Counterintuitive Dispersion Violating Kramers-Kronig Relations in Gain Slabs

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We demonstrate the counterintuitive dispersion effect that the peaks (dips) in the gain spectrum correspond to abnormal (normal) dispersion, contrary to the usual Kramers-Kronig point of view. This effect may also lead to two unique features: a broadband abnormal dispersion region and an observable Hartman effect. These results are explained in terms of interference and boundary effects. Finally, two experiments are proposed for the potential experimental verification.

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The control of pulse propagations in various optical devices has received a lot of attention for many years [1–6], since it is very important for classical and quantum information processing of light. The laws of pulse propagations are completely determined by the reflected and transmitted transfer functions (TFs). These TFs, like the dielectric constants, are directly related to the dispersion relations, and they usually obey the conventional Kramers-Kronig (CKK) relations in the form of Hilbert transforms [7–9]. There have been papers dealing with the critical dispersion phenomena (related to the singular points and/or zeros of these TFs) in certain complicated circumstances, such as a Gires-Tournois interferometer [10], optical "all-pass" filters [11], and lossy or gain-assisted resonator-coupling systems [12–14]. Recently, Stern and Levy studied the flexibly controlling time delays in the integrated atomic cladding waveguide, where the amplitude and the phase of the system's TF are not connected through the CKK relations [15]. The common feature in all of these investigations involves the multiple interference effects of light.

It is well known that multiple interference effects always exist even for a simple dispersive slab, and the interaction between the slab cavity and the dispersive medium may induce the vacuum Rabi splitting effect [16]. It is widely believed that, in gain systems, a gain peak usually corresponds to normal dispersion, while the dip between two gain peaks corresponds to abnormal dispersion. It is worthwhile, therefore, to ask if the interference effect can alter the whole dispersion relation in a gain slab.

The purpose of this Letter is to address the existence of counterintuitive dispersion in gain slabs. It shows that the singular points (SPs) of the reflection and transmission coefficients for a gain slab can be controlled simply by increasing its thickness, while this is impossible for a passive slab. The movement of the SPs into the upper-half complex frequency plane leads to the counterintuitive dispersion effect that the abnormal (normal) dispersions correspond to the peaks (dips) in the gain spectrum, contrary to the usual belief. This novel effect has not been explored in slab systems before. The systems studied here differ from the previous studies as our systems include material dispersion and do not involve any artificial constants. The phenomenon seems counterintuitive from the CKK point of view; however, the causality of the present system is always preserved.

For simplicity, a light pulse $E_i(z=0,t)$ is incident normally on a nonmagnetic slab in vacuum. Here only the transverse-electric plane-wave pulses are present, and the results are similar for the transverse-magnetic plane-wave pulses. The slab is extended from $z=0$ to $z=d$ with the relative complex permittivity $\varepsilon(\omega)$. Note that $E_i(0,t)$ can be decomposed into its Fourier components $\tilde{E}_i(0,\omega)$. From the boundary conditions and Maxwell equations, the reflection and transmission coefficients of the slab, $r(\omega)$ and $t(\omega)$, respectively, are given by [17,18]

$$r(\omega) = -i(1-\varepsilon)\varepsilon^{-1/2}\sin(kd)/g(\omega), \tag{1}$$

$$t(\omega) = 2/g(\omega). \tag{2}$$

Here $g(\omega) = 2\cos(kd) - i(1+\varepsilon)\varepsilon^{-1/2}\sin(kd)$ with $k = \omega\varepsilon^{1/2}/c$ being the complex wave number inside the slab, c is the speed of light in vacuum, and d is the slab's thickness. The coefficients $r(\omega)$ and $t(\omega)$ are usually rewritten into the exponential form [10,19,20] $F(\omega) = e^{\ln F(\omega)} = e^{\ln|F(\omega)| + i\phi_F(\omega)}$, where F denotes r or t, and $|F(\omega)|$ and $\phi_F(\omega)$ are the amplitude and the phase, respectively. In slab systems, $\ln|t(0)| = 0$, but $\ln|r(0)|$ is undefined since $r(0)=0$. Thus, $|t(\omega)|$ and $\phi_t(\omega)$ usually satisfy the CKK relations [7,8]

