Thermalization and Revivals after a Quantum Quench in Conformal Field Theory

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We consider a quantum quench in a finite system of length L described by a $1+1$-dimensional conformal field theory (CFT), of central charge c, from a state with finite energy density corresponding to an inverse temperature $\beta \ll L$. For times t such that $\ell/2 < t < (L-\ell)/2$ the reduced density matrix of a subsystem of length ℓ is exponentially close to a thermal density matrix. We compute exactly the overlap $\mathcal{F}$ of the state at time t with the initial state and show that in general it is exponentially suppressed at large L/β. However, for minimal models with $c < 1$ (more generally, rational CFTs), at times which are integer multiples of $L/2$ (for periodic boundary conditions, L for open boundary conditions) there are (in general, partial) revivals at which $\mathcal{F}$ is $O(1)$, leading to an eventual complete revival with $\mathcal{F} = 1$. There is also interesting structure at all rational values of t/L, related to properties of the CFT under modular transformations. At early times $t \ll (L\beta)^{1/2}$ there is a universal decay $\mathcal{F} \sim \exp(-(\pi c/3)Lt^2/\beta(\beta^2 + 4t^2))$. The effect of an irrelevant nonintegrable perturbation of the CFT is to progressively broaden each revival at $t = nL/2$ by an amount $O(n^{1/2})$.

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The subject of quantum quenches, the time evolution of an extended system, described by a Hamiltonian H, from a pure state $|\psi_0\rangle$ that is not an eigenstate (usually the ground state of some other Hamiltonian H_0), has been of great interest in recent years, both for theoretical reasons and the fact that such coherent evolution may be experimentally realized in ultracold atoms. Important theoretical questions are whether, and in what sense, such systems reach a stationary state and to what extent this can be described by a thermal density matrix. These are difficult to address except in theories which are in some way exactly solvable [1,2] or in the AdS/CFT correspondence, when thermalization has been associated with the formation of a black hole in the bulk [3].

In Ref. [1], the problem was studied for the case when H corresponds to a $1+1$-dimensional conformal field theory (CFT) and $|\psi_0\rangle$ is a particular kind of initial state with short-range correlations and entanglement. It was found that correlation functions of local observables within a subsystem of length ℓ become stationary after a time $\approx \ell/2$ (in units where the speed of propagation is unity), after which they are described by a thermal ensemble at a temperature corresponding to the conserved energy density. At the same time, the entanglement entropy of the subsystem with its complement becomes equal to the Gibbs entropy at the same temperature. These results may be explained within a simple physical picture of pairs of left- and right-moving quasiparticles, initially entangled over a length scale $\sim\beta$, being emitted at $t = 0$ and thereafter moving semiclassically. This general picture has been confirmed in other integrable lattice models, although in these cases the stationary state is a generalized Gibbs ensemble (GGE) rather than a purely thermal state [4].

These considerations have largely been made for the thermodynamic limit, when the total length L of the system is first taken to infinity. In fact the results of Ref. [1] can be straightforwardly extended [5] to the case of finite L as long as $\ell/2 < t < (L-\ell)/2$ (for periodic boundary conditions.) However, for a finite system, the quasiparticle picture also implies quantum recurrence. In a periodic system, an oppositely moving pair of particles will meet again at times that are integer multiples of $L/2$, and this, in the absence of accidental destructive interference, should lead to a revival of the initial state. In open systems with reflecting boundaries, such revivals should occur at multiples of L. In some integrable quantum spin chains [6] and Luttinger liquids [7], such revivals in the expectation values of local observables have indeed been observed and also in the entanglement entropy for a free Dirac fermion [8].

In this Letter, we describe the extension of the methods developed in Ref. [1] to the case of finite systems. We compute exactly the return amplitude or fidelity $\mathcal{F}(t) = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|$ of the quantum state at time t with the initial state, by relating this quantity to the partition function of the CFT on an annulus (or rectangle for open boundary conditions) continued to complex values of its modulus or aspect ratio. Since much is known about these partition functions (in some cases completely) we are able to obtain a number of analytic results. We note in passing that in recent papers [9] a similar quantity has been studied as a function of complex t for various spin chains, and its singularities interpreted as "phase transitions" at finite t. For the case of a CFT studied here, the singularities we find occur close to every rational value of t/L and are simply related to full or partial revivals of the initial state.

