

## *I*-*Q* Relation for Rapidly Rotating Neutron Stars

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We consider a universal relation between moment of inertia and quadrupole moment of arbitrarily fast rotating neutron stars. Recent studies suggest that this relation breaks down for fast rotation. We find that it is still universal among various suggested equations of state for constant values of certain dimensionless parameters characterizing the magnitude of rotation. One of these parameters includes the neutron star radius, leading to a new universal relation expressing the radius through the mass, frequency, and spin parameter. This can become a powerful tool for radius measurements.

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*Introduction.*—General relativity is the cornerstone for our understanding of gravity, one of the fundamental forces of nature. Neutron stars (NSs) are among the best laboratories for testing strong gravity and probing other fundamental interactions (e.g., strong interactions). To date, pulsar observations deliver some of the best tests of general relativity and alternative theories of gravity. In the near future, currently planned or approved observatories such as SKA, ATHENA, LOFT, and NICER will improve these tests by orders of magnitude. Additionally, inspiraling NS binaries are the most likely sources for gravitational wave (GW) detectors such as the Advanced LIGO, which will start operation very soon. To exploit all these ambitious astronomical projects to full extent, a careful and precise modeling is required.

However, a catch of using NSs for gravity tests (see, e.g., Refs. [1,2] for reviews on gravity tests) is our current ignorance about many aspects of their structure. While there is increasing agreement on their very outer layers among various groups, diverse theoretical models for their inner structure are proposed. This is due to the quantum chromodynamical interactions of the matter in regimes not currently accessible by earth-based experiments. For this reason, theoretical predictions are difficult. The issue above raises the question of whether a NS can be used for precision tests of gravity theory at all? For instance, alternative theories of gravity can have experimental signatures similar to finite size effects (internal structure). This was explicitly shown in, e.g., Ref. [3], where the same model star appeared with a different equation of state (EOS) due to the modification of gravity. (This particular modification seems to be undetectable by the discussed universal relations, too [4].) The parameter estimation (spins) through gravitational wave observations would likely be spoiled considerably [5] for similar reasons.

A very important observation, which partly breaks this degeneracy, was recently made in Refs. [5,6]. Relations among various measurable quantities depending on the inner structure of NSs were found to be universal among many proposed NS models. This includes dimensionless quantities related to the moment of inertia, spin-induced quadrupole [7], and tidal-induced quadrupole [8–10] of the NS. A limitation of the work in Refs. [5,6] is the use of the slow rotation approximation, in which these quantities do not depend on the magnitude of rotation, whereas this does not hold true for rapid rotation. However, the slow rotation approximation should be sufficiently accurate for many near-future measurements.

One purpose of the present work is to study an extension of the relation between the moment of inertia ( $\hat{I}$ ) and the spin-induced quadrupole ( $\hat{Q}$ ) beyond the slow rotation approximation. A first study was done in Ref. [11], where the  $\hat{I}$ - $\hat{Q}$  relation was considered as a function of the observationally important (but dimensionful) frequency of rotation. Reference [11] explores at which frequency the  $\hat{I}$ - $\hat{Q}$  relation of Refs. [5,6] is modified. The universality among different NS models seems to be lost for rapid rotation. Contrary to this expectation, we find that when the rotation is characterized by a dimensionless parameter, the universality still holds remarkably well. We consider three different parameters, one based on the angular momentum and two based on the rotation frequency, where one is made dimensionless by the mass and the other by the radius. As a consequence, the universal relation containing the latter can further be used to infer the radius of the NS, making it an effective tool in analyzing astronomical data. We make this explicit by formulating a universal fit of the radius in terms of the mass, pulsar frequency, and spin. On a more theoretical level, we show that even for certain polytropic EOSs the

universality is still present for rapid rotation. This calls for a fundamental explanation using analytic arguments.

