# Conductive Coupling of Split Ring Resonators: A Path to THz Metamaterials with Ultrasharp Resonances

Ibraheem Al-Naib,[1,*] Erik Hebestreit,[2] Carsten Rockstuhl,[2] Falk Lederer,[2] Demetrios Christodoulides,[3]
Tsuneyuki Ozaki,[1] and Roberto Morandotti[1]
[1]*INRS-EMT, 1650 Boulevard Lionel-Boulet, Varennes, Québec J3X 1S2, Canada*
[2]*Institute of Condensed Matter Theory and Solid State Optics and Abbe Center of Photonics,*
*Friedrich-Schiller-Universität, Jena 07743, Germany*
[3]*CREOL, The College of Optics and Photonics, University of Central Florida, Orlando, Florida 32816, USA*

(Received 16 May 2013; revised manuscript received 2 December 2013; published 9 May 2014)

We report on a novel metamaterial structure that sustains extremely sharp resonances in the terahertz domain. This system involves two conductively coupled split ring resonators that together exhibit a novel resonance, in broad analogy to the antiphase mode of the so-called Huygens coupled pendulum. Even though this resonance is in principle forbidden in each individual symmetric split ring, our experiments show that this new coupled mode can sustain quality factors that are more than one order of magnitude larger than those of conventional split ring arrangements. Because of the universality of the metamaterial response, the design principle we present here can be applied across the entire electromagnetic spectrum and to various metamaterial resonators.

DOI: 10.1103/PhysRevLett.112.183903 PACS numbers: 42.25.Bs, 42.70.−a, 78.67.Pt

In recent years the prospect of developing new metamaterial (MTM) systems has been intensively explored. This interest has been mainly driven by a desire to fundamentally alter the electromagnetic response in ways that are otherwise impossible with natural materials. Over the years, various concepts toward engineering the properties of this new class of materials have been suggested [1–5]. In most cases the essential building block of the MTM is a resonant unit cell. Traditional planar MTMs are often based on split ring resonators (SRRs) and their resonances can be excited upon illuminating them at normal incidence with the electric field being either polarized perpendicular or parallel to their gaps. Moreover, SRRs have also been employed as constituents to support magneto-inductive waves [6,7]. Given that such schemes of coupled SRRs tend to affect their spectral response, they could be promising in a great variety of applications of practical relevance, such as slow-light structures and artificial delay lines, just to mention a few.

In this context, the exploration of MTMs at terahertz (THz) frequencies is of cardinal importance since various applications are about to emerge specifically in this frequency range. Pivotal examples are biochemical sensing, imaging, communication, and spectroscopy of different materials. However, the lack of many types of functional devices at this band range still hinders progress in this field. MTMs can play an important role in filling this gap. Typical examples for such functional devices are modulators, high-performance filters, and thin-film sensors [8–13].

The exotic physics observed in metamaterials is generally induced by the resonant nature of their response. Hence, realizing resonators with high quality ($Q$) factors is crucial in order to achieve efficient metamaterial behavior [14]. In general, the $Q$ factors of resonators are limited by both nonradiative (i.e., Ohmic) and radiative losses. The Ohmic losses at terahertz frequencies are quite low since the materials used for the fabrication of MTMs closely resemble perfect metals. Hence, the main challenge in overcoming this hurdle is to ultimately reduce the radiation loss by tailoring the coupling among their unit cells.

By elaborating on the possible strategies to achieve resonances with high $Q$ factors, we can identify two different paths. The first approach consists in asymmetrically coupling two bright resonances, in such a way that they oscillate in an out of phase fashion. In particular, asymmetric double split ring resonators (ASRs) have been suggested as the most promising geometry to achieve high $Q$ factors for planar structures [15]. An ASR usually consists of two parts of metals with slightly different lengths. The radiation losses are extremely low due to the excitation of a dark resonance where currents in the two parts oscillate out of phase. Combined with the very low nonradiative losses of THz metamaterials, the observable $Q$ factor of such resonances is much higher than the value associated with traditional symmetric designs. We note that the asymmetric resonance results from both the geometry and the capacitive and/or inductive coupling of the two constituents. This requires engaging the "meta-atoms" (i.e., each individual ASR) in extremely close (spatial) proximity, in turn rendering the structure extremely sensitive to fabrication errors as the $Q$ factors can drop dramatically for even small deviations from the intended design. Therefore, designing a MTM that supports high $Q$ factors and at the same time is immune against fabrication imperfections in the implemented geometry is indeed a challenging task.