Formulation of the problem.—In principle, the initial state $|\psi_0\rangle$ should be the ground state of a perturbed Hamiltonian $H + \lambda \int \Phi dx$, where λ is a relevant coupling to a local operator Φ which gaps the system, leading to a finite correlation length that we assume is always $\ll L$. In practice, this is too difficult, and instead we assume [1] that $|\psi_0\rangle$ is close in the renormalization group sense to some conformal boundary state $|B\rangle$. However, since such states are scale invariant (and not even normalizable), in order to introduce a finite correlation length we take instead $|\psi_0\rangle \propto e^{-(\beta/4)H}|B\rangle$. This somewhat arbitrary choice was motivated on phenomenological grounds in Ref. [1], but a better argument is to point out that $H \propto \int T_{tt} dx$, where T_{tt} is the local stress tensor, is (often the most leading) an irrelevant operator that acts on the boundary state. β, which initially appears here only as a coupling constant, is in fact chosen so that the mean energy $\langle\psi_0|H|\psi_0\rangle = \pi cL/24(\beta/2)^2$ is the same as that in a thermal state $\pi cL/6\beta^2$ [10]. The effect of modifying the initial state by adding other irrelevant operators may be argued to lead to the stationary state being described by a GGE rather than a purely thermal one [5]. The results, although obtained in a CFT with this particular form of the initial state, should also apply more generally to quenches to the critical point in lattice systems in which predominantly only the low-lying quasiparticle modes are excited; that is, the effective temperature is much less than the bandwidth.

Return amplitude.—With the above choice for $|\psi_0\rangle$, the return amplitude is

$$\mathcal{F} = \left| \frac{\langle B|e^{-(1/4)\beta H} e^{-itH} e^{-(1/4)\beta H}|B\rangle}{\langle B|e^{-(1/4)\beta H} e^{-(1/4)\beta H}|B\rangle} \right| = \left| \frac{Z_{\mathcal{A}}(\frac{1}{2}\beta + it, L)}{Z_{\mathcal{A}}(\frac{1}{2}\beta, L)} \right|, \tag{1}$$

where $Z_{\mathcal{A}}(W, L)$ is the partition function of the CFT on an annulus of width W and circumference L, with conformal boundary conditions corresponding to B on both edges. A great deal is known about the form of $Z_{\mathcal{A}}$ for a CFT [11],

$$Z_{\mathcal{A}}(W, L) = \sum_{\Delta} |B_\Delta|^2 \chi_\Delta(q) = \sum_{\tilde{\Delta}} n_{BB}^{\tilde{\Delta}} \chi_{\tilde{\Delta}}(\tilde{q}), \tag{2}$$

where $q \equiv e^{2\pi i\tau} = e^{-4\pi W/L}$, $\tilde{q} = e^{-2\pi i/\tau} = e^{-\pi L/W}$, and Δ, $\tilde{\Delta}$ label the highest weights of Virasoro representations that propagate across and around the annulus, respectively. $\chi_\Delta(q) = q^{-c/24+\Delta} \sum_{N=0}^{\infty} d_N q^N$ are the characters of these representations, where d_N is their degeneracy at level N. The coefficients B_Δ are the overlaps between the physical states B and the Ishibashi states [12]. The non-negative integers $n_{BB}^{\tilde{\Delta}}$, which for a rational CFT are given by the fusion rules, give the number of states with highest weight $\tilde{\Delta}$ allowed to propagate around the annulus with the given boundary conditions. We assume $n_{BB}^0 = 1$. For minimal CFTs with $c < 1$, there is a finite number of allowed values of Δ and $\tilde{\Delta}$ given by the Kac formula. For more general rational CFTs, the number of different values (mod $\mathbf{Z}$) is still finite, but for a general CFT with $c > 1$ it is infinite, the mean density growing exponentially with $\sqrt{\Delta}$ [13].

