## <span id="page-0-0"></span>Laser-Nucleus Reactions: Population of States Far above Yrast and Far from Stability

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Nuclear reactions induced by a strong zeptosecond laser pulse are studied theoretically in the quasiadiabatic regime where the photon absorption rate is comparable to the nuclear equilibration rate. We find that multiple photon absorption leads to the formation of a compound nucleus in the so-far unexplored regime of excitation energies several hundred MeV above the yrast line. At these energies, further photon absorption is limited by neutron decay and/or induced nucleon emission. With a laser pulse of ≈50 zs duration, proton-rich nuclei far off the line of stability are produced.

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Introduction.—Recent experimental developments in laser physics hold promise to advance the new field of laser-induced nuclear reactions beyond so-far explored territory in nuclear physics. Efforts are under way at both ELI [\[1\]](#page-3-1) and IZEST [\[2\]](#page-3-2) to generate a multi-MeV zeptosecond coherent laser pulse by backward Compton scattering of optical laser light on a sheet of relativistic electrons [\[3,4\].](#page-3-3) Although perhaps somewhat ahead of existing technology, this possibility poses a challenge for nuclear reaction theory that would be confronted with a totally new parameter regime. The central question is, which reactions are expected to occur when an intense high-energy coherent laser pulse hits a medium-weight or heavy target nucleus? What is the difference between this case and other areas [\[5](#page-4-0)*–*8] of laser-matter or laser-nucleus interaction? The answers are relevant also for the layout of future experiments.

In this Letter we provide first semiquantitative theoretical answers to these questions by combining a newly developed method of calculating nuclear level densities at high excitation energies and for large particle numbers [\[9,10\]](#page-4-1) with concepts of nuclear reaction theory. Our work addresses the quasiadiabatic laser-nucleus interaction regime where excitation and relaxation processes are governed by similar time scales. We show that coherent photon absorption by medium-weight and heavy nuclei can produce high excitation with low angular momentum transfer, leading to compound nuclei several hundred MeV above yrast. Our approach renders possible the semiquantitative study of the competition between photon absorption, photon-induced nucleon emission, neutron evaporation, and fission. The latter turns out not to be competitive. With neutron evaporation or photoninduced nucleon emission overtaking photon absorption at energies below the saturation of the latter for medium-weight and heavy nuclei, we expect proton-rich nuclei far from the valley of stability to be produced. Laser-nucleus interaction experiments at ELI or IZEST thus promise to shed light on the structure of such nuclei and the time scales and level densities involved.

To be specific, we consider a laser pulse containing  $N = 10<sup>3</sup> - 10<sup>4</sup>$  coherent photons with mean photon energy  $E_L \approx 1$ –5 MeV and with an energy spread  $\sigma \approx 50$  keV (and a corresponding pulse duration  $\hbar/\sigma \approx 10^{-20}$  s). Nuclei are bound by the strong interaction. As a consequence, the electromagnetic interaction of even such a strong laser pulse with a nucleus is much less violent than the interaction of a medium-intensity optical laser pulse with an atom. In the atomic case, a laser field strong enough to distort the Coulomb potential and set electrons free is characterized by an electric field strength roughly given by the ratio between the ionization potential and the Bohr radius, i.e.,  $\simeq 10^9$  eV/cm. For a corresponding distortion of the nuclear potential, the electric field strength would have to be roughly given by the ratio between the nucleon binding energy and the nuclear radius, i.e., of order  $10^{19}$  eV/cm. Despite the MeV photon energy, even the laser pulse under consideration here does not produce such strong fields, being actually rather weak. A quantitative analysis using the Keldysh parameter [\[11\]](#page-4-2) supports this qualitative argument.

For photons in the few-MeV range, the product of wave number k and nuclear radius R obeys  $k \ll 1$ . In addition, unlike the case of low-lying nuclear excitations [\[5\],](#page-4-0) here available states of all spins allow the use of the dipole approximation. Four energy scales are relevant for the laser-nucleus reaction. In addition to the mean laser photon energy  $E_L \approx 1$ –5 MeV and the energy spread  $\sigma \approx 50$  keV, these are the effective dipole width and the nuclear spreading width. For the effective dipole width of a pulse of coherent photons we use the semiclassical expression  $N\Gamma_{\text{dip}}$  valid for  $N \gg 1$  coherent photons, with the standard nuclear dipole width  $\Gamma_{\text{dip}}$  in the keV range and  $N\Gamma_{\text{dip}}\approx$ 1–5 MeV. In the course of the reaction up to  $N_0 \approx 5 \times 10^2$ photons may be absorbed by the nucleus. We neglect the resulting reduction of N in  $N\Gamma_{\text{dip}}$ . The spreading width  $\Gamma_{\text{sp}}$ , absent in atoms, accounts for the residual nuclear interaction. For excitation energies up to several 10 MeV,  $\Gamma_{\rm{sn}}$  is of the order of 5 MeV [\[12\].](#page-4-3) The nuclear relaxation time