The universal relations are important since they connect several crucial astrophysical parameters. For example, the frequency and the mass are observable in binary pulsar systems using pulsar timing, and the spin (or moment of inertia) might become measurable in the near future; see, e.g., Ref. [12]. The latter two also leave an imprint in the emitted gravitational waves and can be inferred from future detection; see, e.g., Ref. [13]. The radius of a NS is measured using photospheric radius expansion and transiently accreting NSs in quiescence; see Ref. [14] for a review. Both methods yield accuracies of about 10% employing different models for the data analysis; see Ref. [15].

Besides the work in Ref. [11], the (in)validity of the universal relations was investigated in other regimes, too. Nonlinear and dynamical aspects of the tidal-induced quadrupole were included in Ref. [16], where it was found that the universal relations still hold true. In Ref. [17], the effect of magnetic fields was included and it was shown that the universal relations are broken for NSs with a large rotation period ( $\gtrsim 10$  s) and strong magnetic fields ( $\gtrsim 10^{12}$  G) in a twisted-torus field configuration. As an example, Ref. [17] speculates that breaking of the universality might just start to play a role for the slower rotating NS in the double pulsar at the time of merger. However, one should also admit that a rotation period of  $\gtrsim 10$  s implies that the dimensionful quadrupole will be very small and likely irrelevant for most observations. Various other universal properties of NSs were discussed before [18–20] and after [21–23] the discovery of Yagi and Yunes [5,6].

*Rotating NS.*—The spacetime of a rotating NS can be written in the following form (in units where  $G = c = 1$ ):

$$ds^2 = -e^{2\nu} dt^2 + r^2 \sin^2 \theta B^2 e^{-2\nu} (d\phi - \omega dt)^2 + e^{2(\xi-\nu)} (dr^2 + r^2 d\theta^2), \quad (1)$$

where  $\nu$ ,  $B$ ,  $\omega$ , and  $\xi$  depend only on  $r$  and  $\theta$ .

The matter field describing the interior of the NS is modeled by a perfect fluid of the form

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}, \quad (2)$$

where  $\rho$  is the energy density,  $P$  is the pressure, and  $u^\mu$  is the 4-velocity. The model is specified once a particular EOS is given as described in the next section.

To solve the field equations for the rotating NS, we use the RNS code [24,25] (see also Ref. [26] for details on the method and equations) including our own modification implementing the multipole extraction described in Ref. [27]. Note that the RNS code assumes rigid rotation; see Refs. [24,25].

The metric functions, which allow us to define the quadrupole, have the following asymptotic decay [28]

$$\begin{aligned} \nu &= -\frac{M}{r} + \left( \frac{B_0 M}{3} + \nu_2 P_2 \right) \frac{1}{r^3} + \mathcal{O}(r)^{-4}, \\ B &= 1 + \frac{B_0}{r^2} + \mathcal{O}(r)^{-4}, \quad \omega = \Omega \left( 1 + \frac{2I}{r^3} \right) + \mathcal{O}(r)^{-4}, \end{aligned} \quad (3)$$

where  $M$  is the mass of the star,  $P_2$  is a Legendre polynomial, and  $\nu_2, B_0, I$  are real constants,  $I = J/\Omega$  is the moment of inertia,  $J$  being the angular momentum, and  $\Omega \equiv 2\pi f$  is the angular frequency measured by a distant observer (pulsar frequency). The quadrupole moment  $Q$  [27] is then given by

$$Q = -\nu_2 - \frac{4}{3} \left( \frac{B_0}{M^2} + \frac{1}{4} \right). \quad (4)$$

We plan a multipole extraction using source integrals as envisaged in Ref. [29] in the future.

In order to investigate universal relations, we introduce dimensionless quantities as

$$\begin{aligned} a &= \frac{J}{M^2}, & \hat{I} &= \frac{I}{M^3}, & \hat{Q} &= -\frac{Q}{M^3 a^2}, \\ \hat{R} &= \frac{2R}{M}, & \hat{f} &= 200Mf, & \tilde{f} &= 20Rf, \end{aligned} \quad (5)$$

where  $R$  is the equatorial radius of the NS. The dimensionless frequencies are such that  $f = 1$  kHz corresponds to  $\hat{f} \approx 1 \approx \tilde{f}$  for  $M = M_\odot$  or  $R = 15$  km, respectively. Similarly, it holds  $\hat{R} \approx R/\text{km}$  for  $M = 1.4M_\odot$ .

