Dynamical Quantum Phase Transitions in Random Spin Chains

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Using a renormalization group approach, we solve the time evolution of random Ising spin chains with generic interactions starting from initial states of arbitrary energy. As a function of the Hamiltonian parameters, the system is tuned through a dynamical transition, similar to the ground-state critical point, at which the local spin correlations establish true long-range temporal order. In the state with a dominant transverse field, a spin that starts in an up state loses its orientation with time, while in the "ordered" state it never does. As in ground-state quantum phase transitions, the dynamical transition has unique signatures in the entanglement properties of the system. When the system is initialized in a product state, the entanglement entropy grows as $\log(t)$ in the two "phases," while at the critical point it grows as $\log^{\alpha}(t)$, with α a universal number. This universal entanglement growth requires generic ("integrability breaking") interactions to be added to the pure transverse field Ising model.

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Quantum systems can exhibit a great deal of universality at low temperatures due to the structure of ground states and of the critical points separating different states. On the other hand, the time evolution of the same systems, starting from generic states, involves all energies, and it is therefore thought to be much harder, if at all possible, to have sharp transitions in the dynamics. Moreover, closed systems evolving with Hamiltonian dynamics are commonly thought to settle to a thermal equilibrium consistent with the energy density in the initial state. If this is the case, then any sharp transition associated with the long time behavior of observables must correspond to a classical thermal phase transitions in the established thermal ensemble.

But this reasoning can fail in quantum systems with strong randomness. In his original paper on localization, Anderson conjectured that *closed* systems of interacting particles or spins with sufficiently strong disorder would fail to equilibrate [1]. Recently, Basko *et al.* [2] gave new arguments to revive this idea of many-body localization, which has since received further support from theory and numerics [3–7]. An important point is that localized eigenstates, even at macroscopic energies, are akin to quantum ground states in their entanglement properties [8–11]. In particular, it was pointed out that localized eigenstates can sustain different types of quantum order that would not occur in a finite temperature equilibrium ensemble [8].

In this Letter we consider the nature of phase transitions between distinct states of the dynamics. We show that by changing the parameters it is possible to drive such systems through a universal singularity in the time evolution of observables. The sharpness of the transitions and the distinct long time limit of the local order parameter in the two phases are due to many-body localization.

We demonstrate these properties in a quantum Ising spin chain with generic interactions,

$$H = \sum_i [J_i^z S_i^z S_{i+1}^z + h_i S_i^x + J_i^x S_i^x S_{i+1}^x + \ldots]. \quad (1)$$

Here J_i^z, h_i, and J_i^x are uncorrelated random variables and … represents other possible interaction terms that respect the Z_2 symmetry of the model. Without the interaction J_i^x, which we take to be small ($J_i^x \ll J_i^z$, h_i), the Hamiltonian can be mapped to a system of noninteracting fermions. For simplicity, we take the distributions of coupling constants to be symmetric around zero.

The transverse field Ising model (1) undergoes a ground-state quantum phase transition controlled by an infinite randomness fixed point [12]. The transition separates between a quantum paramagnet, when the transverse field is dominant, and a spin ordered state, when the Ising coupling J^z is dominant. Recently, it was pointed out that this transition can also occur in eigenstates with arbitrarily high energy, provided that the system is in the many-body localized phase [8]. However, it is practically impossible to prepare and measure a system in an exact high energy eigenstate. Even if we had a universal quantum computer it is not clear that it could efficiently prepare a generic eigenstate of a prescribed Hamiltonian.

Here we develop a theory of the nonequilibrium transition, focusing on the universal singular effects it has on the time evolution of the system starting from simple initial states of arbitrarily high energy and in the presence of generic interactions. Specifically, we take the initial states to be random Ising configurations of the spins in the S^z basis: $|\psi_{\text{in}}\rangle = |\uparrow\uparrow\downarrow\uparrow, \ldots \downarrow\downarrow\uparrow\rangle$. The theoretical analysis relies on the strong disorder real space renormalization group approach (SDRG) [13,14], which we recently

extended to address the quantum time evolution of random systems [7]. The properties of the transition are elucidated by tracking the time evolution of two quantities: spin correlations and entanglement entropy.

