Quantifying Einstein-Podolsky-Rosen Steering

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Einstein-Podolsky-Rosen steering is a form of bipartite quantum correlation that is intermediate between entanglement and Bell nonlocality. It allows for entanglement certification when the measurements performed by one of the parties are not characterized (or are untrusted) and has applications in quantum key distribution. Despite its foundational and applied importance, Einstein-Podolsky-Rosen steering lacks a quantitative assessment. Here we propose a way of quantifying this phenomenon and use it to study the steerability of several quantum states. In particular, we show that every pure entangled state is maximally steerable and the projector onto the antisymmetric subspace is maximally steerable for all dimensions; we provide a new example of one-way steering and give strong support that states with positive-partial transposition are not steerable.

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Quantum systems display correlations that do not have a counterpart in classical physics. Schrödinger noticed the following consequence of these stronger-than-classical correlations which is now known as Einstein-Podolsky-Rosen (*EPR*) *steering* [1]: Two parties, Alice and Bob, share an entangled state $|\psi_{AB}\rangle$. By measuring her subsystem, Alice can remotely change (i.e., steer) the state of Bob's subsystem in such a way that would be impossible if their systems were only classically correlated. The simplest example of steering is given by the maximally entangled state of two qubits $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Alice can project Bob's system into the basis $\{|a\rangle, |a_\perp\rangle\}$ by making a measurement of her subsystem in the conjugate basis $\{|a\rangle^*, |a_\perp\rangle^*\}$. This feature would be impossible if they shared separable states.

EPR steering was recently given an operational interpretation as the distribution of entanglement by an untrusted party [2]: Alice wants to convince Bob, who does not trust her, that they share an entangled state. Bob, in order to be convinced, asks Alice to remotely prepare a collection of states of his subsystems. Alice performs her measurements (which are unknown to Bob) and communicates the results to him. By looking at the conditional states prepared by Alice, Bob is able to certify if they must have come from measurements on an entangled state. Interestingly, EPR steering is a form of quantum correlation that lies in between entanglement [3] and Bell nonlocality [4], since, on the one hand, not every entangled state is steerable, and, on the other hand, some steerable states do not violate a Bell inequality [2]. Furthermore, similarly to nonlocality, steering can be demonstrated in simple experimental tests through the violation of steering inequalities [5]. In fact, several steering tests have been reported [6,7], including a loophole-free experiment [8].

Apart from the fundamental interest in steering, there is also an applied motivation for studying and implementing it: Steering allows for quantum key distribution (QKD) when one of the parties cannot trust their devices [9]. This result opens a new venue for information-theoretic tasks based on EPR steering that are naturally suited to scenarios where only one party has trust of their device. One big advantage in this direction is that such scenarios are experimentally less demanding than fully device-independent protocols (where both of the parties distrust their devices) [10] and, at the same time, require fewer assumptions than standard QKD scenarios.

Although our understanding of EPR steering has advanced greatly recently, a fundamental question remains open: how to quantify it? Given that a quantum state can be used to demonstrate EPR steering, how "steerable" is it? In the present Letter, we introduce an operationally motivated method to quantify EPR steering of arbitrary finite-dimensional bipartite quantum states. Our quantifier can be calculated by semidefinite programming, allowing one to explore a wide variety of quantum states and measurement scenarios.

We calculate our quantifier for several states of interest in quantum information: entangled pure states, Werner and isotropic states, and bound entangled states (with positive partial transposition). Several interesting results follow from our analysis, such as (i) every entangled pure state is maximally steerable, (ii) the maximally entangled version of Werner states (i.e., the state described by the normalized projector onto the antisymmetric subspace) is maximally steerable, even though in dimensions larger than 2 it is not known to violate any Bell inequality, (iii) we exhibit a new example of one-way EPR steering [11], and (iv) we provide further numerical evidence that bound entangled states are not steerable, hence supporting the extended Peres conjecture [12] recently investigated in Ref. [13]. Finally, we demonstrate the power of using random measurements for steering detection—in some cases, they are more useful than maximally noncommuting observables (mutually unbiased bases).

EPR steering.—Let us begin by describing in more detail the basic setup of a steering scenario. Consider two parties, Alice and Bob, performing measurements on a bipartite state. We will assume that we have no knowledge on the actual implementation of Alice's measurements. All that is assumed is that she can choose to perform one measurement from a set of m choices, each of which has n possible outcomes. On the other hand, Bob's measurements are fully characterized. Thus, he is able to do complete state tomography and give an exact quantum description of his system.