*EOS.*—The interior structure of a NS is modeled by an EOS, giving relations between thermodynamical quantities, such as the energy density  $\rho$  and the pressure  $P$ . Our lack of knowledge of high-density nuclear matter is manifested through a series of candidate EOSs. The dependence of the interior structure of the NS on the respective EOS in turn affects the exterior properties. For example, different EOSs predict different relations between the mass and the radius, and one might expect the same for the moment of inertia and the quadrupole. Note that recently the radius was measured sufficiently accurate with photospheric radius expansion and quiescent low-mass x-ray binaries to yield important constraints on the EOS in a wide range of NS masses [15].

Our selection of realistic (tabulated) EOSs is APR [30], AU (called AV14 + UVII in Ref. [31]), FPS [32], BSK20 [33–35], and SLy [36]. We also include polytropic EOS  $P = K\rho^{1+1/n}$  with polytropic indices  $n = 0.5$  and  $n = 0.6$ , which we denote by p1 and p2, respectively. Here,  $K$  introduces an irrelevant scale, which cancels in the dimensionless quantities.

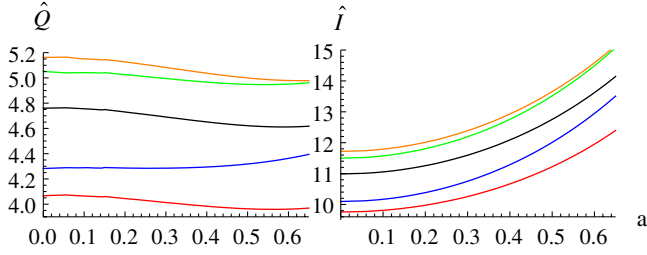


FIG. 1 (color online). Dependence of  $\hat{I}$  and  $\hat{Q}$  on the spin parameter  $a$  for a  $1.4M_\odot$  NS and different EOSs, from top to bottom: BSK20, SLy, APR, FPS, AU.

*Universal  $\hat{I}$ - $\hat{Q}$  relation for arbitrary rotation.*—In the present section, we aim to study the surprising  $\hat{I}(\hat{Q})$  universality beyond the slow rotation approximation. The accuracy of the universality in that approximation is better than 1% [5,6]. Figure 1 indicates a deviation from the slow rotation case within this accuracy at about  $a \sim 0.1$ . We will see that this is indeed the threshold where a modification of the universal relation is required.

We switched to a low order in the angular expansion in the RNS code for  $0.05 < a < 0.15$  in Fig. 1, because the higher orders produce a lot of numeric noise for small  $a$ . The data points at  $a = 0$  were obtained in the slow rotation approximation. We checked that we can use the default order of the angular expansion for  $a > 0.1$  for our desired precision goal. This is an important aspect of our investigation, as we can smoothly connect to the result in Refs. [5,6]. This is computationally challenging, since a large grid size is required to stabilize the result.

In Ref. [11], a deviation from the slow-rotation result greater than 1% showed up for frequencies between 160 and 480 Hz. (However, the deviations become weaker as one approaches the maximum mass of the NS model.) The natural next step is to explore if universality can be extended to this regime and beyond. This requires a suitable dimensionless parameter characterizing rotation, say  $\alpha$ , such that the relation  $\hat{I}(\hat{Q}, \alpha)$  is approximately universal among various EOSs. Indeed, we have defined several natural candidates for such parameters in Eq. (5):  $a$ ,  $\tilde{f}$ , and  $\tilde{f}$ .