First, we show that the spin autocorrelation function $C_z(t) = \langle \psi_{\rm in}|S_i^z(t)S_i^z(0)|\psi_{\rm in}\rangle$ decays to zero in the paramagnetic phase, whereas it saturates to a positive constant in the spin (glass) ordered phase. Second, the growth of entanglement entropy between two halves of the system with time is enhanced at the critical point. Whereas in the two phases the entanglement grows asymptotically as $\log(t)$, it grows as $[\log(t)]^\alpha$ at the critical point, with $\alpha > 1$ a universal exponent. Enhanced entanglement is a hallmark of ground-state quantum criticality. Here the concept is extended to the dynamics at high energy density.

Renormalization group scheme.—The approach we use to describe the time evolution of the random chain at long times was presented in Ref. [7]. It extends the SDRG method, originally formulated to focus on long distance ground-state correlations [13].

Let us review the basic idea of the scheme. The quantum evolution at the shortest times is governed by the largest couplings in the Hamiltonian, which we define as the cutoff scale Ω. Spins affected by much smaller couplings are essentially frozen on that time scale. The dynamics of these slow degrees of freedom at longer times can be described by an effective evolution operator, from which the high frequency scale Ω has been eliminated. Technically, we derive the effective evolution operator perturbatively up to second order in the interaction between the strongly coupled cluster to its neighbors. Repeating the scheme leads to a sequence of effective Hamiltonians $H(\Omega)$ (in the interaction picture) with coupling constant distributions, which flow with Ω. The Hamiltonian $H(\Omega)$ governs the time evolution of the slow degrees of freedom on time scales $t \gtrsim 1/\Omega$. This perturbative scheme becomes better controlled at long times if the flow is toward strong randomness, i.e., wide distributions of coupling constants.

The two types of renormalization group (RG) steps we need to consider are (i) the case where the largest coupling is $J_i^z = \Omega$ and (ii) the case of a large transverse field $h_i = \Omega$. In case (i) the two spins coupled by the large Ising interaction can only flip collectively as a slow degree of freedom, and we therefore join them to make a new effective spin. The effective Hamiltonian connecting the new degree of freedom S_n^α to the rest of the system is

$$H_{\rm eff} = \left(\eta\frac{h_1 h_2}{\Omega} + J_{12}^x\right)S_n^x + \eta J_L S_L^z S_n^z + \eta J_R S_R^z S_n^z + \left(\frac{J_L^x h_1}{\eta\Omega}S_L^x + \frac{J_R^x h_2}{\eta\Omega}S_R^x + \frac{J_L^x J_R^x}{\eta\Omega}S_L^x S_R^x\right)S_n^x, \quad (2)$$

where we denoted the constituent spins of the new effective spin by 1,2 and $\eta = 1(-1)$ if spins 1 and 2 are aligned (antialigned). Because of the random sign of couplings at the outset, the signs of the generated interaction will be unimportant. In addition to the above, the transverse fields on the left and right spins $S_{L,R}$ are slightly renormalized: $h_{L,R} \to h_{L,R} + \eta J_{L,R}^x h_{1,2}/\Omega$.

In case (ii) we have a large transverse field $h_n = \Omega$ on one of the spins, which we denote by n. The effective Hamiltonian for the slow evolution of the spins in its vicinity is

$$H_{\rm eff} = \frac{2J_L^z J_R^z}{\Omega}S_L^z S_n^x S_R^z + \sum_{\alpha=R,L}(J_\alpha^x S_n^x + h_\alpha)S_\alpha^x. \quad (3)$$

Since S_n^x commutes with $H_{\rm eff}$ we can take it as a number $\pm\frac{1}{2}$, which depends on the spin projection along $\hat{x}$. The spin n is eliminated at the expense of having a different effective Hamiltonian $H_{\rm eff}^{\pm}$ operating on initial states with the spin n oriented along the positive or negative $\hat{x}$ axis. This Hamiltonian includes an Ising interaction between the left and right neighbors of n: $\tilde{J}^z = \pm(J_L^z J_R^z/\Omega)$ and a transverse field $\tilde{h}_{L,R} = h_{L,R} \pm \frac{1}{2}J_{L,R}^x$.