A steering experiment can therefore be completely characterized by giving an "assemblage" $\{\sigma_{a|x}\}_{ax}$, the set of unnormalized states which Alice steers Bob into, given her choice of measurement x and outcome a. The assemblage encodes both Alice's conditional probability distribution of her outcomes given her inputs, $P(a|x) = \mathrm{tr}(\sigma_{a|x})$, as well as the conditional states prepared for Bob given Alice's input and outcome, $\hat{\sigma}_{a|x} = \sigma_{a|x}/P(a|x)$. All valid assemblages satisfy the consistency requirements

$$\sum_a \sigma_{a|x} = \sum_a \sigma_{a|x'} \qquad \forall\ x \neq x', \qquad \mathrm{tr}\sum_a \sigma_{a|x} = 1, \quad (1)$$

which encode the facts that Alice cannot signal to Bob and that, without any knowledge about Alice, Bob still holds a valid quantum state. We denote this set of valid assemblages as Σ^S.

In this scenario there is the set of "uninteresting" assemblages, which we shall denote the *unsteerable* assemblages Σ^{US}. These assemblages are those which can be created via classical strategies (i.e., without using entanglement) and can be written in the following form (see Supplemental Material [14]):

$$\sigma_{a|x} = \sum_\lambda D_\lambda(a|x)\sigma_\lambda \qquad \forall\ a, x$$
$$\text{such that } \mathrm{tr}\sum_\lambda \sigma_\lambda = 1, \qquad \sigma_\lambda \geq 0 \qquad \forall\ \lambda, \quad (2)$$

where λ is a (classical) random variable held by Alice, $D_\lambda(a|x)$ are (the extremal) deterministic single-party conditional probability distributions for Alice [i.e., the $D_\lambda(a|x)$ are the deterministic functions from the alphabet of x to the alphabet of a; when there are m inputs and n outcomes, there are precisely n^m such deterministic functions, and hence this is the size of the alphabet of λ], and σ_λ are the states held by Bob. A model (2) is called a *local hidden state* (LHS) model. Any assemblage that cannot be written in the form (2) constitutes a genuine resource in a steering scenario and is called *steerable*. The *steerability* of an assemblage can be demonstrated by the violation of steering inequalities [5].

Given an assemblage $\{\sigma_{a|x}\}_{a,x}$, it is possible to test if it is within the set of unsteerable assemblages, i.e., if $\{\sigma_{a|x}\}_{ax} \in \Sigma^{\mathrm{US}}$, with the following feasibility semidefinite program (SDP) [13]:

$$\text{find } \{\sigma_\lambda\}_\lambda$$
$$\text{such that } \sum_\lambda D_\lambda(a|x)\sigma_\lambda = \sigma_{a|x} \qquad \forall\ a, x,$$
$$\mathrm{tr}\sum_\lambda \sigma_\lambda = 1, \qquad \sigma_\lambda \geq 0 \qquad \forall\ \lambda. \quad (3)$$

In words, if one is able to find a set of positive semidefinite matrices $\{\sigma_\lambda\}_\lambda$ which satisfy all of the above constraints, then the assemblage is unsteerable. Otherwise, it is steerable.

Quantifying EPR steering.—The main result of this Letter is to present an operationally motivated way to measure steerability, which we shall term the *steerable weight*. We will show that this quantity is given by a SDP, which will allow us to calculate it for a wide range of steering scenarios.

The main idea behind the steerable weight is the following. We imagine that Alice, in preparing a given assemblage $\{\sigma_{a|x}\}_{a,x}$, will try to minimize the number of uses of a genuine steerable resource—that is, she will prepare as frequently as possible an unsteerable assemblage $\{\sigma^{\mathrm{US}}_{a|x}\}_{a,x}$ having a decomposition (2) but also sometimes prepare a genuine steerable assemblage $\{\sigma^S_{a|x}\}_{a,x}$, such that on average she prepares the desired assemblage. That is, we decompose the assemblage as

$$\sigma_{a|x} = \mu\sigma^{\mathrm{US}}_{a|x} + (1-\mu)\sigma^S_{a|x} \qquad \forall\ a, x, \qquad 0 \leq \mu \leq 1. \quad (4)$$

We then ask for the maximum μ, denoted by μ^*, for which we can find such a decomposition. The steerable weight is then defined as $\mathrm{SW} = 1 - \mu^*$, i.e., the minimal amount of genuine steerable resource required to reproduce the given assemblage.