The second approach consists of using resonances that cannot be normally excited for a given polarization due to symmetry constraints. In this regime, however, the technique of choice has been to couple a radiative element with a subradiant element. The new meta-atom is now capable of sustaining a bright mode along with a dark mode. Interestingly, such an approach has been previously considered in the literature in an entirely different context, e.g., when dealing with phenomena that bear analogies to electromagnetically induced transparency (EIT) and can be observed in an atomic system [16]. However, in an atomic EIT system, a pump beam is used to couple two energy levels, as opposed to spatial coupling between the radiative and dark atoms in the metamaterial EIT configuration.

Here we report a completely new approach. Specifically, we investigate a unit cell that sustains an antisymmetric resonance of two coupled dark modes. This leads to resonances with highly suppressed radiation losses and, hence, extraordinarily large $Q$ factors. The interaction between the two dark modes is mediated here via a conductive coupling scheme. As we will show, this conductive coupling is also very efficient in restoring the overall symmetry of the unit cell with respect to the illumination. This antisymmetric mode is to a large extent analogous to the antiphase eigenmode observed by Huygens in a classical mechanical arrangement [17]. In principle, a system of two pendulums joined together has two possible steady state eigenmodes: among the two, the quality factor in the case of the antiphase mode is higher than for the in-phase case [18]. This phenomenon is universal and has been observed in physics, chemistry, biology, and even social sciences [19]. A further advantage of the conductive coupling scheme is that it is extremely immune against any kind of imperfections, rendering the fabrication extremely robust and very controllable.

The specific implementation presented in this letter consists of a unit cell made from two SRRs (JSRRs) joined by a microstrip connecting line. It forces a conductive coupling regime between the two SRRs, basically leading to the formation of a supercell. Here, the coupling is clearly much stronger than the conventional inductive and/or capacitive case. When the composite structure is excited with the electric field perpendicular to the gaps of the SRRs, a novel high $Q$ resonance is excited. At the frequency of this resonance, the induced currents in each SRR resemble that of the fundamental resonance of the individual SRR. At this fundamental resonance the currents in each SRR oscillate out of phase. This is by all means surprising since the SRR is mirror symmetric with respect to the excitation and, thus, this fundamental resonance is not expected to be excited by the specific polarization; i.e., the fundamental mode is dark. Traditionally, the fundamental resonance can only be present in an isolated SRR if excited with an electric field parallel to the gaps of the SRRs. In order to excite an eigenmode with an electric field perpendicular to the gap, a gentle break in symmetry via the connecting wire has been employed. Most noticeably, the overall symmetry is only restored by the presence of the microstrip connecting line. It couples the split rings in an antisymmetric fashion such that the dark modes in each SRR oscillate ($\pi$) out of phase. The overall current flowing in the supercell, which can be directly driven by the external illumination, is therefore symmetric and matches the excitation. This suggests that the new resonance is different in nature from the well-known fundamental resonance associated to a single SRR excited by an electric field parallel to the gap.

The geometrical layouts of the traditional SRRs and JSRRs are depicted as insets in Figs. 1(a) and 1(b), respectively. In the JSRRs configuration, two neighboring SRRs are connected electrically, and hence they are engaged via conductive coupling. The dimensions of the SRRs used here are a side length of $l = 24$ $\mu$m, a width of $w = 3$ $\mu$m, a gap of $g = 3$ $\mu$m, and lattice constants $a_x = 140$ $\mu$m and $a_y = 70$ $\mu$m, respectively. In the case of JSRRs, the microstrip line has a length of $2d = 46$ $\mu$m, where $d$ is the distance from the edge of each SRR to the