Note that in case (ii), at this level of approximation, no J^x term is produced between the left and right neighbors. However, such interactions are produced in the more generic Hamiltonian that emerges at intermediate scales after the RG has progressed for some time. Recall that type (i) decimations produce three-spin interaction terms of the form $S_{i-1}^x S_i^x S_{i+1}^x$. This additional term, in turn, may give rise to a four-spin interaction term. In fact, as we show in Ref. [15], all strings of n spin interactions of the form $S_1^x S_2^x \ldots S_n^x$ are produced at some stage of the RG, with coefficients that decay exponentially with the length of the string. The most important effect of the strings is to produce an effective J^x interaction $(J_n^x = J_L^x J_R^x/2\Omega)S_L^x S_R^x$ upon decimating a site at the center of a three-spin interaction.

RG flow and phase diagram.—Depending on the distribution of coupling constants in the model (1), the flow can take one of two directions, leading to distinct dynamical phases. If the disorder in the Ising interaction is dominant, an increasing number of sites will be joined by way of step (i) into an ever-growing cluster (Fig. 1), which would be ultimately frozen in an infinite system. This is the "glass phase." If, on the other hand, the transverse field is dominant, then the typical cluster size is bounded, and at long times essentially all sites will be eliminated by process (ii). This is the "paramagnetic phase."

We obtain the flow of the coupling distributions numerically by operating the RG rules on a long chain of 10^6 sites (see Ref. [15]). Crucially, we find that the added interaction

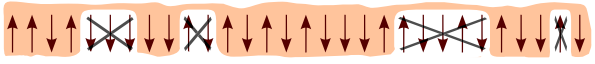
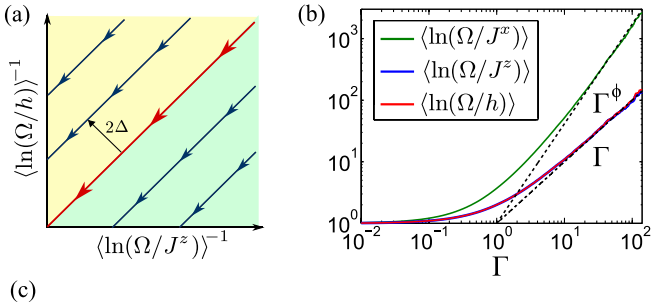


FIG. 1 (color online). Schematic depiction of the spin glass phase. An infinite cluster is formed in the course renormalization.



(c)

Glass $\Delta<0$	Critical $\Delta=0$	Para $\Delta>0$
$\overline{C_z(\infty)} \sim \lvert\Delta\rvert^{2-\phi}$	$\overline{C_z(t)} \sim [1/\ln(\Omega_0 t)]^{2-\phi}$	$\overline{C_z(t)} \sim \ln(\Omega_0 t)/(\Omega_0 t)^{2\Delta}$
$S \sim p_h \ln t$	$S \sim \frac{1}{2}(\ln t)^{2/\phi}$	$S \sim p_h \ln t$

FIG. 2 (color online). (a) RG flow near the dynamical transition. (b) The flow of the averaged scaling variables with Γ at criticality. The interaction term $\langle \ln(\Omega/J^x)\rangle$ scales asymptotically as Γ^ϕ, while the other variables $\langle \ln(\Omega/h)\rangle$ and $\langle \ln(\Omega_0/J^z)\rangle$ scale as Γ. (c) Decay of spin correlations and entanglement entropy growth in the two phases and the dynamical critical point.

terms fall off faster with the RG scale $\Gamma = \ln(\Omega_0/\Omega)$ than do the typical transverse field $h_{\rm typ}$ and Ising coupling $J^z_{\rm typ}$. This, together with the fact that Γ corresponds to the logarithmic time scale of the evolution, implies that the asymptotic scaling at long times near the critical point approaches the scaling of the random transverse field Ising model in the ground state [14]. This flow is illustrated in Fig. 2(a), where the different flow lines are parametrized by the tuning parameter

$$2\Delta = \langle \ln(\Omega/h)\rangle^{-1} - \langle \ln(\Omega/J^z)\rangle^{-1} \equiv \ln\left(h_{\rm typ}/J^z_{\rm typ}\right). \quad (4)$$

For $\Delta > 0$ and $\Delta < 0$ the system flows to the paramagnetic and glass dynamical states, respectively. $\Delta = 0$ is the critical flow line, which ends on the infinite randomness fixed point at the origin.

