Interaction-Induced Quantum Phase Revivals and Evidence for the Transition to the Quantum Chaotic Regime in 1D Atomic Bloch Oscillations

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We study atomic Bloch oscillations in an ensemble of one-dimensional tilted superfluids in the Bose-Hubbard regime. For large values of the tilt, we observe interaction-induced coherent decay and matter-wave quantum phase revivals of the Bloch oscillating ensemble. We analyze the revival period dependence on interactions by means of a Feshbach resonance. When reducing the value of the tilt, we observe the disappearance of the quasiperiodic phase revival signature towards an irreversible decay of Bloch oscillations, indicating the transition from regular to quantum chaotic dynamics.

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The response of a single particle in an ideal periodic potential when subject to an external force constitutes a paradigm in quantum mechanics. As first pointed out by Bloch and Zener the evolution of the wave function is oscillatory in time rather than linear, due to Bragg scattering of the matter wave on the lattice structure [1,2]. Yet, the observation of such Bloch oscillations in condensed-matter lattice systems is hindered by scattering of electrons with crystal defects [3], which causes rapid damping of the coherent dynamics, eventually allowing for electric conductivity [4].

Ensembles of ultracold atoms prepared in essentially dissipation-free optical lattices guarantee long enough coherence times [5] to serve as ideal systems for the observation of Bloch oscillations [6,7]. Furthermore, unprecedented precise control over atom-atom interactions in a Bose-Einstein condensate (BEC), via, e.g., Feshbach resonances [8], allows us to engineer controlled coherent dephasing of the atomic matter wave and even to cancel interactions entirely. Bloch oscillations have been studied with BECs in quasi one-dimensional "tilted" lattice configurations created by a single retro-reflected laser beam with typically thousands of atoms per lattice site [9]. Here, comparatively weak interactions result in strong dephasing observed in a rapid broadening of the atomic wave packet in momentum space [10,11] and are understood by a modification of the wave function in terms of a mean field [12,13]. In contrast, for sufficiently tight confinement when only single spatial orbitals are relevant, the microscopic details determine the quantum coherent time evolution, covered within the Bose-Hubbard (BH) model [14].

In this Letter, we study atomic Bloch oscillations in the one-particle-per-site regime realized with a large ensemble of 1D superfluid Bose gases trapped in an array of “quantum tubes” and subject to a tilted optical lattice. In particular, we observe regular interaction-induced dephasing and revivals of the quantum matter-wave field for strongly tilted lattices [15]. In contrast, for sufficiently small values of the tilt, an irreversible decoherence of the wave function in the presence of interactions is seen, giving a strong indication for the onset of quantum chaos [16].

At ultralow temperatures, much below the lattice band gap, our system is well described by the one-dimensional BH Hamiltonian augmented by a tilt [14,15],

$$\hat{H} = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j + \sum_i\frac{U}{2}\hat{n}_i(\hat{n}_i-1) + E\sum_i i\hat{n}_i. \qquad (1)$$

As usual, $\hat{a}_i^\dagger$ ($\hat{a}_i$) are the bosonic creation (annihilation) operators at the ith lattice site, $\hat{n}_i = \hat{a}_i^\dagger\hat{a}_i$ are the number operators, J is the tunnel matrix element, and U is the on-site interaction energy. The linear energy shift from site to site is denoted by E. Let us first discuss the case of a strong force $E \gg J$. For $U = 0$ the energy spectrum is the famous equidistant Wannier-Stark ladder, which gives rise to a single frequency, $f_B = E$ [17], contributing to the dynamics of any arbitrary initial wave packet [18]. This is the origin of Bloch oscillations for a wave packet in momentum space that is periodically driven across the first Brillouin zone and Bragg reflected at the zone edge with the Bloch frequency f_B [19]. An intermediate interaction energy $U \approx J$ causes splitting of the degenerate energy levels of the Wannier-Stark ladder into a regular pattern of energy bands and results in quasiperiodic coherent decay and revival of the Bloch oscillations with a new fundamental frequency $f_{\rm rev} = U$ [15,16]. This can be understood with the previous assumption of a strong tilt ($E \gg J$) for which the eigenstates of a single atom in the lattice coincide with the localized Wannier states. The Stark localization of the wave function together with the discreteness of the site occupation number leads to an evolution of the mean atomic momentum

$$\langle p\rangle(t) \propto \exp\{-2n[1-\cos(2\pi f_{\rm rev}t)]\}\sin(2\pi f_B t), \qquad (2)$$

where n denotes the mean occupation number per lattice site [15,20]. Note the close analogy of the dynamical evolution to the experiment of Ref. [21]. For the sake of completeness, we stress that for sufficiently strong interaction energy ($U \gg J$) and sufficiently small tilt ($E \ll U$), the ground state is a Mott insulator for one-atom commensurate filling, which shows resonant tunneling dynamics when subject to a sudden tilt [22,23].