$$\ln |t(\omega)| = -\frac{\omega^2}{\pi} P \int_{-\infty}^{\infty} \frac{\phi_t(\nu)}{\nu(\nu^2 - \omega^2)} d\nu, \tag{3}$$

$$\phi_t(\omega) = \frac{\omega}{\pi} P \int_{-\infty}^{\infty} \frac{\ln |t(\nu)|}{\nu^2 - \omega^2} d\nu. \tag{4}$$

Here P denotes the Cauchy integral principal value. That is to say, there are no SPs in the upper-half complex frequency ($\tilde{\omega} = \omega' + i\omega''$) plane. Since the slab material, $\varepsilon(\omega)$, obeys the Kramers-Kronig relations, from the comparison of Eqs. (1) and (2), we can conclude that the SPs in $r(\tilde{\omega})$ should be the same as those in $t(\tilde{\omega})$, except for a zero point at $\tilde{\omega} = 0$. However, here we show the existence of the SPs in the upper-half $\tilde{\omega}$ plane in a gain slab (see the following derivations). It should be emphasized again that although the existence of the SPs in the upper-half $\tilde{\omega}$ plane breaks the CKK relations, the causality of the gain slab system is always preserved.

For a complex ε, when there are SPs in both $r(\tilde{\omega})$ and $t(\tilde{\omega})$, it means $g(\tilde{\omega}) = 0$ in the complex domain. In order to obtain an intuitive picture, ε is assumed to be a complex constant [21]. Under this assumption, the SP, obtained from the solution of $g(\tilde{\omega}) = 0$, is located at

$$\omega' = \frac{c}{d|\varepsilon|}[n_r(m\pi - \alpha) - n_i\beta], \tag{5}$$

$$\omega'' = -\frac{c}{d|\varepsilon|}[n_i(m\pi - \alpha) + n_r\beta], \tag{6}$$

where $\alpha = \tan^{-1}[2n_i/(|\varepsilon| - 1)]$, $\beta = \tanh^{-1}[2n_r/(1 + |\varepsilon|)]$, and $n_r = \mathrm{Re}(\sqrt{\varepsilon})$ and $n_i = \mathrm{Im}(\sqrt{\varepsilon})$ are the real and imaginary parts of the complex refractive index, respectively. Here the integer m can be estimated by $m = [(1/\pi)(\mathrm{Re}(kd) + \alpha)]$, where "$[\cdot]$" is a round function. In order to reveal the condition of the SPs in the upper-half $\tilde{\omega}$ plane, three cases of the slab are discussed below.

Case 1.—For a lossless slab ($n_i = 0$), from Eq. (6), it is found that $\omega'' = -(c\beta/dn_r) < 0$, since $n_r > 0$ for non-magnetic materials. Thus the CKK relations should be always valid in this case.

Case 2.—For a lossy slab ($n_i > 0$), it can be proved that $\omega'' < 0$ is always satisfied. This means that the CKK relations are still valid for any lossy dielectric slab. Therefore, for a passive slab, there is no possibility to obtain the novel dispersion beyond the CKK relations.

Case 3.—For a gain slab ($n_i < 0$), it can be proved that $\omega'' > 0$, when the following condition is satisfied:

$$|n_i|\left[m\pi + \tan^{-1}\left(\frac{2|n_i|}{|\varepsilon| - 1}\right)\right] > n_r\beta. \tag{7}$$

Here m is controlled by the slab thickness d. In Fig. 1, we plot the different situations satisfying the condition (7) under three values of m. It is clear that as m increases, the limitation on n_i gradually disappears. Thus, for any $n_i < 0$, as long as m is large enough, the inequality (7) can be always satisfied. A large $|n_i|$ requires a small m for satisfying the condition (7). Therefore, for the gain slab with a thickness larger than a critical value, the CKK relations are inapplicable [22].