Figure 2(b) shows that the typical interaction $J^x_{\rm typ} \equiv \Omega \exp\left(-\langle \ln(\Omega/J^x)\rangle\right)$ falls off as $\exp(-a\Gamma^\phi)$ at criticality, with $\phi = (1+\sqrt{5})/2$ the golden ratio. This is compared to the slower decay as $\exp(-a'\Gamma)$ of the typical Ising coupling J^z and transverse field h. As a result, the interaction J^x does not feed back onto the flow of the other coupling constants. But in spite of being irrelevant, we will show that in the course of their flow the interactions do have a dramatic effect on the growth of the entanglement entropy.

Another dangerous effect of the interaction, particular to the dynamical systems, is that they can potentially destabilize the localized state by mediating resonances between modes that oscillate with similar frequencies $\Omega \pm \delta\Omega$ at remote sites on the chain. Though this process is not included in the SDRG, we argued in Ref. [7] that for sufficiently strong disorder such resonances do not proliferate and are therefore irrelevant. This is because the energy mismatch of potential resonant sites found within a range L scales as $1/L$, whereas the effective interaction between such sites is suppressed as $e^{-L/\xi}$.

Spin decay.—An important question about the dynamics is how the local spin orientation, fixed in the initial state, is disordered at long times. This is quantified by the spin autocorrelation function $C_z(t) = \langle \psi_{\rm in}|S^z_i(t)S^z_i(0)|\psi_{\rm in}\rangle = \langle S^z_i(t)\rangle\langle S^z_i(0)\rangle$ (since $|\psi_{\rm in}\rangle$ is assumed to be an eigenstate of S^z_i) or its disorder average $\overline{C_z(t)}$.

Within the SDRG scheme described above, a spin loses its orientation when the cluster it belongs to is decimated due to a large transverse field. Therefore, the average moment of a spin is directly related to the probability that it will not be decimated by the time of measurement: $\overline{C_z(t)} \sim \frac{1}{2}(N_r/N_0)$, where N_r is the number of original spins, which belong to undecimated clusters, and N_0 is the total number of spins.

Exactly the same ratio enters the calculation of the ground-state magnetization density in the standard SDRG scheme [14,16]. Hence, we can read off the results and translate them to the time evolution. In the paramagnetic Griffiths phase leading up to the critical point, we have $N_r/N_0 \sim \Gamma \exp(-2\Delta\Gamma)$ (see also Ref. [17]). In our case $\Gamma = \ln(\Omega_0 t)$ and we see that the spin autocorrelation decays as $\overline{C_z(t)} \sim \ln(\Omega_0 t)/(\Omega_0 t)^{2\Delta}$ [18] At criticality ($\Delta = 0$), the power law reverts to the logarithmic decay $\overline{C_z(t)} \sim [1/\ln(\Omega_0 t)]^{2-\phi}$. Finally, in the "glass" phase, dominated by J^z, one cluster grows to include a finite fraction of the original spins and is never decimated. Hence, in an infinite system the autocorrelation function saturates to a positive constant at long times, which serves as the order parameter of the glass state. Using the analogy with the ground-state magnetization density, we can determine the onset of the order parameter at the critical point as $\overline{C_z(\infty)} \sim |\Delta|^{2-\phi}$.

Growth of entanglement entropy.—While the initial state is a nonentangled product state, correlations between the two halves are generated in the time evolution. This leads to growth of the entanglement entropy $S = -{\rm tr}\rho_A \log_2 \rho_A$ between two halves of the chain, A and B.

The main contribution to the entanglement growth (when $J^x \neq 0$) comes from decimation of a site with a large transverse field. Entanglement is generated because the effective Hamiltonian acting on the nearby spins, $H^\pm_{\rm eff}$, depends on the orientation of the decimated spin along the x axis. Since the spin initially points along z, and therefore a superposition of projections on x, the two distinct evolutions occur in parallel, thus producing entanglement after a delay time $t_{\rm ent} \sim 1/J^x_{\rm typ}$, set by $H^+_{\rm eff} - H^-_{\rm eff}$.