Our experiment starts with a BEC of typically 8×10^4 Cs atoms prepared in the internal hyperfine ground state $|F = 3,\ m_F = 3\rangle$ and trapped in a crossed-beam optical dipole trap [24,25]. The sample is levitated against gravity by a magnetic field gradient of $|\nabla B| \approx 31.1$ G/cm. The BEC is loaded adiabatically into an optical lattice of three mutually orthogonal retro-reflected laser beams at a wavelength of $\lambda_l = 2\pi/k = 1064.5$ nm within 500 ms. During the lattice loading, the scattering length is $a_s = 115a_0$. At the end of the ramp, the final lattice depth is $V_{x,y} = 30E_R$ in the horizontal and $V_z = 7E_R$ ($J = 52.3$ Hz) in the vertical direction, where $E_R = 1.325$ kHz is the photon recoil energy. This creates an array of about 2000 vertically oriented 1D Bose-Hubbard systems ("tubes") at near unity filling that are decoupled over the time scale of the experiment. The residual harmonic confinement along the vertical z direction is measured to $\nu_z = 16.0(0.1)$ Hz. We now ramp a_s slowly (with $\approx 1.5\ a_0$/ms) to values of typically 0 to $90a_0$ by means of a Feshbach resonance [26], and thereby prepare the 1D systems near the many-body ground state for an on-site interaction energy U of typically 0 to 400 Hz. This constitutes the initial condition for the observation of Bloch oscillations. Bloch oscillations are then initiated by quickly applying a gravity-induced tilt $E = 1740(4)$ Hz through a reduction of the magnetic levitation field, giving a Bloch period $T_B \equiv 1/f_B = 575(1)$ μs. After a variable hold time t_h we switch off the lattice beams within 300 μs, remove the tilt, and allow the sample for a free levitated expansion of 30 ms to measure the atomic momentum distribution by standard time-of-flight absorption imaging. During expansion, a_s is set to zero to avoid any additional broadening due to interactions.

First, we study a noninteracting sample by setting $a_s = 0a_0$ to quantify the effect of the residual harmonic trapping potential. Time-of-flight absorption images spanning one Bloch oscillation cycle are shown in Figs. 1(a)–(c) after $t_h = 0$, 7, and 14 T_B. Note that the aspect ratio has been adapted for better visualization to compensate the faster expansion transversal to the orientation of the tubes. The typical linear motion of the atomic wave packet through the first Brillouin zone together with Bragg reflections at the zone edge shows a slow dephasing due to the harmonic confinement. The mean atomic momentum $\langle p\rangle$ extracted from time-of-flight images is depicted in Fig. 1(d) as a function of t_h. We model the dephasing effect of the harmonic trapping potential by a local variation of the Bloch frequency over the extent of the initial wave packet [27].

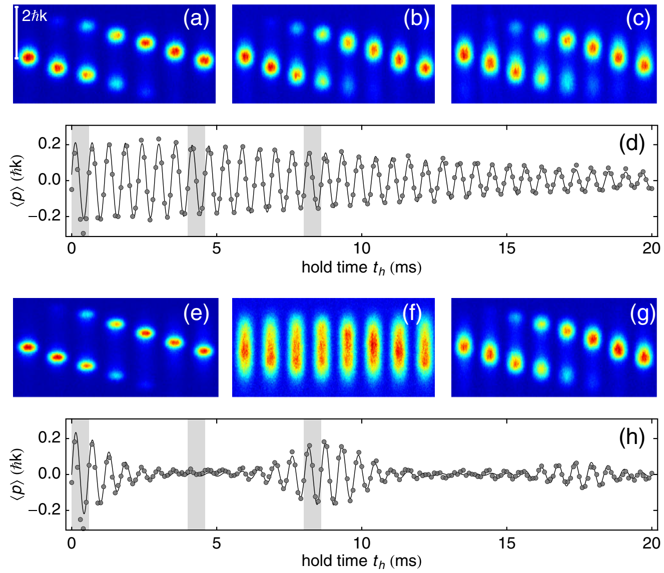

FIG. 1 (color online). Interaction-induced collapse and revival of atomic Bloch oscillations in a large ensemble of 1D tubes. The lattice depth along the tubes is $V_z = 7E_R$ ($J = 52.3$ Hz). Time-of-flight absorption images of one Bloch cycle are shown after $t_h = 0$, 7, 14 T_B for zero interaction (a–c) and for $a_s = 21.4(1.5)a_0$ (e–g), respectively. The vertical bar in (a) indicates the extent of the first Brillouin zone. Full time evolution of the mean atomic momentum is shown for zero interaction (d) and for $a_s = 21.4(1.5)a_0$ (h). Solid lines are fits to the data using the analytic model function (see text). The shaded areas indicate the data points shown in the time-of-flight pictures.