The main properties of the characters that we need is that they are holomorphic in the upper half τ plane and that they transform linearly under a representation of the modular group $\mathrm{SL}(2, \mathbf{Z})$, generated by $S: \tau \to -1/\tau$ and $T: \tau \to \tau + 1$. The first property ensures that the continuation to $\tau = (-2t + i\beta)/L$ implied in Eq. (1) makes sense, and the second will allow us to relate the values of $\mathcal{F}(t)$ at different times to those back in the principal domain where $\tau \to i\infty$ and the series are rapidly convergent.

Universal short time behavior.—Note that $\tilde{q} = \exp(-2\pi L(\beta - 2it)/(\beta^2 + 4t^2))$. For $t^2 \ll L\beta$, $|\tilde{q}| \ll 1$, and so the sum on the rhs of Eq. (2) is dominated by its first term $\tilde{q}^{-c/24}$. After normalizing by the denominator in Eq. (1), this gives the first main result

$$\mathcal{F}(t) \sim \exp(-(\pi c/3)Lt^2/\beta(\beta^2 + 4t^2))(1 + O(|\tilde{q}|^\alpha)), \tag{3}$$

which shows a decay, initially faster than exponential, to a plateau value that is, however, exponentially small in L/β. The power α in the correction term is the smallest nonzero value of $\tilde{\Delta}$ such that $n_{BB}^{\tilde{\Delta}} \geq 1$ or 2. We stress that this result should hold for *any* CFT.

Revivals.—$t = nL/2$ corresponds to $\tau \approx -n$, and we may then relate the value of $Z_{\mathcal{A}}$ at this point to that near $\tau = 0$ and then as $\tau \to i\infty$ using the transformation properties of the characters. This gives, in the limit $L/\beta \to \infty$,

$$\mathcal{F}(nL/2) = \sum_{\Delta} |B_\Delta|^2 (\mathbf{T}^n \mathbf{S})_{\Delta,0} = \sum_{\tilde{\Delta}} n_{BB}^{\tilde{\Delta}} (\mathbf{S}\mathbf{T}^n\mathbf{S})_{\tilde{\Delta},0},$$

where $\mathbf{S}$ and $\mathbf{T}$ are the corresponding matrices according to which the characters transform. It follows that as long as these are finite dimensional (as for the minimal models or more generally a rational CFT), the value of $\mathcal{F}(t)$ at $t = nL/2$ is, therefore, finite (although, as we shall see below, it may accidentally vanish). At times within $(L\beta)^{1/2}$ of this there is a similar decay to that in Eq. (3) with t replaced by $|t - nL/2|$. If M is the lowest common denominator of all the $\{\Delta\}$, then, since all the energy gaps of H (of even parity) are quantized in units of $4\pi/LM$, there must always be complete revival ($\mathcal{F} = 1$) at multiples of $t = ML/2$. For the minimal models, the Kac formula implies that in $M \sim 24/(1 - c)$ and, therefore, in general the time for a complete revival diverges as $c \to 1-$. We also find (numerically) that in the same limit the return amplitude at any fixed revival time goes to zero exponentially fast. A similar result should hold for other sequences of rational CFTs with a maximal value of c.

Structure at rational values of t/L.—Although finite values of $\mathcal{F}$ occur only at integer values of $2t/L$, in fact, there is interesting universal structure near every rational value. This is because the characters are singular at $\tau = 0$, and the modular group maps this to every rational point $\tau = n/m$ on the real line. This is mapped to $\tau = 0$ by applying $ST^{n_1}ST^{n_2}\ldots$, where $(n_1, n_2, \ldots)$ are the integers appearing in the continued fraction expansion of n/m. However, the nearby point $\tau = n/m + i\beta/L$ is mapped to $\tau \approx im^2(\beta/L)$, and so we find, after normalizing with the denominator of Eq. (1),

$$\mathcal{F}(nL/2m) \propto (e^{-2\pi L/\beta})^{(c/24)(1-1/m^2)}. \qquad (4)$$

Once again, at nearby values of t, this is modified in a manner similar to Eq. (3). A more careful analysis also shows that the correction terms may be neglected only for $m \ll (L/\beta)^{1/2}$ so that for a fixed β/L the structure near only a finite number of rational values will be evident. This result, however, shows that if we define a "large deviation function" $-\lim_{L/\beta\to\infty}(\beta/2\pi L)\log\mathcal{F}(t)$, it is a sum of delta functions of strength $\propto 1/m^2$ at each rational value n/m of $2t/L$, on top of the uniform plateau value $c/24$. This structure may be understood in the quasiparticle picture as being due to the simultaneous emission at $t = 0$ of entangled pairs of particles separated by distances that are integer divisors of L. An example is illustrated in Fig. 1.