We extend the fit in Ref. [5] by a dependence on  $a$  or  $\tilde{f}$  as

$$\log \hat{I} \approx \sum_{i,j} \mathcal{A}_{ij} a^i \log^j \hat{Q} \approx \sum_{i,j} \mathcal{B}_{ij} \tilde{f}^i \log^j \hat{Q}, \quad (6)$$

where the coefficients are given in Table I. We used around 30k data points for the regime  $0.1 < a < 0.6$ ,  $0.2 < \tilde{f} < 1.2$ ,  $1.5 < \hat{Q} < 15$ . The deviation from these fits is maximally  $\sim 1\%$  (independent of the EOS) and on average  $\sim 0.3\%$ . Figure 2 shows the accuracy of the fit for the selected EOS. At the time of writing this Letter, we became aware of Ref. [22], where a similar fit with  $a$  as a parameter was given but the discussion therein focused on other universal relations.

TABLE I. Numerical coefficients for the fits of Eqs. (6) and (7).

$i =$	0	1	2	3	4
$\mathcal{A}_{i0}$	1.35	0.3541	-1.871	3.034	-1.860
$\mathcal{A}_{i1}$	0.697	-1.435	8.385	-14.75	10.05
$\mathcal{A}_{i2}$	-0.143	1.721	-9.343	18.14	-12.65
$\mathcal{A}_{i3}$	0.0994	-0.8199	4.429	-8.782	6.100
$\mathcal{A}_{i4}$	-0.0124	0.1348	-0.7355	1.460	-1.008
$\mathcal{B}_{i0}$	1.35	0.1570	-0.3244	0.09399	0.02863
$\mathcal{B}_{i1}$	0.697	-0.6386	1.509	-0.6932	0.05381
$\mathcal{B}_{i2}$	-0.143	0.7711	-1.636	0.8434	-0.1210
$\mathcal{B}_{i3}$	0.0994	-0.3594	0.7482	-0.3079	0.06019
$\mathcal{B}_{i4}$	-0.0124	0.05788	-0.1140	0.05262	-0.03466
$\mathcal{C}_{i0}$	3.081	-0.1108	0.3402		
$\mathcal{C}_{i1}$	0.6266	-0.01873	0.08047		
$\mathcal{C}_{i2}$	-0.009608	0.01382	-0.02374		

Note that the polytropes were not included in the data for the fit but are contained in Fig. 2. It is well known that one can approximate the EOS of a NS with polytropes in the range  $n \sim 0.5 \dots 1$ ; see, e.g., Ref. [18]. Typically, the tabulated EOSs have an  $n$  value closer to 0.5 in the core, and then it increases up to 1.0. Keeping this in mind, we found that for  $n \lesssim 0.6$  the polytrope is in perfect agreement with our fits (Fig. 2), whereas the  $n = 1$  polytrope has greater deviation. This was observed in the slow rotation approximation in Ref. [6], too. Therefore, polytropes can be an ideal toy model to investigate the underlying mechanism of the universality analytically (see, e.g., Ref. [37]).

Subsequently, we discuss three choices of the dimensionless parameters and their implications for the universality of the  $\hat{I}$ - $\hat{Q}$  relation.

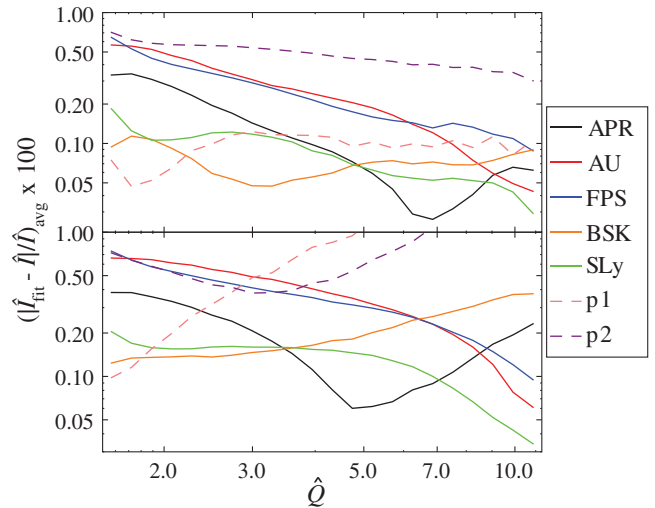


FIG. 2 (color). Percentage deviation of the  $\hat{I}$ - $\hat{Q}$ - $a$  (top) and  $\hat{I}$ - $\hat{Q}$ - $\tilde{f}$  (down) fits with respect to data points, averaged over the  $a$  (top) or  $\tilde{f}$  (down) direction. The deviation is almost constant in the  $a$  or  $\tilde{f}$  direction.