Let us compute the contribution of this process to the growth of entanglement entropy. A spin decimated at time t_1 near the AB interface entangles with its neighbors by the time $t = t_1 + t_{\rm ent}$. The space between the decimated spin and its neighbors at t_1 contains many spins that had already been decimated. These spins are associated with a smaller

delay time and, by the observation time t, must also be fully entangled with each other. Hence, the entanglement entropy at time t is the number of such spins $S(t) \sim l_{\Gamma_1} p_h(\Gamma_1)$. Here l_{Γ_1} is the separation between the surviving spins at that stage of the RG in units of the original spins and $p_h(\Gamma_1)$ is the fraction of those spins that had been decimated by a large h. At long times p_h is simply a constant equal to $1/2$ at criticality (see Ref. [15] for details). The time-length scaling $l(\Gamma)$ is well known from ground-state results [14]. But we must find the relation between Γ_1 and Γ.

The fact that J^x decays rapidly in time ensures that $t_{\mathrm{ent}}(t_1) = 1/J^x_{\mathrm{typ}}(t_1) \gg t_1$. Therefore, $t \approx t_{\mathrm{ent}}(t_1)$ or, equivalently, $\Gamma \approx \Gamma_1 + \langle \ln(\Omega_1/J_x)\rangle \approx \langle \ln(\Omega_1/J_x)\rangle$. The precise relation between Γ_1 and Γ now depends on whether or not we are at the critical point.

At criticality we found [see Fig. 2(b)] that $\langle \ln(\Omega_1/J_x)\rangle \sim \Gamma_1^{\phi}$ and therefore $\Gamma_1 \sim \Gamma^{1/\phi}$. Plugging this into the expression for the entropy, we have

$$S = \frac{1}{2} l_{\Gamma_1} \sim \frac{1}{2}\Gamma_1^2 \sim \frac{1}{2}\Gamma^{2/\phi} \sim \frac{1}{2}(\ln t)^{2/\phi}. \tag{5}$$

The universality stems from the fact that the exponent depends only on the universal asymptotic flow at long times and not on the precise initial distribution and on how it is renormalized at short times. Strictly speaking, our result pertains to models in the symmetry class of (1). However, as suggested by the result of [7] concerning a model with different symmetry [$U(1)$], entanglement growth as $(\ln t)^{2/\phi}$ may be a more general characteristic of dynamical infinite randomness fixed points.

Close to the transition, the system will display the critical behavior up to the crossover time scale $t_* = \Omega_0^{-1} \exp(|\Delta|^{-1})$ [14]. To derive the entanglement growth at later times, we note that the interactions scale as $\langle \ln(\Omega_1/J^x)\rangle \sim e^{2|\Delta|\Gamma_1}$ in both phases (see Ref. [15]). On the other hand, we found $\langle \ln(\Omega_1/J^x)\rangle \approx \Gamma$ and therefore $2|\Delta|\Gamma_1 = \ln\Gamma$. The length scale grows with the same exponential rate [16] $l_{\Gamma_1} \sim e^{2|\Delta|\Gamma_1} = \Gamma$. Substituting into the expression for the entanglement entropy, we have

$$S^{\Delta\neq 0}_{J^x\neq 0} = p_h l_{\Gamma_1} \sim p_h \Gamma \sim p_h \ln t. \tag{6}$$

In both cases the interaction-induced growth of the entanglement entropy begins after a delay $t_d \approx 1/J^x_0$, where J^x_0 is the typical value of the interaction at the outset.

The logarithmic growth of entanglement in generic localized phases has been observed in numerical simulations [5,6,19]. A heuristic argument for this behavior was given in Refs. [20,21].

In a system of length L the entanglement entropy, both on and off criticality, saturates to a value linear in L: $S(t) = p_h l_{\Gamma_1} \to p_h L$. However, this extensive entropy does not imply thermalization of the system. Since p_h, the fraction of large field decimations, is less than 1, the saturation value of the entropy is smaller than the expected thermal entropy of 1 unit ($\log_2 2$) per spin (for generic initial states with energy in the middle of the many-body spectrum). In fact, p_h increases monotonically as the system is tuned from the glass to the "paramagnetic" phase.

Discussion.—The absence of thermalization can be associated with the emergence of local conserved quantities, whose value is constrained by the initial state. In our case these would be operators that involve S^z_i since the spins have a well-defined z projection at the outset.