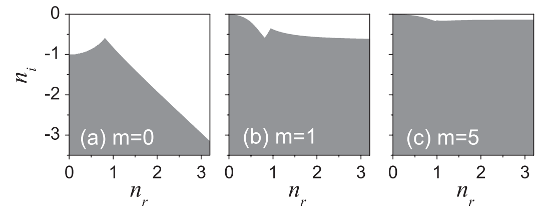

FIG. 1. The gray area obeying the condition (7) under (a) $m = 0$, (b) $m = 1$, and (c) $m = 5$.

Now let us present the picture of how the SPs move into the upper-half $\tilde{\omega}$ plane. Here the slab medium is a single Lorentz model: $\varepsilon(\omega) = 1 + M/(\omega - \omega_0 + i\gamma)$ with $M > 0$ for a gain medium and ω_0 a constant. We use the numerical method to obtain the SPs from solving $g(\tilde{\omega}) = 0$ in the complex domain. Figure 2(a) demonstrates the evolutions of the main SPs as the thickness increases. Note that those slowly moving SPs are not shown in Fig. 2(a). Initially the SPs are in the lower-half plane ($\omega'' < 0$). It is seen that the SPs will move spirally from the side of $\omega' - \omega_0 > 0$ into the other side of $\omega' - \omega_0 < 0$ with increasing d. On the $\omega'' \sim d$ projection plane, it is found that the minimal critical thickness d_c in this example is about 29.5 μm (denoted by the highlight red arrow). For $d < d_c$, no SPs are in the upper-half $\tilde{\omega}$ plane, while for $d > d_c$, more and more SPs move into the upper-half $\tilde{\omega}$ plane. Therefore, it is expected that the behavior of the whole dispersion for the slab will change dramatically.

Figures 2(b)–2(d) show the typical changes of the amplitude and phase of both $r(\omega)$ and $t(\omega)$ and the locations of the corresponding SPs under three different cases: Fig. 2(b), $d \approx 22.5$ μm ($< d_c$); Fig. 2(c), $d \approx 31.5$ μm ($> d_c$); and Fig. 2(d), $d \approx 70.5$ μm ($\gg d_c$). When $d < d_c$, in Fig. 2(b), the phase curves ($\phi_{r,t}$) (i.e., dispersion) for both $r(\omega)$ and $t(\omega)$ are normal inside the gain peaks, which are in agreement with the normal dispersion properties for an ordinary gain medium. Actually, there are infinite SPs that are splitted from $\tilde{\omega} = -i\gamma$ (when $d = 0$), see the inset in the right-side figure of Fig. 2(b). However, in Fig. 2(c), when $d > d_c$, $\phi_{r,t}$ become anomalous near ω_0, since one SP moves into the upper-half $\tilde{\omega}$ plane. The inverse effect of the dispersion seems counterintuitive from the usual accepted point of view that the normal dispersion usually corresponds to the peak in the gain spectrum. When $d \gg d_c$, see Fig. 2(d), $\phi_{r,t}$ may have the broad abnormal dispersion region, compared with the original width (γ) of the single-Lorentz dispersion. This is a unique feature that the slab becomes a completely fast-light device in the whole spectrum. Here three SPs appear in the upper-half $\tilde{\omega}$ plane, see the right side of