As we show in Supplemental Material [14], μ^* is given by the solution to the following SDP:

$$\max\ \mathrm{tr}\sum_\lambda \sigma_\lambda$$
$$\text{such that } \sigma_{a|x} - \sum_\lambda D_\lambda(a|x)\sigma_\lambda \geq 0 \qquad \forall\ a, x,$$
$$\sigma_\lambda \geq 0 \qquad \forall\ \lambda. \quad (5)$$

This is crucial, as efficient numerical algorithms to evaluate SDPs are available. Furthermore, the dual of program (5), given by

$$\min\ \mathrm{tr}\sum_{ax} F_{a|x}\sigma_{a|x}$$
$$\text{such that } \mathbb{1} - \sum_{ax} D_\lambda(a|x)F_{a|x} \leq 0 \qquad \forall\ \lambda,$$
$$F_{a|x} \geq 0 \qquad \forall\ a, x, \tag{6}$$

provides an additional operational meaning to the steerable weight (Supplemental Material [14])—as the minimal possible violation of any linear steering inequality (given here by the $F_{a|x}$) which takes only positive values, and for which all unsteerable assemblages achieve the minimum value of 1. Thus, given an assemblage, the solution of (6) provides an optimal linear steering inequality to test its steerability.

Steerable weight of quantum states.—In what follows, we compute the steerable weight for several examples of quantum states and measurements.

(i) *Pure entangled states.* In Supplemental Material [14], we show that every pure entangled state is *maximally steerable*. This can be shown by appealing to the dual characterization of the steerable weight as given by (6). In particular, we show that, by performing two suitably chosen von Neumann measurements, Alice can create an assemblage for Bob which maximally violates an appropriately defined steering inequality and that this implies SW = 1.

(iia) 2×2 *Werner states.*—Next, we consider the two-qubit Werner state $\rho = p|\psi^-\rangle\langle\psi^-| + (1-p)\mathbb{1}_2 \otimes \mathbb{1}_2/4$, where $|\psi^-\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$ is the singlet state and $\mathbb{1}_n$ is the n-dimensional identity operator [15]. We first consider the assemblage created when Alice performs measurements of the three Pauli operators X, Y, and Z. We find that for the singlet ($p = 1$), the assemblage is maximally steerable [in accordance with (i) above]. As p decreases, we find as expected that the steerable weight decreases monotonically and, furthermore, that the assemblage becomes unsteerable (i.e., has SW = 0) exactly when $p = 1/\sqrt{3}$, coinciding with the point where the singlet stops violating the steering inequality $\langle XX\rangle + \langle YY\rangle + \langle ZZ\rangle \leq \sqrt{3}$ [5]. Consider now that Alice chooses a given number k of random measurements; i.e., she chooses k directions at random on the Bloch sphere to measure along. For $k = 4$–10, we sampled over 1000 randomly generated assemblages for various values of p. In Fig. 1, we show, as a function of p, the largest steerable weight among the randomly generated assemblages, for each k. First, for all p we see that (except for the end points where the assemblage is either maximally steerable or completely unsteerable) as k increases, so does the steerable weight. Furthermore, we see that we can demonstrate steerability for Werner states with $p < 1/\sqrt{3}$ as we increase k, surpassing the limit for three measurement steering inequalities and approaching the $p = 1/2$ steerable limit calculated in Ref. [2]. Finally, for the case of ten measurements we also give, as insets, the distribution of steerable weight over the 1000 random assemblages for different values of p. We observe that as p increases the distribution of steerable weights becomes increasingly peaked around the maximum value, indicating that random measurements become increasingly effective in this regime.

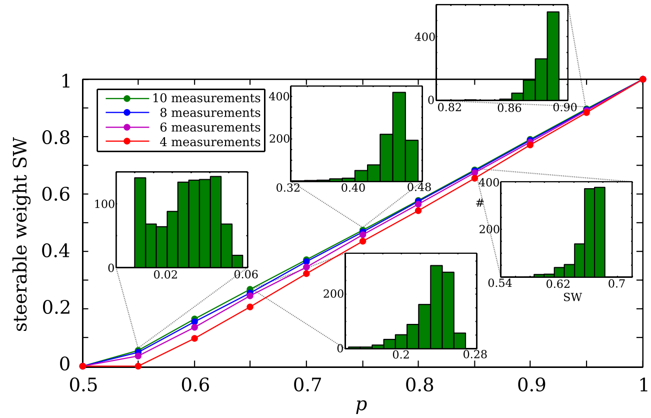

FIG. 1 (color online). Plot of steerable weight against parameter p of two qubit Werner states, for varying numbers of random measurements. For each value of p and a given number of measurements, we plot the biggest steerable weight among 1000 randomly generated assemblages. Inset: For the case of ten measurements, we also show the distribution of steerable weights over the ensemble of assemblages.