 $\hbar/\Gamma_{\rm{sn}}$  in which the compound nucleus reaches statistical equilibrium and the mean time for dipole absorption  $\hbar/N\Gamma_{\text{dip}}$  are both much shorter than the duration  $\hbar/\sigma$  of the laser-induced nuclear reaction.

The laser-nucleus interaction is characterized by three regimes. (i) In the perturbative regime  $N\Gamma_{\text{dip}} \ll \Gamma_{\text{sp}}$ , single excitation of the collective dipole mode plays the dominant role [\[6\].](#page-4-4) The experimental signal for the laser-nucleus interaction in this regime is the nonexponential decay in time of the compound nucleus [\[13\].](#page-4-5) (ii) In the sudden regime ( $N\Gamma_{\text{dip}} \gg \Gamma_{\text{sp}}$ ) the residual interaction is irrelevant. Nucleons are excited independently of each other and are emitted from the common average potential. For sufficiently long pulse duration, the nucleus evaporates. (iii) The quasiadiabatic regime  $(N\Gamma_{\text{dip}} \approx \Gamma_{\text{sp}})$  forms the topic of this Letter and arguably is physically the most interesting one since it leads to excitation energies far above the yrast line and to nuclei far beyond the valley of stability. For  $N\Gamma_{\text{dip}} \approx \Gamma_{\text{sp}}$  nuclear equilibration is as fast as single-photon absorption. The binding energy of a nucleon  $E_b \approx 8$  MeV being larger than the photon energy  $E_L$ considered here, the dipole excitation energy  $E_L$  of the nucleon is shared almost instantaneously with several or many other nucleons. This equilibration mechanism is absent in atoms. Only absorption of a large number of photons leads to significant induced particle emission or significant neutron evaporation from the nucleus. Because of these inherently nuclear processes without atomic counterpart, the theoretical methods used here to describe the laser-nucleus interaction do not relate to the strong-field approximation [\[14\]](#page-4-6) known from atomic physics. Nuclear photon absorption may rather be treated in a manner analogous to nucleon-induced precompound reactions [\[15\]](#page-4-7), i.e., in terms of a (set of) time-dependent master equation(s). In this Letter we use a simplified version of such an approach.

Quasiadiabatic regime.—In this regime, the nucleus (almost) attains statistical equilibrium between two subsequent photon absorption processes. Consecutive absorption of  $N_0 \gg 1$  dipole photons by an (almost) equilibrated compound nucleus leads to high excitation energies  $N_0E_L$ . Explicit calculation shows that the average nuclear spin is given by  $J = \hbar \sqrt{N_0}$ . Therefore, the laser-induced reactions open access to the regime of states with small spin far above open access to the regime of states with small spin far above the yrast line, not accessible to reactions induced by heavy ions, see Fig. [1](#page-1-0). Since  $J/\hbar$  is only of order 10 even for several hundred absorbed photons (see the inset of Fig. [1](#page-1-0)), we neglect spin in what follows.

We simplify the description further and assume that between two subsequent photon absorption processes, nuclear equilibration is complete. Without that assumption, the process must be described in terms of a time-dependent master equation. That does not seem justified at this early stage of theoretical and experimental development. Photon absorption at excitation energy  $E$  is then governed by the

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FIG. 1 (color online). Qualitative illustration of two regimes of nuclear excitation. The yrast line defines the minimum energy of a nuclear state with a certain angular momentum. Heavy-ion collisions preferentially excite states close to the yrast line (region depicted by hatched area). Multiple absorption of coherent multi-MeV dipole photons involves small transfer of angular momentum and leads to compound states several hundred MeV above yrast (thick arrow). The inset shows the angular-momentum distributions for  $N_0 = 30$ , 100 and 200 absorbed dipole photons.

effective absorption rate of an equilibrated compound nucleus and given by  $(N\Gamma)_{\text{eff}}(E) = N\Gamma_{\text{dip}}\rho_{\text{acc}}(E)/\rho_{\text{acc}}(E_g)$ . Here  $\rho_{\text{acc}}(E)$  is the density of accessible states and  $E_g$  is the energy of the ground state. We have used the expression for  $\rho_{\rm acc}(E)$  based on the Fermi-gas model given in Ref. [\[10\]](#page-4-8). Our results show that  $(N\Gamma)_{\text{eff}}(E)$  slowly decreases with increasing  $E$ . This supports our assumption that the spreading width  $\Gamma_{sp}$  does not change significantly with excitation energy.

