more than an order of magnitude, as seen by the blue histograms in Fig. 4(a). Figure 4(b) shows a typical decrease in error during the closed loop optimization. As $\bar{\mathcal{F}}$ is being maximized, $\Phi$, computed for comparison, also increases. The corresponding number of required evaluations of $\bar{\mathcal{F}}$ for each realization is shown in Fig. 4(c).

*Robustness.*—Unlike pure open-loop techniques, the robustness of Ad-HOC is limited by the reliability of fidelity estimation. In the previous examples the sampling of the integral in Eq. (1) introduces noise into the fidelity estimation. Noise would also be present in an experiment but for different reasons. Here is further investigated the effect of noise on convergence. We consider the fidelity $\Phi$ which can be computed without introducing noise. A noiseless run of closed-loop optimization is compared to one with noise artificially added by a depolarizing channel [1]. Both optimizations are shown in Fig. 5, they converge at the same speed until the noisy case halts. This termination results from the increase in fidelity, averaged over several iterations, being smaller than the noise threshold $\Delta\Phi_{\text{th}}$ (see Supplemental Material [36]). The calibration protocol can no longer determine if the changes made to the pulses improve $\Phi$ and halts. This is illustrated in Fig. 5(b), showing the difference between successive iterations of fidelity of the worst pulse $\Phi_{\text{w.}}$ in the NM simplex.

In conclusion we have proposed adaptive hybrid optimal control (Ad-HOC), a protocol for overcoming model imperfection and incompleteness afflicting the design of control pulses for quantum systems. The protocol is efficient and can be applied to almost arbitrary quantum control experiments as it can be used with any fidelity measure that captures the essence of the desired time evolution. We showed that noise does not affect convergence speed but rather the terminal fidelity. Therefore higher fidelity can be gained by increasing the estimation precision. The closed-loop part of Ad-HOC has been demonstrated in [26].

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[1] M. A. Nielsen and I. L. Chuang, *Quantum Computing and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[2] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. **85**, 2733 (2000).
[3] S. Rice and M. Zhao, *Optical Control of Molecular Dynamics* (Wiley, New York, 2000).
[4] P. Brumer and M. Shapiro, *Principles of the Quantum Control of Molecular Processes* (Wiley, New York, 2003).
[5] A. Spörl, T. Schulte-Herbruggen, S. J. Glaser, V. Berg-holm, M. J. Storcz, J. Ferber, and F. K. Wilhelm, Phys. Rev. A **75**, 012302 (2007).
[6] N. Khaneja, T. Reiss, C. Kehlet, T. Schulte-Herbrüggen, and S. J. Glaser, J. Magn. Reson. **172**, 296 (2005).
[7] S. Glaser, T. Schulte-Herbrüggen, M. Sieve-king, O. Sch-edletzky, N. Nielsen, O. Sorensen, and C. Griesinger, Science **280**, 421 (1998).
[8] T. Boreneman, M. Hurliman, and D. Cory, J. Magn. Reson. **207**, 220 (2010).
[9] M. Lapert, G. Ferrini, and D. Sugny, Phys. Rev. A **85**, 023611 (2012).
[10] R. Schmidt, A. Negretti, J. Ankerhold, T. Calarco, and J. T. Stockburger, Phys. Rev. Lett. **107**, 130404 (2011).
[11] P. Doria, T. Calarco, and S. Montangero, Phys. Rev. Lett. **106**, 190501 (2011).
[12] A. Castro, J. Werschnik, and E. K. U. Gross, Phys. Rev. Lett. **109**, 153603 (2012).
[13] R. Bücker, J. Grond, S. Manz, T. Berrada, T. Betz, C. Koller, U. Hohenester, T. Schumm, A. Perrin, and J. Schmiedmayer, Nat. Phys. **7**, 608 (2011).
[14] G. Jaeger and U. Hohenester, Phys. Rev. A **88**, 035601 (2013).
[15] R. Nigmatullin and S. G. Schirmer, New J. Phys. **11**, 105032 (2009).
[16] S. Montangero, T. Calarco, and R. Fazio, Phys. Rev. Lett. **99**, 170501 (2007).
[17] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm, Phys. Rev. Lett. **103**, 110501 (2009).