(iib) *d-dimensional Werner states.*—We now consider the steerable weight of arbitrary dimension Werner states, defined as a convex combination of the normalized projector onto the antisymmetric subspace (A_d) and the normalized identity in $\mathbb{C}^d \otimes \mathbb{C}^d$ [15]:

$$\rho_W^d = \eta\frac{A_d}{N_A} + (1-\eta)\frac{\mathbb{1}_{d^2}}{d^2}, \tag{7}$$

where $N_A = \mathrm{tr}(A_d) = d(d-1)/2$. This state is steerable (for projective measurements) if and only if $\eta > 1 - 1/d$ [2]. Curiously, for $d \geq 3$ no Bell inequality violation is known for this state (notice that it has a local hidden variable model for projective measurements if $\eta \leq 1 - 1/d$ [15]). Using the steerable weight, we find that for $d = 3$ measuring mutually unbiased bases (MUBs) generates unsteerable assemblages (see Supplemental Material [14]), while for $d = 4$ measuring MUBs demonstrates maximal steering (with SW = 1). However, if Alice performs d random measurements onto the d-dimensional Werner state with $\eta = 1$, she always produces a maximally steerable assemblage to Bob (i.e., SW = 1)—see the demonstration in Supplemental Material [14]. This is interesting for numerous reasons. First, it contradicts the intuition that maximally noncommuting observables are the best candidates for demonstrating steering and shows the power of randomly chosen measurements. Second, it demonstrates the existence of maximally steerable mixed states. Finally, since no Bell violation is known for Werner states with $d \geq 3$, they are good candidates for states which are maximally steerable yet Bell local.

(iii) *Erasure state and one-way steering.*—We now consider the qubit erasure state ρ_p^{er}:

$$\rho_p^{\mathrm{er}} = p|\psi^-\rangle\langle\psi^-| + (1-p)|2\rangle\langle 2| \otimes \mathbb{1}/2, \tag{8}$$

so called as it can be produced by sending Alice's half of a singlet through an erasure channel with parameter p, where $|2\rangle$ is the flag state. The erasure state has, for $p \leq 1/k$, a k-symmetric extension [16] on Alice's side. As we show in Supplemental Material [14], similarly to the case of nonlocality [17], it follows that the assemblages created when Alice performs k or less measurements [including positive-operator-valued-measure (POVM) measurements with an arbitrary number of outcomes] are unsteerable. However, on the contrary, if we send Bob's qubit through the erasure channel, so that he holds the flag, we find that the state is steerable for all $p \neq 0$ and that this can be demonstrated with only two projective measurements for Bob. Thus, for any arbitrary number k of POVM measurements for Alice, the erasure state with $p = 1/k$ is an example of a state which is unsteerable from Alice to Bob but steerable from Bob to Alice, with only the need for two measurements on Bob. This example complements the first demonstration of one-way steering presented in Ref. [11], where an example was given which works for projective measurements on Alice (including the case of infinitely many measurements) and requires 13 measurements for Bob.

(iv) *Bound entangled states.*—Finally, we can use our quantifier to gather evidence on the Peres conjecture, that no bound entangled state can violate a Bell inequality. Since steering is a form of quantum correlation which is easier to demonstrate than nonlocality, the steerability of bound entangled states may shed light on whether one may expect them to be nonlocal also. In particular, if it is the case that bound entangled states are unsteerable, then it immediately follows that they can never produce nonlocal correlations. Here we provide further numerical evidence of this fact, complementing the recent numerical evidence given in Ref. [13].

We have considered MUBs, spin, and random measurements applied to several families of bound entangled states and could not find a single instance where steering is observed. The families of states we have explored are the (a) 3×3 unextendible product basis states [18]; (b) both the 3×3 Horodecki states [19,20]; (c) the family of (4,4) edge states [here, (4,4) refers to the fact that the state is rank 4 and the partial transpose is rank 4, respectively] of Ref. [21]; (d) the (5,5) edge state of Ref. [22]; (e) the family of (5,5) edge states of Ref. [21]; (f) the (6,6) edge state of Ref. [23]; (g) the max realignment state of Ref. [22]; and (h) the family of Bell diagonal states from Ref. [24] for $d = 3$ and 4.