The fact that $\langle S^z_i\rangle$ does not decay to zero in the glass phase implies that this operator is closely related to a true conserved quantity. Specifically, on sites i that will eventually join the infinite cluster, S^z_i has overlap of order 1 with a conserved quantity $\tilde{S}^z_i$. The form of these operators, consistent with scaling near the critical point, is $\tilde{S}^z_i = AS^z_i + B\exp(-C\Delta^{\phi} n)\hat{O}_{i,n}$, where $O_{i,n}$ are strings of n spins and $A, B, C \sim 1$. The decay of nonlocal terms stems from the finite correlation length $\xi \sim 1/\Delta^{\phi}$ in the glass phase. Near the critical point these operators have low density on the chain proportional to $\Delta^{2-\phi}$. Spins on other sites have a finite but exponentially suppressed overlap with true conserved quantities.

At the critical point and in the paramagnetic state, S^z_i is no longer a quasiconserved operator. However, there are other local conserved quantities. For example, the products $S^z_1 S^z_2$ of spins belonging to the same decimated cluster are quasiconserved in the same sense as above. Therefore, the entropy does not reach the maximal thermal value in either phase. However, deep in the paramagnetic phase almost all decimations are of a single spin, and there are essentially no conserved quantities that contain S^z_i. Then, for initial states with well-defined S^z on sites, the entropy is not constrained.

Conclusions.—Using a real space RG method formulated in real time, we developed a theory of a dynamical quantum phase transition between distinct many-body localized phases of a quantum spin chain. The two phases, a simple paramagnet and a spin glass, are separated by an infinite randomness fixed point. The spin glass is characterized by long range temporal order in the spin; that is, $\langle S^z_i(t) S^z_i(0)\rangle$ saturates to a finite value. The saturation value onsets as $\Delta^{2-\phi}$ and serves as an order parameter of the dynamical phase. We note that a paper, which was submitted in parallel to this one, explores a similar dynamical transition from the complementary perspective of dynamical response in a system prepared at equilibrium [22].

Unlike the spin correlations, the growth of entanglement entropy following the quench is dramatically affected by interactions. While without interactions the entropy saturates in the two phases, the interactions lead to (delayed) logarithmic growth of the entanglement entropy, which is enhanced to $\ln^{2/\phi} t$ at criticality. In a finite system the entropy grows to an extensive value but with smaller

entropy density than it would reach in thermal equilibrium. An infinite set of emergent conserved quantities whose value is constrained by the initial state prevents the system from thermalizing.

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[1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[2] D. Basko, I. Aleiner, and B. Altshuler, Ann. Phys. (Amsterdam) **321**, 1126 (2006).
[3] V. Oganesyan and D. A. Huse, Phys. Rev. B **75**, 155111 (2007).
[4] A. Pal and D. A. Huse, Phys. Rev. B **82**, 174411 (2010).
[5] M. Žnidarič, T. Prosen, and P. Prelovšek, Phys. Rev. B **77**, 064426 (2008).
[6] J. H. Bardarson, F. Pollmann, and J. E. Moore, Phys. Rev. Lett. **109**, 017202 (2012).
[7] R. Vosk and E. Altman, Phys. Rev. Lett. **110**, 067204 (2013).
[8] D. A. Huse, R. Nandkishore, V. Oganesyan, A. Pal, and S. L. Sondhi, Phys. Rev. B **88**, 014206 (2013).
[9] M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **111**, 127201 (2013).
[10] B. Bauer and C. Nayak, J. Stat. Mech. (2013) P09005.
[11] Y. Bahri, R. Vosk, E. Altman, and A. Vishwanath, arXiv:1307.4092.
[12] D. S. Fisher, Phys. Rev. B **50**, 3799 (1994).
[13] C. Dasgupta and S.-k. Ma, Phys. Rev. B **22**, 1305 (1980).
[14] D. S. Fisher, Phys. Rev. Lett. **69**, 534 (1992).
[15] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.217204 for more details of the calculations.
[16] D. S. Fisher, Phys. Rev. B **51**, 6411 (1995).
[17] F. Igloi and C. Monthus, Phys. Rep. **412**, 277 (2005).
[18] Far from the critical point, a model with bounded disorder also admits a normal paramagnetic phase where the spin autocorrelations would decay exponentially in time.
[19] G. D. Chiara, S. Montangero, P. Calabrese, and R. Fazio, J. Stat. Mech. (2006) P03001.
[20] M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **110**, 260601 (2013).
[21] D. A. Huse and V. Oganesyan, arXiv:1305.4915.
[22] D. Pekker, G. Refael, E. Altman, E. Demler, and V. Oganesyan, Phys. Rev. X **4**, 011052 (2014).