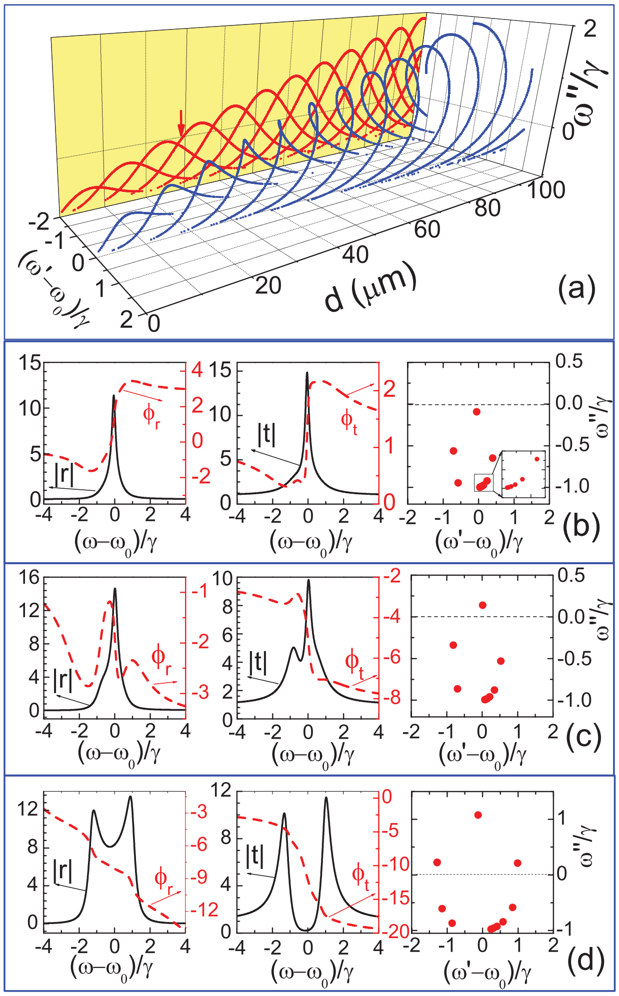

FIG. 2 (color). (a) Evolutions of the SPs with increasing d. The points on the $\omega'' \sim d$ plane are the projection of the SPs for the easy observation. The critical thickness is denoted by the highlight red arrow. Here it only demonstrates the main SPs that are moving quickly. (b)–(d) Typical changes of the amplitude and phase of $r(\omega)$ (left) and $t(\omega)$ (middle) and the locations of the SPs in the $\tilde{\omega}$ plane (right) under three cases: (b) $d \approx 22.5\ \mu m$ ($< d_c$), (c) $d \approx 31.5\ \mu m$ ($> d_c$), and (d) $d \approx 70.5\ \mu m$ ($\gg d_c$). The inset of the right-side figure in Fig. 2(b) shows a series of SPs that are nearly overlapped and are hard to distinguish. The slab medium has $\varepsilon(\omega) = 1 + M/(\omega - \omega_0 + i\gamma)$ with $\omega_0 = 10^{14}$ Hz, $M = 10^{-4}\omega_0$, and $\gamma = 2 \times 10^{-4}\omega_0$.

Fig. 2(d). The dispersion in Fig. 2(d) is also totally different from that of the Rabi splitting effect [16]. In fact, as the number of the SPs in the upper-half $\tilde{\omega}$ plane increases, the anomalous dispersion regions in $r(\omega)$ and $t(\omega)$ may become much broader. The counterintuitive dispersion in Figs. 2(c)–2(d) implies that the CKK relations for $r(\omega)$ and $t(\omega)$ are no longer valid for $d > d_c$.

When the slab consists of a gain double-Lorentz model $\varepsilon(\omega) = 1 + M/(\omega - \omega_0 + \Delta + i\gamma) + M/(\omega - \omega_0 - \Delta + i\gamma)$ with $M > 0$, ω_0 a constant, and $\Delta > 0$, if d is larger than a critical value, there are the similar counterintuitive dispersion effects that the dip between two gain peaks corresponds to the normal dispersion, while the two peaks correspond to the abnormal dispersion regions. Meanwhile, the very broad abnormal dispersion region is observable with suitable parameters.