In all cases, we concentrated only on cases were Alice has as many measurements as computationally feasible for the collection of statistics (in this case, six measurements). After sampling 1000 times in each case, we were unable to produce a single assemblage which was steerable. Clearly, it remains to extend this approach by both collecting more data, with more measurements, and also considering more families of bound entangled states.

Comparison with entanglement and nonlocality.—As mentioned previously, steering can be seen as an intermediate scenario between the entanglement scenario and the Bell nonlocality scenario. In the former case, one trusts both parties and hence can give an exact and complete quantum description of the state ρ_{AB} held by Alice and Bob. In the latter case, one does not trust either party and has access only to the measured statistics $P(ab|xy)$ related to measurement choices x and y of Alice and Bob and the corresponding outcomes a and b.

The entanglement problem refers to deciding if a given state ρ_{AB} is separable, i.e., admits a decomposition of the form $\rho_{AB} = \sum_\lambda p(\lambda)\sigma_A^\lambda \otimes \sigma_B^\lambda$, where $p(\lambda)$ is a probability distribution over the shared random variable λ and σ_A^λ and σ_B^λ are states for Alice and Bob, respectively. In the nonlocality case, one is interested in deciding if a given probability distribution is local, i.e., if it admits a decomposition of the form $P(ab|xy) = \sum_\lambda p(\lambda)P_\lambda(a|x)P_\lambda(b|y)$, where $p(\lambda)$ is a probability distribution over the shared random variable λ and $P_\lambda(a|x)$ and $P_\lambda(b|y)$ are probability distributions for Alice and Bob, respectively.

In all three cases, there is a way to test whether the given state, assemblage, or probability distribution lies in the set of separable states, unsteerable assemblages, or local distributions, respectively. While we have seen that steerability can be decided by using a SDP, for the case of quantum states separability can be checked by membership in a convergent hierarchy of SDPs, checking for k symmetric extensions of the given state [16], and probability distribution membership within the set of local distributions can be checked by a linear program [4].

As far as quantification is concerned, entanglement and nonlocality can also be measured by finding optimal decompositions minimizing the weight on the "expensive" part. This is the so-called *best separable approximation* (BSA) of entangled states [25] and the EPR2 decomposition of probability distributions [26]. Our results suggest that the steerable weight sometimes behaves as the BSA and sometimes as the EPR2 decomposition. For instance, every entangled two-qubit pure state is maximally entangled according to the BSA, while it is not maximally nonlocal according to the EPR2 [27,28]. Another (possible) difference with nonlocality is the fact that the 3×3 Werner state is steerable, while its nonlocality properties are still unknown. On the other hand, bipartite bound entangled states are conjectured to be local states (i.e., with zero nonlocality according to the EPR2 decomposition). Here we find evidence that this is also true for EPR steering.

Finally, notice that, although random measurements can also be used to detect nonlocality [29], they are not known to provide any advantage over MUBs in this case. As we have seen, random measurements can detect (even maximal) steering for cases where MUBs are useless. Furthermore, they allow us to detect the steering of two-qubit Werner states very close to their LHS limit of $p = 1/2$, then providing an interesting and scalable alternative to the previous measurement strategy based on Platonic solids [6].

Conclusion.—In this Letter, we have proposed the first method to quantify the steering power of quantum states or, more precisely, of assemblages obtained by measurements on quantum states. This quantifier can be calculated by using a SDP, which allowed us to estimate the steerable weight of several quantum states. We saw that the steerable weight behaves sometimes like the entanglement weight and some other times like the nonlocal weight. This confirms, in a quantitative way, that steering is an intermediate resource in between entanglement and nonlocality. Interestingly, we have seen that mutually unbiased bases are not always the best choice of measurements to demonstrate steering.

Our study motivates several open questions. Is it the case that bound entangled states are unsteerable? If this is the case, then the Peres conjecture would indeed be true. By using the insight that the Peres conjecture might be even stronger than previously anticipated, could this suggest alternative ways of looking for a proof? We know that nonlocality can be superactivated; is the same also true for steering? Finally, in this study we have highlighted the power of random projective measurements. Could it be the case that going beyond projective measurements to general POVM measurements could provide even stronger tests of steering?

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