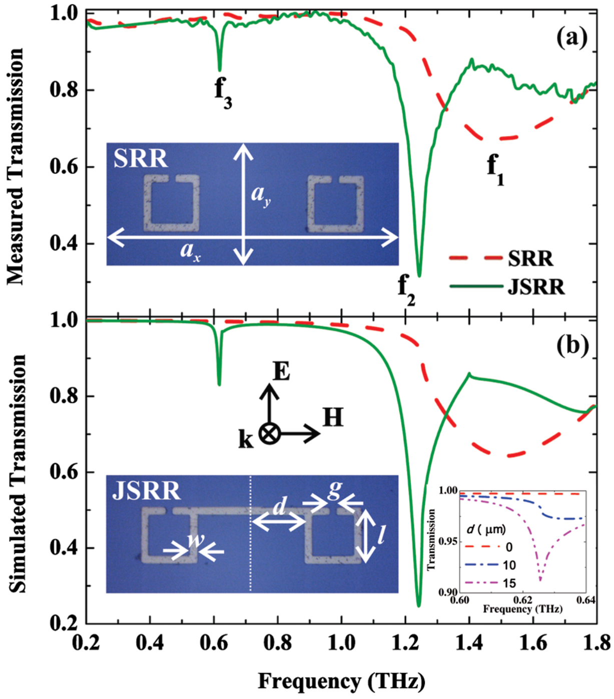

FIG. 1 (color online). (a) Measured and (b) simulated transmission amplitudes for the SRR and JSRR structures, respectively. The picture insets show microscopy images of the SRR and JSRR structures with the relevant dimensions, for an excitation featured by the $E$-field oscillating perpendicular to the gaps. The right bottom inset shows the new resonance, which is emerging when we extend the strip lines from both sides of the coupled SRRs. More details are available in the Supplemental Material [21] in Fig. S3.

middle of the unit cell. The arrows in the inset of Fig. 1(b) depict the polarization of the electric field illumination, which is perpendicular to the gaps of the SRRs. The structures are fabricated on a thick high resistivity silicon substrate with a refractive index of $n = 3.42$. The metallization consists of a thin 15 nm chrome buffer layer and of 200 nm of gold deposited on top. A conventional terahertz time domain spectroscopy setup has been used to perform the measurements presented here [20]. It is based on Ti: sapphire laser driven photoconductive antennas for both emission and detection of the terahertz pulses. After a measurement scan was performed, a bare Si wafer was measured as a reference and the transfer function was calculated as the ratio of the corresponding Fourier spectra. All simulations were carried out using the frequency domain solver of the commercial simulation software CST MICROWAVE STUDIO.

Figure 1 shows the measured and simulated transmission spectra for the traditional SRRs (dashed lines) and joint SRRs (solid lines) structures. The measurements are in excellent agreement with the simulations. Traditional SRRs exhibit a very broad dipole resonance at $f_1 = 1.5$ THz. The bandwidth (BW) at FWHM is 514 GHz and hence the $Q$ factor ($f_r$/BW) is 2.9. The $Q$ factor is small since the net electric dipole moment is large (as shown in Fig. S1 in the Supplemental Material [21]) and the mode can be properly excited with free space radiation. We notice that the dipole resonance of the JSRRs at $f_2$ is redshifted to 1.24 THz with a bandwidth of 184 GHz and a $Q$ factor of 6.7. More interestingly, as previously anticipated, a new, very sharp resonance is excited at $f_3 = 0.617$ THz. The bandwidth of this resonance is only 15 GHz, and therefore the $Q$ factor is 41. It is more than an order of magnitude higher than that of the fundamental resonance $f_1$ of the SRRs. To identify the polarization dependence we have studied the transmission of both structures with the electric field being polarized horizontally along the gaps of the SRRs. Here, the inductive-capacitive ($LC$) resonance of the SRRs is excited. In this case, large quality factors were not observed. In such a configuration the radiation losses are too high, which in turn prevents the occurrence of sharp resonances. The $Q$ factors of the $LC$ mode for the SRRs and the JSRRs are 10.25 and 2.5, respectively. This corresponds to only one fourth of the value for the other polarization in the case of the JSRRs configuration.