[18] D. Egger and F. K. Wilhelm, Supercond. Sci. Technol. **27**, 014001 (2014).
[19] J. A. Nelder and R. Mead, Computer Journal **7**, 308 (1965).
[20] M. Hellgren, E. Räsänen, and E. K. U. Gross, Phys. Rev. A **88**, 013414 (2013).
[21] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008).
[22] R. S. Judson and H. Rabitz, Phys. Rev. Lett. **68**, 1500 (1992).
[23] S. Rosi, A. Bernard, N. Fabbri, L. Fallani, C. Fort, M. Inguscio, T. Calarco, and S. Montangero, Phys. Rev. A **88**, 021601 (2013).
[24] C. Brif, R. Chakrabarti, and H. Rabitz, New J. Phys. **12**, 075008 (2010).
[25] M. J. Biercuk, H. Uys, A. P. VanDevender, N. Shiga, W. M. Itano, and J. J. Bollinger, Nature (London) **458**, 996 (2009).
[26] J. Kelly *et al.*, Phys. Rev. Lett. **112**, 240504 (2014).
[27] R. Barends *et al.*, Nature (London) **508**, 500 (2014).
[28] F. Motzoi, J. M. Gambetta, S. T. Merkel, and F. K. Wilhelm, Phys. Rev. A **84**, 022307 (2011).
[29] P. Owrutsky and N. Khaneja, Phys. Rev. A **86**, 022315 (2012).
[30] B. Khani, S. T. Merkel, F. Motzoi, J. M. Gambetta, and F. K. Wilhelm, Phys. Rev. A **85**, 022306 (2012).
[31] J.-S. Li and N. Khaneja, Phys. Rev. A **73**, 030302(R) (2006).
[32] F. F. Floether, P. de Fouquieres, and S. G. Schirmer, New J. Phys. **14**, 073023 (2012).
[33] E. Magesan, J. M. Gambetta, B. R. Johnson, C. A. Ryan, J. M. Chow, S. T. Merkel, M. P. da Silva, G. A. Keefe, M. B. Rothwell, T. A. Ohki, M. B. Ketchen, and M. Steffen, Phys. Rev. Lett. **109**, 080505 (2012).
[34] J. M. Chow, J. M. Gambetta, L. Tornberg, J. Koch, L. S. Bishop, A. A. Houck, B. R. Johnson, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. **102**, 090502 (2009).
[35] E. Magesan, J. M. Gambetta, and J. Emerson, Phys. Rev. Lett. **106**, 180504 (2011).
[36] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.240503, which includes Refs. [37,38].
[37] T. E. Skinner, T. O. Reiss, B. Luy, N. Khaneja, and S. J. Glaser, J. Magn. Reson. **163**, 8 (2003).
[38] J. M. Gambetta, F. Motzoi, S. T. Merkel, and F. K. Wilhelm, Phys. Rev. A **83**, 012308 (2011).
[39] M. Mariantoni, H. Wang, T. Yamamoto, M. Neeley, R. C. Bialczak, Y. Chen, M. Lenander, E. Lucero, A. D. O'Connell, D. Sank, M. Weides, J. Wenner, Y. Yin, J. Zhao, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Science **334**, 61 (2011).
[40] A. Galiautdinov, A. N. Korotkov, and J. M. Martinis, Phys. Rev. A **85**, 042321 (2012).
[41] S. Machnes, U. Sander, S. J. Glaser, P. de Fouquières, A. Gruslys, S. Schirmer, and T. Schulte-Herbrüggen, Phys. Rev. A **84**, 022305 (2011).
[42] E. Lucero, R. Barends, Y. Chen, J. Kelly, M. Mariantoni, A. Megrant, P. O'Malley, D. Sank, A. Vainsencher, J. Wenner, Y. Y. T. White, A. N. Cleland, and J. M. Martinis, Nat. Phys. **8**, 719 (2012).
[43] A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, Phys. Rev. A **86**, 032324 (2012).