In order to understand these counterintuitive effects, we now discuss the reflected and transmitted group delays of a narrow-spectrum pulse ($\Delta\omega \ll \omega$), defined by [23–25] $\tau_{r,t} = \partial\phi_{r,t}(\omega)/\partial\omega$. Figures 3(a) and 3(b) demonstrate respectively the changes of both τ_r and τ_t as a function of d in the slab of a gain single-Lorentz medium. The amplitude and phase for $r(\omega)$ and $t(\omega)$ are correspondingly shown in Figs. 3(c) and 3(d). Both τ_r and τ_t have transition characteristics from positive to negative at differen ω. When d is small, the initial dispersion of the slab near ω_0 is normal so that both τ_r and τ_t are positive, since it is normal dispersion for the gain slab with a single Lorentz medium. However, when d is large enough, both τ_r and τ_t can be negative due to the violation of the CKK relations for $r(\omega)$ and $t(\omega)$ as the SPs move into the upper-half $\tilde{\omega}$ plane. This is a distinct effect from the previous studies [26] where only τ_r can be reversed due to the slab's resonances. The oscillation features in τ_r and τ_t due to the slab's resonances have also been studied theoretically and experimentally [18,25]. As $d > d_c$, τ_r tends to be a constant (independent of the slab's thickness). This is also known as the Hartman effect [27,28], and the corresponding reflected phase also tends to a constant. However, there is no Hartman effect for τ_t since ϕ_t increases initially, then decreases and becomes negative as d increases. From the

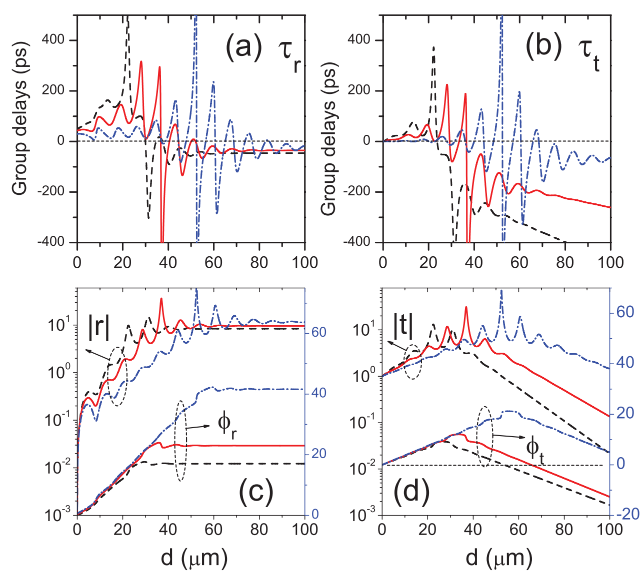

FIG. 3 (color). The changes of (a) τ_r and (b) τ_t as a function of d at different frequencies, and the corresponding changes of the amplitude and phase for (c) $r(\omega)$ and (d) $t(\omega)$. Dashed, solid, and dash-dotted curves denote the cases of $\omega = \omega_0$, $\omega_0 + 0.4\gamma$, and $\omega_0 + 0.8\gamma$, respectively. Other parameters are the same as in Fig. 2.

comparison between Figs. 3(a)–3(b) and Figs. 3(c)–3(d), the transition behaviors in τ_r and τ_t are quantitatively in agreement with the maximal phases in ϕ_r and ϕ_t, respectively. Meanwhile, $|r(\omega)|$ goes to a constant value but $|t(\omega)|$ goes to zero for a large d [see Figs. 3(c)–3(d)]. Thus, it is easy to measure τ_r but not τ_t in experiments for verifying this phenomenon.