To demonstrate the effect of atom-atom interactions on the dynamics, we repeat the above measurement now with the sample prepared at $a_s = 21.4(1.5)a_0$, corresponding to $U = 102(8)$ Hz. Analogously to the previous measurement, time-of-flight images of full Bloch oscillation cycles after $t_h = 0$, 7, and 14 T_B are shown in Figs. 1(e)–(g). In contrast to the noninteracting sample, we observe a rapid initial decay of the Bloch dynamics, resulting in a spreading of the atomic cloud over the entire Brillouin zone; see Fig. 1(f). Following the dynamical evolution we find a near perfect recovering of the Bloch oscillations, as evident from Fig. 1(g). This arises from the strong coherent dephasing of the initial atomic wave packet in the presence of atom-atom interactions discussed above, leading to a subsequent high contrast matter-wave phase revival. The mean atomic momentum $\langle p\rangle$ as a function of t_h is plotted in Fig. 1(h) and fit by the model function Eq. (2), including the overall decaying envelope discussed above to account for the residual harmonic trap [27]. We do such measurements at different values for a_s. The values for $f_{\rm rev}$ and f_B extracted from the two-mode fit function are depicted in Fig. 2(a). While f_B is not affected by interactions, $f_{\rm rev}$ increases linearly with a_s and is in good agreement with the prediction for U calculated from lowest-band Wannier functions [14]. Two additional data sets showing the evolution of the atomic sample in the tilted lattice are shown in Figs. 2(b) and 2(c) for intermediate and comparatively strong interaction energy, respectively, in order to demonstrate the experimental robustness of the quantum phase revival. While we find clear separation between the time scales given by f_B and f_{rev} and observe four distinct decay and revival periods of the matter-wave packet in Fig. 2(b), Bloch and revival periods start to mix when U is increased to $\approx E/3$ in Fig. 2(c). Interestingly, the model function, Eq. (2), still provides a surprisingly precise fit to the data. Note that our technique allows the direct measurement of the Bose-Hubbard interaction parameter U in the superfluid regime down to very small values, limited only by the residual external confinement.

So far, we have restricted the discussion to large values of the tilt. In the remainder of this letter, we study Bloch oscillations in the regime when all energy scales in the BH Hamiltonian are of comparable magnitude, $E \approx J \approx U$. Consequently, it is impractical to assign a meaningful set of quantum numbers to the energy levels in a perturbative approach. Instead, the energy spectrum emerges densely packed and requires a statistical analysis, revealing the onset of quantum chaos in a Wigner-Dyson distribution of the energy level spacings for sufficiently small E [16]. The transition from the regular to chaotic regime is predicted to become manifest in a rapid irreversible decoherence of Bloch oscillations within a few oscillation cycles. To probe this regime, we prepare the sample in a more shallow lattice $V_z = 4E_R$ ($J = 114.2$ Hz) at a fixed value of $U = 106(8)$ Hz, and we now vary E. Two example data sets are shown in Fig. 3(a) taken at $E = 855(15)$ Hz and 266(5) Hz, respectively, which show very different qualitative behaviors. For the strongly forced lattice we clearly identify the regular decay and revival dynamics described above. In contrast, for smaller E the revival, expected to appear at $t_h = 1/U$, is missing. Instead, we observe a single, rapid irreversible decay of Bloch oscillations. This observation is quantified in two ways. First, we fit the initial decay in our data to a decaying sinusoid with an envelope $\propto \exp(-t/\tau)$. The extracted Bloch frequency f_B and the exponential decay time τ as a function of E are plotted in Figs. 3(b) and 3(c) (circles). Second, we extract the amplitude of the revival δp in momentum space [28] and show it in Fig. 3(c) (triangles). As expected, f_B is directly given by E. Further, τ is found to be constant for $E \gtrsim 600$ Hz; it quickly increases and finally saturates for $E \lesssim 400$ Hz. The revival signal δp exhibits an opposite behavior and decreases with decreasing tilt.