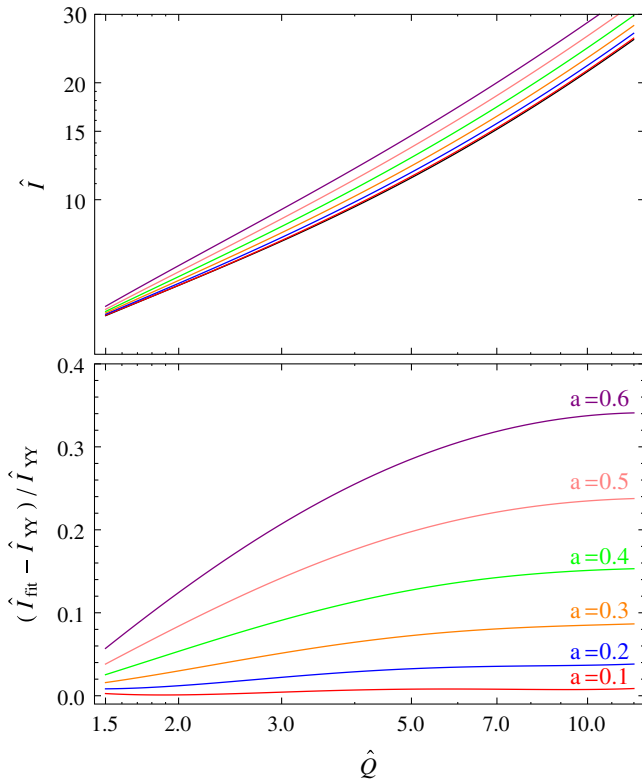


FIG. 3 (color online). Dependence of the  $\hat{I}$ - $\hat{Q}$  relation on the spin parameter  $a$  (upper plot) and the relative deviation from the slow rotation result  $I_{\text{YY}}$  (lower plot). The curve for  $a = 0.1$  is almost identical to  $I_{\text{YY}}$ .

(1)  $a = J/M^2$  as dimensionless parameter.—This parameter is the natural choice and works best for the proposed universality. For fixed  $a$ , the  $\hat{I}$ - $\hat{Q}$  relation depends on the EOS within less than 1%. However, it depends on  $a$ ; see Fig. 3. A simultaneous measurement of  $\hat{I}$ ,  $\hat{Q}$ , and  $a$  must be consistent with these curves if general relativity holds. This can be used to test strong-field gravity independent of assumptions on the EOS.

(2)  $\hat{f} \propto Rf$  as dimensionless parameter.—Again, universality is found for constant  $\hat{f}$ , see Fig. 4. In principle, this can be used to (indirectly) constrain the radius  $R$  of the NS once the dimensionless  $\hat{I}$ ,  $\hat{Q}$ , and the dimensionful  $f$  (pulsar frequency) are known. This is an important prospect of this Letter, since a direct measurement of  $R$  is difficult. But the mass-radius relation contains invaluable information about the EOS. Thus, although the  $\hat{I}$ - $\hat{Q}$  relation is universal among EOSs, it can still be useful to constrain them.

(3)  $\hat{f} \propto Mf$  as dimensionless parameter.—This is not independent from the parameter choice (1). From Eq. (5) along with the definition of  $I$ , it can easily be seen that  $\hat{I} \propto a/\hat{f}$ . Interestingly, the lines of constant  $\hat{f}$  look quite different from Fig. 3. Instead, they qualitatively resemble Fig. 1 in Ref. [11] but display good universality now.