The consecutive absorption of  $N_0$  photons terminates prior to the end of the laser pulse whenever  $(N\Gamma)_{\text{eff}}(E)$ is equal to the largest one of four widths: the width  $(N\Gamma)_{ind}(E)$  for induced dipole emission, the width  $\Gamma_n(E)$ for neutron evaporation, the width  $(N\Gamma)_{\text{cnt}}(E)$  for induced nucleon emission, and the width  $\Gamma_f$  for induced fission. The expressions for these four widths involve the density  $\rho_A(E)$ of spin-zero states of the target nucleus with mass number A at excitation energy E, or the density  $\rho_{\text{acc}}(E)$  of accessible states. For  $\rho_A(E)$  we use the expressions given in Ref. [\[10\]](#page-4-8). These are valid for high excitation energies E and for  $A \gg 1$ and depend on the density  $\rho_1(\varepsilon)$  of bound single-particle states. We have used two continuous forms for  $\rho_1(\varepsilon)$ :

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$$
\rho_1^{(1)}(\varepsilon) = \frac{2A}{F^2} \varepsilon, \qquad \rho_1^{(2)}(\varepsilon) = \frac{3A}{F^3} \varepsilon^2, \tag{1}
$$

with  $\rho_1^{(1)}(\varepsilon)$   $[\rho_1^{(2)}(\varepsilon)]$  used for mass number  $A = 100$ <br> $[A - 200$  respectively! The range of the single-particle  $[A = 200,$  respectively]. The range of the single-particle spectrum is  $0 \le \varepsilon \le V$  with  $V = 45$  MeV while the Fermi energy F was taken as  $F = 37$  MeV. For  $A = 100$  $[A = 200]$  the A-particle level density  $\rho_A(E)$  reaches its

maximum at an excitation energy  $E_{\text{max}} = (2/3)A(V-F)$ 533MeV [at  $E_{\text{max}} = (3/4)A(V-F) = 1200 \text{MeV}$ , respec-tively]. The expressions [\(1\)](#page-1-1) for  $\rho_1(\varepsilon)$  were also used [\[10\]](#page-4-8) to calculate  $\rho_{\text{acc}}(E)$  and the density of accessible continuum states  $\rho_{cont}(E)$  introduced below.

Induced dipole emission.—Probability conservation in the master equation implies  $(N\Gamma)_{ind}(E) = (N\Gamma)_{eff}(E) \times$  $\rho_A(E - E_L)/\rho_A(E)$ . The ratio  $\rho_A(E - E_L)/\rho_A(E)$  is very small at excitation energies in the 10 MeV range but increases steeply with  $E$ . Absorption and induced emission become equal at the maximum  $E_{\text{max}}$  of  $\rho_A(E)$ . Substantial excitation of the compound nucleus by dipole absorption beyond  $E_{\text{max}}$  is impossible because induced dipole emission overcompensates absorption. In the absence of all other decay mechanisms, the nuclear occupation probability would hover in a set of states with excitation energies close to  $E_{\text{max}}$  until the laser pulse terminates.

Neutron evaporation.—From the Weisskopf formula we have  $\Gamma_n(E) = (2\pi)^{-1} \int_{E_g(A-1)}^{E-E_n} dE' \rho_{A-1}(E') / \rho_A(E)$ . Here E is the excitation energy,  $E_n = V - F$  is the binding energy of the last neutron, and  $E_q(A - 1)$  [ $\rho_{A-1}(E)$ ] is the groundstate energy [the level density, respectively] of the nucleus with mass number  $A - 1$ . In accordance with our semiquantitative approach we have taken all transmission coefficients in the Weisskopf formula (i.e., the transmission probabilities into the individual open neutron channels) equal to unity. To calculate  $E_q(A-1)$  and  $\rho_{A-1}(E)$  we have used the single-particle level densities in Eqs. [\(1\)](#page-1-1) with  $A \rightarrow (A - 1)$ . Because of its dependence on level densities,  $\Gamma_n(E)$  rises steeply with excitation energy. The point of intersection with the curve for  $(N\Gamma)_{\text{eff}}(E)$  defines the neutron-evaporation limit of excitation by dipole absorption.