We interpret our data as a strong indication for the transition from the regular to the quantum chaotic regime. For large $E/J \gtrsim 6$, we find a coherent dephasing and

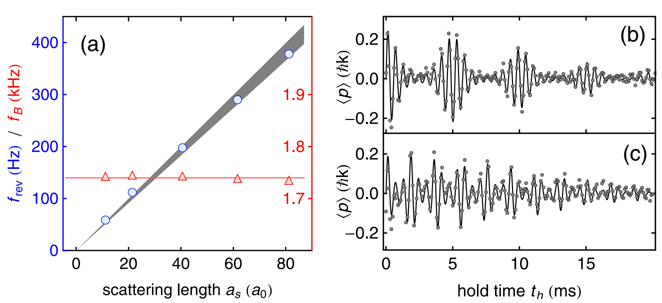

FIG. 2 (color online). Dependence of the revival frequency f_{rev} on the on-site interaction energy U. (a) Extracted f_{rev} (circles) and Bloch frequency f_B (triangles) from model fits of Eq. (2) to time traces $\langle p \rangle(t_h)$ as shown in Fig. 1 as a function of a_s. Error bars are smaller than the data points. The shaded gray area depicts the calculated U, taking into account a 5% error on the lattice depth. The solid line denotes a constant fit to the data for f_B. Mean atomic momentum as a function of hold time for $a_s = 40.6(1.5)a_0$ (b), giving $U = 194(10)$ Hz, and $a_s = 111.5(1.5)a_0$ (c), giving a comparatively large $U = 533(15)$ Hz. Solid lines are fits to the data based on Eq. (2).

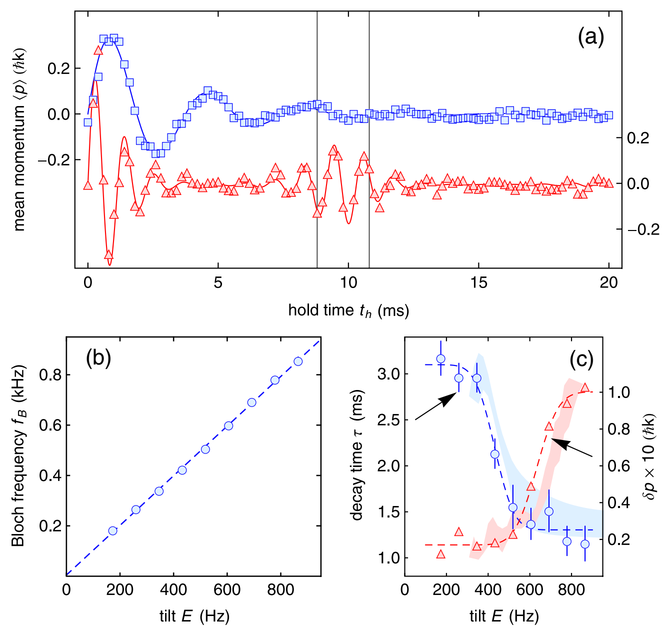

FIG. 3 (color online). Transition from regular to chaotic dynamics when varying the tilt E. (a) Mean atomic momentum as a function of hold time for $E = 855(15)$ Hz (triangles) and $E = 266(5)$ Hz (squares) at $a_s = 26.9(1.5)a_0$. Here, $V_z = 4E_R$ ($J = 114.2$ Hz), giving $U = 106(8)$ Hz. Solid lines show fits to the data based on the analytic model function, Eq. (2), in the strong-forcing limit and an exponentially damped sinusoidal in the chaotic regime. The two data sets are offset for clarity. Bloch frequency f_B (b) and exponential decay time τ (c) as a function of E (circles) extracted from exponentially damped sinusoidal fits to the initial decay in data sets as depicted in (a). Triangles in (c) depict the amplitude of the revival δp as a function of E. The dashed line in (b) is a linear fit to the data. The dashed lines in (c) are error-function fits to the data to guide the eye. The shaded areas indicate the prediction from numerical simulations with $1 \leq n \leq 1.4$ [27].