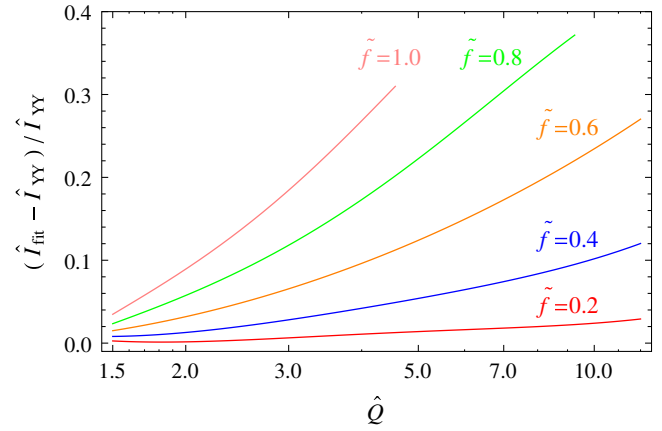


FIG. 4 (color online). EOS-independent deviation of the  $\hat{I}$ - $\hat{Q}$  relation from the slow rotation limit  $I_{\text{YY}}$  for fixed  $\tilde{f}$ .

*Combination of relations.*—The aforementioned relations can of course be combined. In particular, one can solve relation (1) for  $\hat{Q}$  (i.e., use it to “measure”  $\hat{Q}$ ), eliminate it from relation (2), and obtain an  $\hat{I}$ - $\hat{f}$ - $\hat{f}$  relation. This is useful, as  $Q$  is most difficult to measure. Next, identities among the dimensionless quantities ( $a \propto \hat{I}\hat{f}$ ,  $\hat{f} \propto \hat{R}\hat{f}$ ) allow a reformulation as an  $a$ - $\hat{R}$ - $\hat{f}$  relation, which we fit as

$$\log \hat{R} \approx \sum_{i,j} C_{ij} a^i \log^j \frac{a}{\hat{f}}. \quad (7)$$

The result is depicted in Fig. 5. The maximal deviation in Fig. 5 is about 2% for our selection of EOS. However, Eq. (7) does not fit very well to the data generated using the polytropes discussed above. Hence, one must expect an increasing deviation if further EOSs are included in the future. Still, this relation should put tight constraints on  $R$  if  $a$ ,  $f$ , and  $M$  are known. We find that a reformulation of the slow rotation fit given in Eq. (15) of Ref. [21] is in good

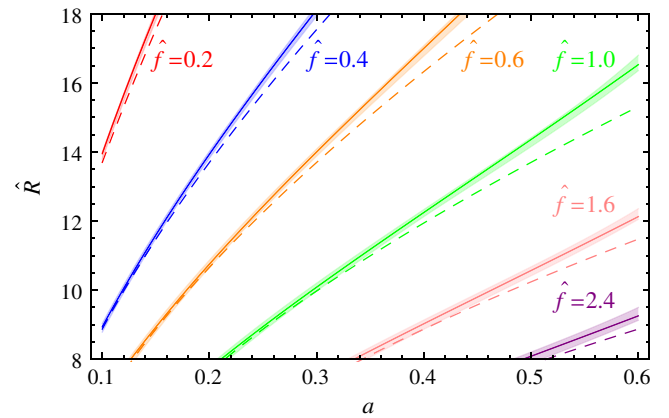


FIG. 5 (color online). Fit of Eq. (7) including deviations from our data (shaded area) and the slow rotation fit from Ref. [21] (dashed lines).

agreement with our Eq. (7)—surprisingly even in the rapid rotation regime. Since our fit uses the observable mass and equatorial radius, this extrapolation of the slow-rotation case is indeed astonishing. Other combinations of the relations can be studied; e.g., one can try to formulate a mass measurement (given a radius) or eliminate the frequency (for pure GW observations).

The universal relation in Eq. (7) can either be used to measure or constrain the radius or to improve the accuracy of the radius measurement. If future x-ray observatories increase the accuracy of the radius measurement sufficiently, then this relation can be used to test fundamental physics. This is what makes universal relations, its combinations, and reformulations so powerful: One can use them to infer unobservable properties or to test gravity if all quantities entering the relations are observable. The diversity of upcoming instruments (see Introduction) makes this even more interesting.

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