The physical mechanism for such counterintuitive dispersion effects occurring can be explained in terms of the multiple interference and boundary effects in the gain slab. The total electric field inside the slab is expressed as [29] $\tilde{E}(z,\omega) = \tilde{E}_i(0,\omega)[Ae^{ikz} + Be^{-ikz}]$ for $0 \le z \le d$, where $A = (1+\varepsilon^{-1/2})e^{-ikd}/g(\omega)$ and $B = (1-\varepsilon^{-1/2})e^{ikd}/g(\omega)$ from the boundary conditions at $z = 0$ and d. For $d < d_c$, both the forward and backward fields are of the same order. For $d \to d_c$, both A and B reach their critical values since $g(\tilde{\omega})$ vanishes in the complex domain. For $d_c \ll d < \infty$, $A \approx (1+\varepsilon^{-1/2})e^{-i2kd}/(1-\Omega) \to 0$ with $\Omega = (\varepsilon^{1/2}+\varepsilon^{-1/2})/2$, while $B \approx (1-\varepsilon^{-1/2})/(1-\Omega)$. This means that for $d \gg d_c$, there is only the existence of the backward field ($\sim e^{-ikz}$) and the forward field ($\sim e^{ikz}$) is suppressed [30]. The backward field leads to the negative transmitted phase. Although the slab medium is normal, the total dispersion becomes anomalous due to the boundary effect. Meanwhile, for $d_c \ll d < \infty$, $r(\omega) \to (1+\varepsilon^{1/2})/(1-\varepsilon^{1/2})$ and $t(\omega) \approx -4\varepsilon^{1/2}e^{-ikd}/(1-\varepsilon^{1/2})^2$ [31]. Therefore, both $|r(\omega)|$ and ϕ_r tend to be constants, and $|t(\omega)|$ decays and ϕ_t also decreases and even becomes negative due to the term e^{-ikd}. The steady value of τ_r as a function of ω is plotted in Fig. 4 for two cases. There is a remarkable effect that τ_r is completely reversed with the results of the semi-infinite cases [32]. The steady value of τ_r for the gain single-Lorentz model slab becomes negative (anomalous dispersion). For the slab of the double-Lorentz model [33], the steady value of τ_r between two gain peaks becomes positive (normal dispersion) while it is negative (anomalous dispersion) for the gain peaks.

Can one verify these counterintuitive effects? Here two potential experiments are discussed. First, the slab is a gain single-Lorentz medium, $\varepsilon(\omega) = 1+\omega_p^2/(\omega^2-\omega_0^2+i\gamma\omega)$, with $\gamma/\omega_p = 10^{-2}$ and $\omega_p/\omega_0 = 5\times 10^{-6}$ for the $5d[7/2]_3^0 - 6p[3/2]_1$ transition (wavelength = 3.507 μm) of Xe atomic gas as suggested in Refs. [34,35]. Under these parameters, the d_c of the slab is about 20.06 mm (at $\omega = \omega_0$), which is estimated from Eq. (6) and is an accessible thickness. When $d \sim 140$ mm, τ_r reaches its steady value in the range about $(-10, -75)$ ns within $\omega \in (\omega_0 - \gamma, \omega_0 + \gamma)$, [like Fig. 4(a)]. The transmitted pulse is completely forbidden since $t(\omega) \to 0$. Second, the slab may be a gain double-Lorentz medium, which was realized by using the atomic Cs vapor slab with the Raman scheme [33]. The experimental data in Ref. [33] are $\omega_0/2\pi = 3.5\times 10^{14}$ Hz, $M/2\pi = 2.262$ Hz, $\Delta/2\pi = 1.35$ MHz, and $\gamma/2\pi = 0.46$ MHz. In this situation, d_c is about 732.18 mm (at $\omega = \omega_0 + \Delta$), which is hard to realize. Fortunately, the

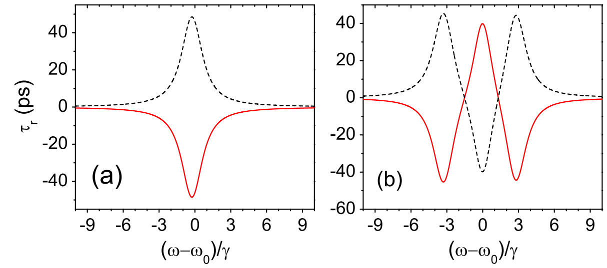

FIG. 4 (color online). The steady value of τ_r in the gain slab with (a) $\varepsilon(\omega) = 1 + M/(\omega-\omega_0+i\gamma)$, and (b) $\varepsilon(\omega) = 1 + M/(\omega-\omega_0+\Delta+i\gamma) + M/(\omega-\omega_0-\Delta+i\gamma)$, with $\Delta = 3\gamma$. The dashed curves for the semi-infinite cases (see Ref. [32]). Other parameters are the same as in Fig. 2.