Induced nucleon emission.—The occupation probability of single-particle states above the Fermi energy increases with increasing excitation energy. Dipole absorption by nucleons in such states may lead to direct particle emission into the continuum. In analogy to the expression for  $(N\Gamma)_{\text{eff}}(E)$ , we have  $(N\Gamma)_{\text{cnt}}(E) = N\Gamma_{\text{dip}}\rho_{\text{cnt}}(E)/\rho_{\text{acc}}(E_q)$ . As in the case of  $\rho_{\text{acc}}(E)$ , the Fermi-gas model was used [\[10\]](#page-4-8) to calculate the density of accessible continuum states  $\rho_{\text{cnt}}(E)$ . While for the calculation of  $\rho_{\text{acc}}(E)$  only bound single-particle states (with energies  $\varepsilon$  < V) are taken into account, for  $\rho_{\text{cnt}}(E)$  only particle-instable single-particle states with energies  $\varepsilon \geq V$  are used. The density of these states was determined by a fit to results given in Ref. [\[16\]](#page-4-9). We have not attempted to determine the ratio of protons to neutrons emitted in the process. This ratio is expected to depend on the height of the Coulomb barrier.

Induced fission.—According to the Bohr-Wheeler for-mula [\[17\]](#page-4-10) modified by friction [\[18\]](#page-4-11),  $\Gamma_f$  decreases monotonically with increasing friction constant  $β$ . We use the maximum value  $\Gamma_f = [\hbar \omega_1/(2\pi)] \exp\{-E_f/T\}$  attained at  $\beta = 0$ . Here  $\omega_1$  is the frequency of the inverted harmonic oscillator that osculates the fission barrier at its maximum,

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FIG. 2 (color online). Various widths (in MeV) versus excitation energy (in MeV) (a) for  $A = 100$  and (b) for  $A = 200$ . The solid red line depicts  $(N\Gamma)_{\text{eff}}$ , the dotted pink line  $(N\Gamma)_{\text{ind}}$ , the long-dashed green line  $(N\Gamma)_{\text{cnt}}$ , the dash-dotted light blue line  $\Gamma_f$ , and the short-dashed dark blue line  $\Gamma_n$ , respectively. For  $\Gamma_f$  we have used in the calculations  $\omega_1 = 4$  MeV and  $E_f = 10$  MeV (4 MeV) for  $A = 100$  ( $A = 200$ , respectively).

 $E_f$  is the height of the fission barrier, and T is the nuclear temperature given by  $1/T = (d/dE) \ln \rho_A(E)$ . Very little is known about the temperature dependence of  $\omega_1$  and  $E_f$ and of the Strutinsky shell corrections [\[19\]](#page-4-12). Therefore, our estimate for  $\Gamma_f$  is more qualitative than for the other widths.

Results.—Figure [2](#page-2-0) shows the five widths (in MeV) for  $A = 100$  and  $A = 200$  versus excitation energy E (in MeV) and for  $N\Gamma_{\text{dip}} = 5$  MeV. We note that  $(N\Gamma)_{\text{eff}}(E)$  decreases slowly as  $E$  increases. At the maxima  $E_{\text{max}}$  of the level density  $\rho_A(E)$  given above,  $(N\Gamma)_{\text{eff}}(E)$  intersects with  $(N\Gamma)_{ind}(E)$ . In all cases considered the point of intersection of  $(N\Gamma)_{\text{eff}}(E)$  with either  $\Gamma_n(E)$  or  $(N\Gamma)_{\text{cnt}}(E)$  lies significantly below  $E_{\text{max}}$ . Thus, nuclear excitation by dipole absorption is always limited by neutron evaporation or induced nucleon emission. The fission width is always smaller than the other widths and does not terminate dipole absorption, even though we have chosen the unrealistically large value  $\omega_1 = 4$  MeV. For  $A = 100$  neutron evaporation is the dominant process irrespective of the value of  $N\Gamma_{\text{dip}}$ . For  $A = 200$ , however, the competition between neutron evaporation and induced nucleon emission is decided by  $N\Gamma_{\text{dip}}$ . In the case of Fig. [2\(b\)](#page-2-0)  $(A = 200 \text{ and }$  $N\Gamma_{\text{dip}} = 5$  MeV), photon absorption is terminated by

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FIG. 3 (color online). The solid red line [dashed green line] gives the energy  $E_x$  versus  $N\Gamma_{\text{dip}}$  where the curves for  $(N\Gamma)_{\text{eff}}$  and  $\Gamma_n$  [for  $(N\Gamma)_{\text{eff}}$  and  $(N\Gamma)_{\text{cnt}}$ , respectively] intersect for  $A = 200$ for  $A = 200$ .

