Comment on "Phase Coexistence and a Critical Point in Ultracold Neutral Plasmas"

In Ref. [1], the authors declare a liquid-vapor-like phase transition in the ultracold neutral (UCN) plasmas by using the state equation derived from the Helmholtz free energy, which is given by [1]

$$F = \frac{q^2}{2}\sum_{i\neq j}^{N_i}\frac{\exp[-k_D|\boldsymbol{r}_i-\boldsymbol{r}_j|]}{|\boldsymbol{r}_i-\boldsymbol{r}_j|} - \frac{2\pi q^2 N_i n_i}{k_D^2} - \frac{1}{2}q^2 k_D N_i + \sum_{\alpha=i,e} N_\alpha k_B T_\alpha[\ln(n_\alpha \lambda_\alpha^3) - 1], \quad (1)$$

where q is the ionic charge, N_i is the total number of ions, k_B is the Boltzmann constant, $\lambda_i = \sqrt{h^2/2\pi m k_B T_i}$ and $\lambda_e = \sqrt{h^2/2\pi m k_B T_e}$ are the thermal wavelengths of ions and electrons, respectively, $k_D = \sqrt{4\pi q^2 n_e/k_B T_e}$ is the reciprocal of the electron Debye radius, n_i is the density of ions, and n_e is the density of electrons. The state equation suggests the relationship between the ion pressure and the reciprocal of the particle density, which resembles the relationship of van der Waals gas. Following the well-known Maxwell construction, the isotherm gives a fact of the phase coexistence of UCN plasmas. In the present Comment, we point out that the phase transition of the UCN plasmas cannot be derived from this method.

There is a basic fault in Ref. [1], being that the authors neglect the variation of the density of electrons, which causes mistakes in the following three aspects. First, it is wrong to write Eq. (1) in the form similar to the free energy of the van der Waals gas. Considering that k_D is inversely proportional to $V^{1/2}$ ($k_D = \sqrt{4\pi q^2 N_e/k_B T_e V}$, where V is the volume of the UCN plasmas and N_e is the total number of electrons) and the approximation that $|\boldsymbol{r}_i - \boldsymbol{r}_j|$ is approximated by $a = (3V/4\pi N)^{1/3}$ [1], the dependency of the free energy of the UCN plasmas on V is different from that of the free energy of van der waals gas on V. Second, it is inappropriate to give a state equation of UCN plasmas by using the definition of ion pressure [1]:

$$p_i = \frac{n_i}{V}\left(\frac{\partial F}{\partial n_i}\right)_{T_i,T_e,n_e}. \quad (2)$$

In fact, the ion pressure in Ref. [1] has no practical physical meaning in the UCN plasmas. Imagining that the UCN plasmas are put in a container, Eq. (2) indicates a pressure imposed on the container in the compression process in which the electronic density remains unchanged all of the time. Obviously, this process breaks the electric neutrality of the UCN plasmas, $n_i = n_e = n$. Third, in Fig. 1 in Ref. [1], the authors choose a constant $n_e = 5\times 10^9$ cm^{-3} but make n_i varying from 10^9 cm^{-3} to 10^{10} cm^{-3}. At any point on the isotherms where n_e and n_i are not equal, the derivation of the electric field and the free energy in the beginning of Ref. [1] is not appropriate.

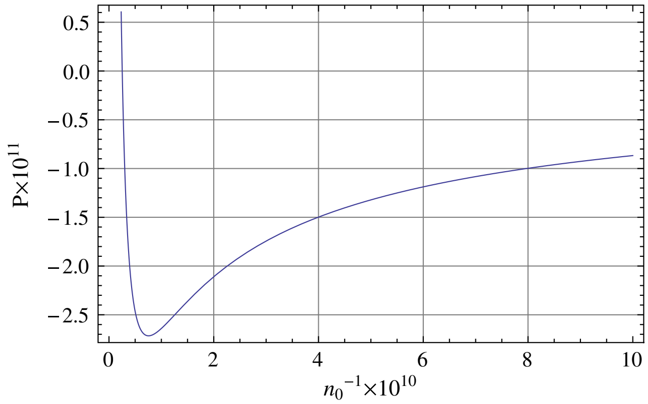

FIG. 1 (color online). The variation of P against n_0^{-1} for $T_i = 1$ K and $T_e = 40$ K. For $n_0^{-1} < 0.76\times 10^{-10}$, the reduced pressure P decreases monotonously with n_0^{-1}.

The correct state equation can be derived from a formal approach. The pressure of the UCN plasmas is given by

$$p = -\left(\frac{\partial F}{\partial V}\right)_{T_i,T_e,N}. \quad (3)$$

Choosing the dimensionless quantities $P = p(36\pi q^6/k_B^4 T_e^4)$ and $n_0 = n_i(36\pi q^6/k_B^3 T_e^3)$, considering the approximation that the double sum term in the Eq. (1) can be replaced by $(2\pi q^2 N_i/3k_D^3 a)e^{-k_D a}$ [1], we arrive at a formula

$$P = -\frac{1}{36}n_0^{5/6}(1+n_0^{1/6})e^{-n_0^{1/6}} + \frac{1}{3}n_0^{3/2} + n_0\left(1+\frac{T_i}{T_e}\right). \quad (4)$$

Equation (4) cannot give an isotherm like Fig. 1 in Ref. [1]. Figure 1 in the present Comment indicates nothing about phase coexistence by any means. In fact, it is easy to verify that the equation $\partial P/\partial(1/n_0) = 0$ has only one solution at most. For typical parameters, $T_i = 1$ K and $T_e = 40$ K [2], the state of UCN plasmas is similar to the state of a gas for most values of the density n ($n > 9.2$ cm^{-3}).

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