In order to understand the physical mechanism behind these spectral responses in greater detail, we have calculated the current distributions at the resonance for both structures at the three resonance frequencies, $f_1$, $f_2$, and $f_3$. They are shown in Fig. 2. The surface current distribution of the traditional SRRs at the dipole frequency $f_1$ [Fig. 2(a)] clearly shows an in-phase excitation in both arms of the two individual SRRs. Currents are identical in both unit cells, indicating that any possible capacitive or inductive coupling is negligible for such large distances. In essence, the resonances are strongly coupled to free space, in turn leading to very high radiation losses and very low $Q$ factors. At the dipole frequency $f_2$ of the JSRRs, the current is characterized by three nodes along the entire structure. Although the incident polarization is such that only the currents in the side arms of the SRRs can be directly driven, currents leak out into the central microstrip connecting line [Fig. 2(b)]. This causes an overall deformation of the current distribution also inside each individual SRR, making it possible for a fraction of the current to flow in the bottom arm of the SRRs. The current in one half of the supercell has to be necessarily compensated by the current flowing in the opposite direction in the other half of the supercell. This is required to ensure the overall symmetry of the geometry with respect to the illumination. But since the field distribution of the resonance now possesses components that cannot radiate into the far field due to symmetry constraints, the radiation losses of the entire structure are smaller when compared to the isolated SRRs. This is for obvious reasons beneficial towards reducing the line width of the resonance. According to the usual classification of the resonances sustained by SRRs, we identify this as a third order resonance with respect to the supercell; i.e., the current in the entire structure has three amplitude nodes. Moreover, multipole analysis has been carried out and is presented in the Supplemental Material (Fig. S1) [21]. Such analysis shows that the individual traditional SRR configuration response is dominated by the electric dipole moment. However, the response of the new resonance characteristic of the joint SRR structure has contributions from an electric dipole moment, necessary to excite such a mode, together with a magnetic and an electric quadrupole moment. Furthermore, a phenomenological simplified model has been introduced in the Supplemental Material [21] and the results support the interpretation above (Fig. S2).

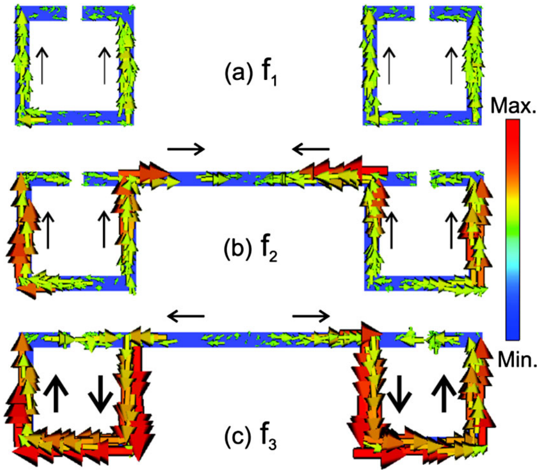

FIG. 2 (color online). Current distribution for (a) the dipole resonance for the SRRs structure at $f_1 = 1.5$ THz, (b) the dipole resonance for the JSRRs structure at $f_2 = 1.24$ THz, and (c) the new resonance for the JSRRs structure at $f_3 = 0.617$ THz.

From symmetry considerations it is clear that the structure should support also an additional lower order mode that is characterized by only a single amplitude node in the current. This mode corresponds exactly to the sharp resonance at $f_3$, as shown in Fig. 2(c). A single node in the current is observed in the vertical plane of the supercell (which is symmetric with respect to the middle of the microstrip line). The current distribution in each SRR corresponds to that of the ordinary *LC* mode, which cannot be excited if the incident electric field is polarized perpendicular to the gap, as previously mentioned. Therefore, the resonance we excite in the individual SRRs is dark. This excitation is only possible because of a small break in symmetry due to the presence of the microstrip connecting line. Nonetheless, it is even more important to notice that the currents [Fig. 2(c)] in the two SRRs forming the supercell flow ($\pi$) out of phase. Since radiation into the far field is mediated only by the SRRs but not by the central connecting wire element due to symmetry considerations (as only currents that are parallel to the incident electric field will radiate into the far field), the resonance has a strongly reduced net electric dipole moment. This drastically suppresses the radiation losses and raises the $Q$ factor dramatically. To a certain extent these reduced radiation losses come at the expense of a lower excitability, but the resonance itself is well established and can be clearly observed. We note that similar current distributions have been observed very recently in 3D structures at microwave frequencies [26].