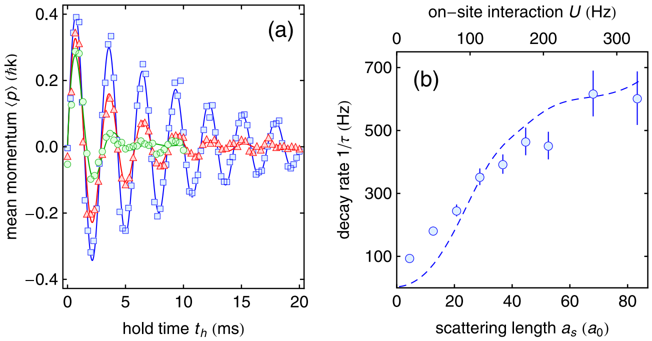

FIG. 4 (color online). Dependence of the irreversible decay of Bloch oscillations on atom-atom interactions in the chaotic regime. (a) Mean atomic momentum as a function of hold time for $a_s = 4.5(1.5)a_0$ (squares), $a_s = 20.8(1.5)a_0$ (triangles), and $a_s = 83.4(1.5)a_0$ (circles) at $E = 346(10)$ Hz. Here, $V_z = 4E_R$ ($J = 114.2$ Hz). Solid lines are exponentially damped sinusoidal fits to the data. (b) Exponential decay rate $1/\tau$ as a function of a_s extracted from data sets, as shown in (a). The dashed line shows a prediction from numerical simulations for a homogeneous system without a trap.

revival of Bloch oscillations with a characteristic dephasing time scale that is solely given by U and not affected by the strength of the tilt, as expected from Eq. (2). With decreasing E, the revival continuously disappears, accompanied by a significant change of τ, indicating the onset of a regime where the system dynamics decay irreversibly for $E/J \lesssim 3$. A spectral analysis of the tilted BH model for our system parameters ($n \gtrsim 1$) reveals the onset of Wigner-Dyson statistics in agreement with the observed crossover in τ [27]. In addition we compare our data in Fig. 3(c) to results obtained from numerical simulations of Bloch oscillations within the BH model, indicated by the shaded areas. We emphasize again that the irreversible decay is due to interaction-induced decoherence in the system arising from the multitude of avoided crossings in the chaotic level structure of the many-body energy spectrum and ought to be contrasted from a coherent dephasing, as observed for $E \gg J$ [16]. In this sense, the atomic ensemble itself acts as the bath responsible for decoherence of the quantum many-body system [29,30].

Finally, we report on a last set of measurements to emphasize the role of interactions on the irreversible decay discussed above. We study Bloch oscillations for a fixed tilt $E = 346(10)$ Hz at $V_z = 4E_R$ ($J = 114.2$ Hz) and vary U. Three example data sets are shown in Fig. 4(a). We identify an overall irreversible decay indicative of the chaotic regime. Moreover, the decay rate strongly depends on the interaction strength and decreases with decreasing U. This is expected from the noninteracting limit $U \to 0$, for which the system turns regular again. Note that the observation of decay rates $\lesssim$70 Hz is currently limited by the overall dephasing due to the presence of the harmonic trap discussed above. We extract the decay rate $1/\tau$ as before and plot it as a function of U in Fig. 4(b). Our data reveal a saturating monotonic increase of $1/\tau$ with U. Scaling of the decoherence rate with U is expected to change from a quadratic (regular regime) to a square-root (chaotic regime) dependence [31], indicated by the dashed line in Fig. 4(b) [27]. However, a precise experimental confirmation requires compensation of the harmonic trap to allow for longer observation times of the Bloch oscillations and will be the issue of a forthcoming experiment.

In conclusion, we have studied Bloch oscillations in the context of a strongly interacting many-body system properly described within the Bose-Hubbard model. In this regime the "granularity" of matter in combination with strong atom-atom repulsion causes coherent quasiperiodic decay followed by a high contrast quantum phase revival of the Bloch oscillating matter-wave field. The revival period is entirely determined by the interaction strength and thus provides a direct, precise measure for the on-site interaction energy in the superfluid regime, in contrast to a related technique to measure U in a Mott insulator [32]. For practical applications we point out that the phase revivals effectively extend the observation time of Bloch oscillations even in the presence of interactions with potential prospects to e.g., precision force measurements [33,34]. Further, we have investigated the Bloch dynamics of the interacting atomic ensemble as a function of the applied tilt and found clear evidence for the transition from regular to quantum chaotic dynamics. Our results may open the experimental study of quantum chaos in such systems, including its implication on the decoherence and thermalization of interacting 1D quantum many-body systems [29,30,35]. Moreover, quantitative studies on the parameter dependence of the transition from the regular to the quantum chaotic regime are of interest [36].

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[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.193003 for theoretical background on the transition to the chaotic regime, including a statistical analysis of the energy spectrum and time-dependent numerical simulations of the tilted BH Hamiltonian, and for details on modeling the effect of the harmonic trap.
[28] We evaluate $\delta p = \sqrt{\sum_i (\langle p \rangle_i - \overline{\langle p \rangle})^2 / (N-1)}$ where the summand is over the sample of N data points in the range $8.8\ \mathrm{ms} \leq t \leq 10.8\ \mathrm{ms}$, and $\overline{\langle p \rangle}$ denotes the mean over the sample.
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