Example: Ising CFT.—Many of these features are present in the simplest minimal CFT, corresponding to the scaling limit of the Ising model with $c = \frac{1}{2}$. There are three distinct conformal boundary states, corresponding to the scaling limits of free and fixed boundary conditions on the Ising spins. In the last two cases [11], corresponding to a quench in the transverse field Ising model to the critical point from the ground state in a large longitudinal field or from the ordered phase,

$$Z_{\mathcal{A}}^{\text{fixed}} = \frac{1}{2}\chi_0(q) + \frac{1}{2}\chi_{1/2}(q) + \frac{1}{\sqrt{2}}\chi_{1/16}(q) = \chi_0(\tilde{q}).$$

At the recurrence times $t = nL/2$, we find by applying $\mathbf{T}^n$ and then $\mathbf{S}$

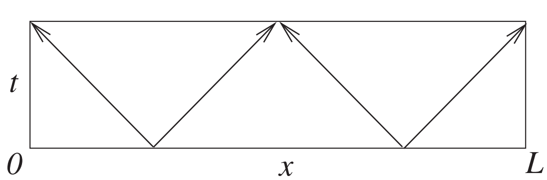

FIG. 1. Quasiparticle configuration leading to the feature in the return amplitude at $t = L/4$ for periodic boundary conditions. The pairs emitted a distance $L/2$ apart must be correlated, leading to an exponential suppression.

$$Z_{\mathcal{A}}^{\text{fixed}} = \frac{1}{2}\chi_0(q') + \frac{1}{2}e^{i\pi n}\chi_{1/2}(q') + \frac{1}{\sqrt{2}}e^{i\pi n/8}\chi_{1/16}(q)$$
$$\sim \left[\frac{1}{4}(1 + e^{i\pi n}) + \frac{1}{2}e^{i\pi n/8}\right]\chi_0(\tilde{q}'),$$

where $q' = e^{-2\pi\beta/L}$, $\tilde{q}' = e^{-2\pi L/\beta}$, and we have retained only the dominant term in the second step. For n odd, this gives $\mathcal{F}(nL/2) = \frac{1}{2}$, whereas for n even we get $|\cos(\pi n/16)|$. There is complete revival at $t = 8L$, whereas at $t = 4L$ the coefficient vanishes, leaving a much smaller term $O((e^{-2\pi L/\beta})^{1/16})$.

On the other hand, for free boundary conditions [11], corresponding to a quench from the disordered phase in zero longitudinal field,

$$Z_{\mathcal{A}}^{\text{free}} = \chi_0(q) + \chi_{1/2}(q) = \chi_0(\tilde{q}) + \chi_{1/2}(\tilde{q}).$$

At $t = nL/2$, we get $\chi_0(q') + (-1)^n\chi_{1/2}(q')$, so for n even there is complete revival, but for odd n, $\chi_0 - \chi_{1/2} = \sqrt{2}\chi_{1/16}(\tilde{q}')$, so again the revival is suppressed. The above expression may also be written as $Z_{\mathcal{A}}^{\text{free}} = q^{-1/48}\prod_{k=0}^{\infty}(1 + q^{k+1/2})$, which explicitly shows the structure near rational values of $2t/L$. This is illustrated in Figs. 2 and 3.