Having discussed the transmission response and the current distributions of the JSRRs, we will now briefly evaluate the effect of the length of the microstrip connecting line. To better visualize the results, the transmission at resonance and the $Q$ factors as a function of the length of the microstrip connecting line have been extracted by sweeping the length of the connecting line ($d_x = 2d$) from 6 to 84 $\mu$m in steps of 6 $\mu$m each. The results are shown in Fig. 3. Increasing the length lowers the coupling in a very deterministic and well controllable manner. It can be seen that a certain minimum length of the connecting line is required to observe the new resonance. Once established, the $Q$ factor significantly rises with increasing length. For the present geometry, we observe an optimal response that occurs at a length of approximately 42 $\mu$m. Increasing the length even further causes a decline of the $Q$ factor that might look on a first glance surprising. Nevertheless, it has to be kept in mind that essentially the analyzed structure is periodic. For excessively long connecting lines the SRRs in adjacent supercells tend to couple to each other. This intercell coupling increases nonradiative losses, which not only lower the transmission but also broaden the resonance, hence resulting in lower $Q$ factors. Further analysis and simulations have been included in the Supplemental Material [21] in order to explain the steps used in the design of the joint SRR configuration as shown in Fig. S3. They demonstrate how adding a little microstrip line to an individual SRR leads to the excitation of a new eigenmode. However, it is quite weak in terms of sharpness and depth. Furthermore, our numerical analysis reveals how coupling via the microstrip line between the joint SRRs enhances both the sharpness and the depth of the resonance drastically.

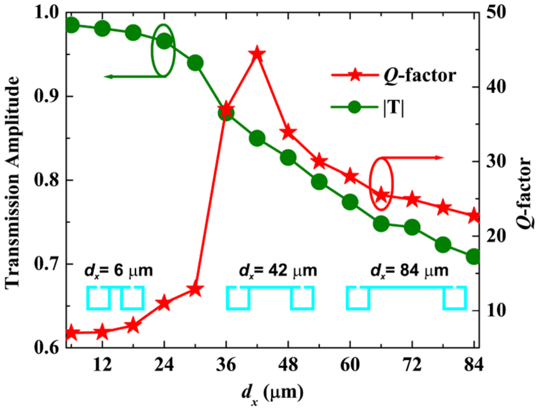

FIG. 3 (color online). Transmission level (left scale) and $Q$ factor (right scale) obtained by varying the length of the microstrip line $d_x$.

To conclude, we proposed a new unit cell for a MTM that supports extremely sharp resonances. The unit cell consists of two conductively coupled SRRs in broad analogy to the double-pendulum Huygens system, since each pendulum oscillation can be compared to the dipole mode of an individual SRR. The double SRRs are separated by a significant distance (in relation to the proposed geometry). Very high $Q$ factors as large as 41 were experimentally observed and numerically confirmed. The specific mode that allows the observation of such sharp resonances has been explored in detail. We have shown that it is based on the asymmetric coupling of two dark modes that are sustained by the individual SRRs. The MTMs proposed and analyzed here can be used in various applications, e.g., for the realization of slow-light structures and artificial delay lines. Finally, the design principles we have introduced can be applied to other geometries of MTMs and can be also implemented over a broader frequency domain.

The authors would like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC) for the financial support. I. A. would like to thank Dr. Alessandro Salandrino for useful discussions. C. R. and F. L. would like to thank the German Federal Ministry of Education and Research (project PhoNa).

---

[*] Corresponding author.
ibraheemalnaib@gmail.com
Present address: Department of Physics, Engineering Physics and Astronomy, Queen's University, Kingston, ON K7L 3N6, Canada

[1] H.-T. Chen, W. J. Padilla, J. M. O. Zide, A. C. Gossard, A. J. Taylor, and R. D. Averitt, Nature (London) **444**, 597 (2006).

[2] V. M. Shalaev, Nat. Photonics **1**, 41 (2007).

[3] H.-T. Chen, J. F. O'Hara, A. K. Azad, A. J. Taylor, R. D. Averitt, D. B. Shrekenhamer, and W. J. Padilla, Nat. Photonics **2**, 295 (2008).

[4] B. Luk'yanchuk, N. I. Zheludev, S. A. Maier, N. J. Halas, P. Nordlander, H. Giessen, and C. T. Chong, Nat. Mater. **9**, 707 (2010).