value M can be controlled by increasing the effective atomic density difference of states $|F=4, m=-4\rangle$ and $|F=4, m=-2\rangle$, which are the hyperfine magnetic sublevels of the ground state $6S_{1/2}$ of Cs atomic gas [33]. When M increases 100 times, then d_c becomes about 4.837 mm, and in this case τ_r reaches its steady value in the range between ~ -320 to ~ 280 ns within $\omega \in (\omega_0 - 5\gamma, \omega_0 + 5\gamma)$ for $d \approx 40$ mm, like the solid curve of Fig. 4(b). Other candidates may also be possible, such as in the double-lambda scheme in the Rb atomic vapor [36] and in the slab consisting of a negative-refraction active dense gas of atoms [37].

In summary, the counterintuitive effects on the dispersion are demonstrated when the gain slab's thickness is larger than a critical value, and the condition for the appearance of the counterintuitive dispersion is presented. It should be noted that, for observing these novel dispersion effects, one can also increase the gain instead of changing the slab's thickness. Here the CKK relations of the TFs are violated due to the breaking of the "passivity" [9,11]. But the system's causality is still preserved, because the impulse response function has no component that appears earlier than the input impulse. In particular, optical precursors always travel at the speed of light in both normal and abnormal dispersive media [38]. Two unique features (the broadband abnormal dispersion region and the observable Hartman effect) are predicted, and these properties are significant to observe the superluminal propagations [39,40]. The physical mechanism of the counterintuitive effects is explained as a result from the domination of the backward wave in the gain slab due to interference and boundary effects. Finally, two possible experiments (using Xe and Cs atomic media) are suggested to verify the effects. These results are very important to obtain the novel dispersion, beyond the CKK relations, in gain slabs and even other structures. This may become the basis of future research.

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[21] If ε is a complex-frequency function, the solution of $g(\tilde{\omega}) = 0$ in the complex $\tilde{\omega}$ domain has to be obtained numerically. The solution presented here can give us a very good estimation on the critical thickness in gain slabs.

[22] It should be pointed out that the modified Kramers-Kronig relations may be recovered by using the Blaschke product [20] when the singular point moves into the upper-half complex frequency plane.

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[30] Here $g(\omega)$ can be rewritten as $g(\omega)=(1+\Omega)e^{-ikd}+(1-\Omega)e^{ikd}$. For $A \approx (1+\varepsilon^{-1/2})e^{-i2kd}/(1-\Omega)$ for a large d, it still decays faster than the exponential-increasing term e^{ikz} due to the gain ($n_i < 0$); therefore, the forward wave is reduced in this situation.

[31] We can rewrite $r(\omega)$ and $t(\omega)$ as $r(\omega)=(1-\varepsilon)(1-e^{i2kd})/[(1+\varepsilon^{1/2})^2-(1-\varepsilon^{1/2})^2e^{i2kd}]$ and $t(\omega)=4\varepsilon^{1/2}e^{ikd}/[(1+\varepsilon^{1/2})^2-(1-\varepsilon^{1/2})^2e^{i2kd}]$. When d increases, the term e^{i2kd} will dominate. Thus, $r(\omega)\to(1+\varepsilon^{1/2})/(1-\varepsilon^{1/2})$ and $t(\omega)\approx-4\varepsilon^{1/2}e^{-ikd}/(1-\varepsilon^{1/2})^2$ for $d_c \ll d < \infty$.

[32] For a semi-infinite medium, only the forward field exists since there is only one interface. In a semi-infinite medium, $r(\omega)=(1-\varepsilon^{1/2})/(1+\varepsilon^{1/2})$, which is completely different from the case of the gain slab for $d_c \ll d < \infty$. Meanwhile $t(\omega)\to\infty$ with increasing the propagation distance inside the semi-infinite medium.

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