Open boundary conditions.—Suppose now that the system is open with conformal boundary conditions B' at $x = \pm L/2$. (We may also introduce an extrapolation length ℓ_0 in order to smooth out this condition, but this only has the effect of changing L to $L + 2\ell_0$ and we shall ignore it.) Then $Z_{\mathcal{A}}$ in Eq. (1) is replaced by $Z_{BB'}$, the partition

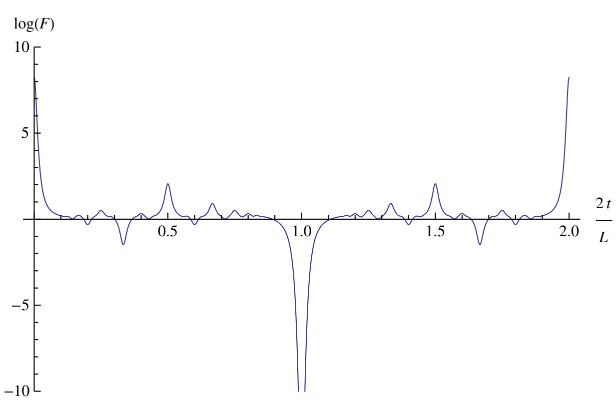

FIG. 2 (color online). Log of the return amplitude for the Ising CFT starting from a disordered state for $0 < 2t/L < 2$, with $\pi\beta/L = 0.1$. The vertical axis has been shifted so as to expose the mean plateau behavior. This shows the initial Gaussian decay and revival at $t = L$. The negative peak at $t = L/2$ is due to destructive interference between two kinds of quasiparticles. Smaller Gaussian peaks are seen at rational values with small denominators. The positive peaks are mapped by the modular group to the initial peak, and the negative ones are mapped to the feature at $2t/L = 1$.

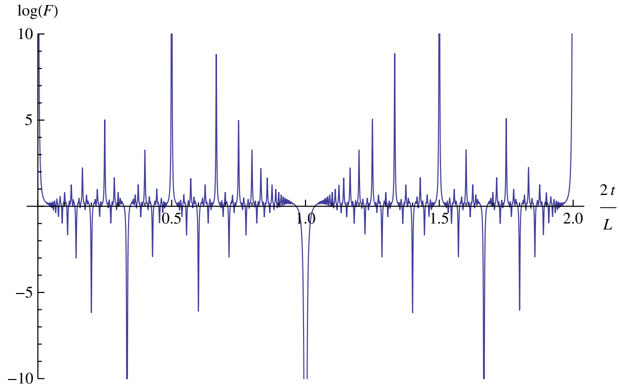

FIG. 3 (color online). Same as above with $\pi\beta/L = 0.01$. Now there is structure at more rational values, and we see the predicted $1/m^2$ dependence of the heights of nearby peaks with denominators m.

function for a $W \times L$ rectangle. In the special case when $B = B'$, the CFT partition function is known exactly [14],

$$Z_{BB}(W, L) = L^{c/4}\eta(q)^{-c/2} = W^{c/4}\eta(\tilde{q})^{-c/2},$$

where now $q = e^{-2\pi W/L}$, $\tilde{q} = e^{-2\pi L/W}$, and $\eta(q) = q^{1/24}\prod_{n=1}^{\infty}(1 - q^n)$. Setting $W = \frac{1}{2}\beta + it$ we see that $\mathcal{F}(t)$ now recurs with period L. This exact revival may be traced to the fact that, although the spatial boundary conditions may allow other states corresponding to Virasoro representations with $\Delta \neq 0$, the initial condition selects only those which are descendants of the identity. An example for the Ising CFT would be to consider a quench from the disordered phase in a system with free boundary conditions on the Ising spins at $x = \pm\frac{1}{2}L$. However, the result holds for any CFT, whether it is rational or not. The same property, of exact revival at multiples of π/L, will also occur irrespective of the initial state if B' is such that $n^{\Delta}_{B'B'} = \delta_{\Delta,0}$. Such boundary conditions are known to exist for all the minimal models, for example, fixed boundary spins in the Ising model. The modular properties of $\eta(q)$ imply that there is structure near all rational multiples of t/L, similar to the case of periodic boundary conditions.