[5] H. O. Moser, and C. Rockstuhl, Laser Photonics Rev. **6**, 219 (2012).

[6] E. Shamonina, V. A. Kalinin, K. H. Ringhofer, and L. Solymar, J. Appl. Phys. **92**, 6252 (2002).

[7] N. Liu, S. Kaiser, and H. Giessen, Adv. Mater. **20**, 4521 (2008).

[8] J. F. O'Hara, R. Singh, I. Brener, E. Smirnova, J. Han, A. J. Taylor, and W. Zhang, Opt. Express **16**, 1786 (2008).

[9] I. A. I. Al-Naib, C. Jansen, and M. Koch, Appl. Phys. Lett. **93**, 083507 (2008).

[10] H.-T. Chen, J. F. O'Hara, A. K. Azad, and A. J. Taylor, Laser Photonics Rev. **5**, 513 (2011).

[11] H. Tao, L. R. Chieffo, M. A. Brenckle, S. M. Siebert, M. Liu, A. C. Strikwerda, K. Fan, D. L. Kaplan, X. Zhang, R. D. Averitt, and F. G. Omenetto, Adv. Mater. **23**, 3197 (2011).

[12] I. Al-Naib, R. Singh, C. Rockstuhl, F. Lederer, S. Delprat, D. Rocheleau, M. Chaker, T. Ozaki, and R. Morandotti, Appl. Phys. Lett. **101**, 071108 (2012).

[13] J. F. O'Hara, W. Withayachumnankul, and I. Al-Naib, J. Infrared Millimeter Waves **33**, 245 (2012).

[14] H. Tao, A. C. Strikwerda, M. Liu, J. P. Mondia, E. Ekmekci, K. Fan, D. L. Kaplan, W. J. Padilla, X. Zhang, R. D. Averitt, and F. G. Omenetto, Appl. Phys. Lett. **97**, 261909 (2010).

[15] V. A. Fedotov, M. Rose, S. L. Prosvirnin, N. Papasimakis, and N. I. Zheludev, Phys. Rev. Lett. **99**, 147401 (2007).

[16] S. Zhang, D. A. Genov, Y. Wang, M. Liu, and X. Zhang, Phys. Rev. Lett. **101**, 047401 (2008); R. Singh, I. A. I. Al-Naib, Y. Yang, D. R. Chowdhury, W. Cao, C. Rockstuhl, T. Ozaki, R. Morandotti, and W. Zhang, Appl. Phys. Lett. **99**, 201107 (2011).

[17] T. Birch, *The History of the Royal Society of London for Improving of Natural Knowledge, from Its First Rise* (Gale ECCO, London, 1756), Vol. 2, p. 19.

[18] L. P. Pook, *Understanding Pendulums: A Brief Introduction* (Springer, New York, 2011), 1st ed., p. 49.

[19] M. Rosenblum, and A. Pikovsky, Contemp. Phys. **44**, 401 (2003).

[20] D. Grischkowsky, S. Keiding, M. van Exter, and Ch. Fattinger, J. Opt. Soc. Am. B **7**, 2006 (1990).

[21] See Supplemental Material, which includes Refs. [22–25], at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.183903 for details concerning multipole analysis, the phenomenological model, and a step-by-step design procedure.

[22] S. Mühlig, C. Menzel, C. Rockstuhl, and F. Lederer, Metamaterials **5**, 64 (2011).

[23] C. Rockstuhl, C. Menzel, S. Mühlig, J. Petschulat, C. Helgert, C. Etrich, A. Chipouline, T. Pertsch, and F. Lederer, Phys. Rev. B **83**, 245119 (2011).

[24] C. Menzel, T. Paul, C. Rockstuhl, T. Pertsch, S. Tretyakov, and F. Lederer, Phys. Rev. B **81**, 035320 (2010).

[25] T. Paul, C. Menzel, W. Śmigaj, C. Rockstuhl, P. Lalanne, F. Lederer, Phys. Rev. B **84**, 115142 (2011).

[26] V. A. Fedotov, A. V. Rogacheva, V. Savinov, D. P. Tsai, and N. I. Zheludev, Sci. Rep. **3**, 2967 (2013).