induced nucleon emission. Figure [3](#page-3-4) shows the dependence of the intersection points of the various widths on the value of  $N\Gamma_{\text{dip}}$ . While  $\Gamma_n$  is independent of  $N\Gamma_{\text{dip}}$ , both  $(N\Gamma)_{\text{eff}}(E)$  and  $(N\Gamma)_{\text{cnt}}(E)$  depend linearly on  $N\Gamma_{\text{dip}}$ . Therefore, the intersection point of the two latter curves is fixed at  $E_x = 901$  MeV. At  $N\Gamma_{\text{dip}} = 2.73$  MeV, neutron evaporation and induced nucleon emission exchange their roles in limiting dipole excitation.

If neutron evaporation dominates over induced nucleon emission, a single highly excited daughter nucleus with mass number  $(A - 1)$  is produced. Our values for  $\rho_{A-1}(E)$ show that for  $A = 100$  ( $A = 200$ ) the spectrum of evaporated neutrons falls off (nearly) exponentially with energy and less than 10% of the emitted neutrons have energies in excess of 20 MeV (25 MeV, respectively). Therefore, absorption of only a few photons suffices to excite the daughter nucleus to energies where another neutron is emitted. The chain continues. A laser pulse of sufficient duration thus opens the possibility to reach proton-rich nuclei far off the line of stability.

Our neglect of proton evaporation may not be justified for nuclei where proton binding energies are small, especially for proton-rich nuclei. Moreover, protons may be emitted in significant numbers when induced emission of protons and neutrons dominates over neutron evaporation. At the end of the laser pulse we expect a fixed number of nucleons to be emitted. The resulting final-product nuclei are in highly excited states and have fixed mass number but a distribution in proton numbers that ranges from the valley of stability to a very proton-rich nucleus. The exact details depend on the competition between neutron and proton emission and are beyond the scope of this Letter.

Discussion and conclusions.—We have studied theoretically the interaction of a strong coherent zeptosecond laser pulse with medium-weight and heavy nuclei. The comparison with atom-laser reactions shows that in the nuclear case the interaction is comparatively weak so that we always deal with multiphoton excitations. A novel aspect of the nuclear case is the important role played by the residual interaction, which drives the nucleus towards statistical equilibrium. Combined with the fact that dipole absorption dominates all other multipoles, this leads in the quasiadiabatic case to compound nucleus excitation energies far above yrast.

The main uncertainty in our calculations is due to the various level densities that determine the five widths. However, each width actually depends on a ratio of many-body level densities taken at nearly the same energies and/or mass numbers. Such ratios are much less sensitive to details of the single-particle level density  $\rho_1(\varepsilon)$  in Eqs. [\(1\)](#page-1-1) than the many-body level densities themselves. Therefore, we expect our results not to change drastically when other values for  $\rho_1(\varepsilon)$  are used. Such values could be obtained, for instance, from a temperature-dependent Hartree-Fock calculation. Nevertheless, it would be unreasonable to expect that our results define the critical energies more precisely than to within several 10 MeV. Within such errors it seems reasonably safe to say that neutron evaporation and induced nucleon emission (and not induced dipole emission) terminate photon absorption, and that nuclear fission is irrelevant (except perhaps for the heaviest nuclei not considered here). The competition between neutron evaporation and induced nucleon emission is so narrow for  $A = 200$ , however, that either of these processes may dominate.

Typical maximum excitation energies depend on the intensity and duration of the laser pulse. With  $E<sub>L</sub>$  the mean energy per photon,  $N_0 = E/E_L$  photons must be absorbed to reach the high excitation energies E of up to  $\approx 1000$  MeV shown in Figs. [2](#page-2-0) and [3](#page-3-4). Even larger values of  $N_0$  are needed to reach nuclei far off the line of stability. The time for the total absorption process is roughly  $N_0 \hbar / (N\Gamma_{\text{dip}})$ , and the laser pulse duration must then obey  $\hbar/\sigma \geq N_0 \hbar/(N\Gamma_{\text{dip}})$ . Hence, large values of N, values of  $E_L$  in the 5 MeV range, and values of  $\sigma$  in the 10 keV range or below are desirable to exploit the full potential of the process and reach the region far above yrast. If the laser pulse lasts long enough, nuclei far off the line of stability are produced. Then, laserinduced nuclear reactions promise insight into the structure of proton-rich nuclei.

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