One-point functions.—Analytic results for the one-point function of a local operator Φ in a finite annulus or rectangle are available in only a few cases [15]. The simplest is that discussed above, a $W \times L$ rectangle with the same conformal boundary condition on each edge. Since this geometry is conformally equivalent to the upper half plane, where the one-point function decays as a power Δ_Φ of the distance from the real axis, it follows that in the rectangle $\langle\Phi(\tau, x)\rangle$ is a completely universal function of the coordinates and (W, L), for any CFT. The simplest and most useful form is then found by taking Φ to be the exponential of a massless scalar field, for which the method of images may be used. The result, after continuing to real times and taking x to be at the midpoint for simplicity, is

$$\langle\Phi(t, 0)\rangle \propto \prod_{m=-\infty}^{\infty}\left[\frac{\mathrm{ch}(\frac{2\pi}{\beta}(t - (m + \frac{1}{2})L))\mathrm{ch}(\frac{2\pi}{\beta}mL)}{\mathrm{ch}(\frac{2\pi}{\beta}(t - mL))\mathrm{ch}(\frac{2\pi}{\beta}((m + \frac{1}{2})L))}\right]^{\Delta_\Phi}.$$

This shows an exponential decrease, as for the infinite system, until $t \approx \frac{1}{2}L$, followed by a symmetrical recovery to the initial value at $t = L$. Note that there is no signal of the fine structure that occurs in the overlap at rational t/L, but the quasiparticle picture suggests that this should show up in the higher-point functions.

Nonintegrable perturbations.—The simple picture of partial and exact revivals at multiples of $L/2$ (in a periodic system) in a pure CFT is clearly a consequence of the integrable structure imposed by the Virasoro algebra. In any realistic critical system, H will contain irrelevant terms that in general spoil the integrability. In general, their effect is very difficult to quantify. However, some progress is possible for an irrelevant perturbation of the form $\delta H = \lambda\int T\bar{T}dx$, which for many systems is the most important scalar irrelevant operator. In a periodic system of size L, in first-order perturbation theory it causes a shift $\sim(\lambda/L^3)(-c/24 + \Delta)^2$ in a level whose unperturbed energy is $\sim(4\pi/L)(-c/24 + \Delta)$, but to this order, the degeneracies remain. Thus, the characters $\chi_\Delta(q)$ appearing in the first expression in Eq. (2) are replaced by

$$\chi_\Delta(q) \to \sum_N d_N q^{-(c/24)+\Delta+N+(\lambda/L^2)[-(c/24)+\Delta+N]^2}.$$

Writing the quadratic term as a Gaussian integral $\propto \int d\xi q^{\xi^2 + i[(\lambda^{1/2})/2L]\xi[(-(c/24)+\Delta+N]}$, we see that we may take the expressions for $\mathcal{F}$ evaluated within the pure CFT at times $t(1 + O(\lambda^{1/2}\xi/L))$ and integrate them against $q^{\xi^2} \sim e^{-2\pi(\beta+2it)\xi^2/L}$. This will lead to an $O(n^{1/2}\lambda^{1/2})$ broadening of the revival peak at $t = nL/2$. (There is also a ξ-dependent shift in β, which makes the peaks asymmetrical.) At $O(\lambda^2)$ the degeneracies are split, leading to a new time scale $O(L^5/\lambda^2)$ beyond which we would expect to see complete decoherence.

Discussion.—$1 + 1$-dimensional CFTs in a finite system have spectral gaps that (at zero momentum, in periodic systems) are integer multiples of $4\pi/L$, which naturally leads to revivals at times which are multiples of $L/2$. However, the spectrum is purely of this form only for very special initial states (of the Ishibashi form, and in general these are unphysical.) For minimal models (more generally, rational CFTs), the spectrum still contains only a finite number of rational multiples of $4\pi/L$, leading to partial revivals and then full revival at some multiple M of $L/2$. This statement is true irrespective of the detailed form of the initial state, although the detailed results for the amplitudes of the partial revivals presented here do depend on this. For irrational CFTs, on the other hand, an infinite number of such states is needed to form the physical states.

In this case, it is unlikely that a complete revival is possible, but this leaves open the question of whether finite partial revivals may occur. The behavior of the minimal models as $c \to 1-$ suggests that this is not the case. For holographic CFTs, this is consistent with the idea that a sufficiently energetic collapsing shell of matter in AdS_3 will irreversibly form a black hole [16]. From this point of view, our example of open boundary conditions, where there is exact periodicity for any CFT, must always correspond to the subcritical case where no black hole forms. We note that in $2+1$ dimensions partial revivals have been observed in simulations [17].

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