Did BICEP2 See Vector Modes? First *B*-Mode Constraints on Cosmic Defects

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Scaling networks of cosmic defects, such as strings and textures, actively generate scalar, vector, and tensor metric perturbations throughout the history of the Universe. In particular, *vector* modes sourced by defects are an efficient source of the cosmic microwave background *B*-mode polarization. We use the recently released BICEP2 and POLARBEAR *B*-mode polarization spectra to constrain properties of a wide range of different types of cosmic strings networks. We find that in order for strings to provide a satisfactory fit on their own, the effective interstring distance needs to be extremely large—spectra that fit the data best are more representative of global strings and textures. When a local string contribution is considered together with the inflationary *B*-mode spectrum, the fit is improved. We discuss implications of these results for theories that predict cosmic defects.

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Observations of the cosmic microwave background (CMB) radiation have established a compelling case for an inflationary beginning of our Universe [1–6]. Inflation resolved the monopole, the flatness, and the horizon problems and, as a bonus, provided a mechanism for generating small fluctuations in the metric of space-time [7–13]. CMB temperature anisotropies measured by COBE, WMAP, and Planck are in spectacular agreement with the predictions of simplest inflationary models [14–16]. The same models also predict a scale-invariant spectrum of gravitational waves [7] that can be imprinted in the CMB temperature and polarization [17–19]. A smoking gun of the gravity waves is the so-called *B*-mode pattern of polarization [20–23]. Recent data from BICEP2 [24] provide tantalizing evidence for this signal at an amplitude that is consistent with predictions of the simplest inflationary models. The *B*-mode signal seen by BICEP2 can also contain contributions from other sources. The purpose of this Letter is to examine the implications of BICEP2 as well as the recently released POLARBEAR results [25] for another potential source of *B* modes: cosmic strings.

Particle theory suggests that the Universe went through a series of symmetry-breaking phase transitions as it expanded and cooled. Cosmic defects, such as monopoles, strings, domain walls, and textures, could form in these phase transitions and potentially survive until the time of last scattering and even today [26,27]. Cosmic strings were actively studied as an alternative to an inflation mechanism for generating the structure in the Universe [28]. Eventually, it became apparent that the CMB and matter power spectra predicted by cosmic strings were distinctly different from what was observed, and they have been ruled out as the main seed for structure formation [29]. However, in models with multiple scalar fields, strings can form at the end of inflation [30,31] and contribute a small amount of power to the CMB temperature anisotropy [32–36]. Such scenarios include supersymmetric grand unified models [37–39] and brane inflation [40–44].

Among the predicted signatures of cosmic defects is the CMB *B*-mode polarization on subdegree angular scales [45–53,55]. The nature of cosmological perturbations generated by defects is qualitatively different from those set by inflation. The latter sets the initial conditions for the metric and matter inhomogeneities that subsequently evolve unperturbed. Defects, on the other hand, actively generate scalar, vector, and tensor perturbations throughout the history of the Universe [46,56,57]. Because vector modes quickly decay when not actively sourced, they are completely negligible in the inflationary mechanism. But, for defects, they are comparable to scalar modes and can be an efficient source of the CMB *B*-mode polarization. Tensor modes, or gravity waves, are also produced by defects but with a lower impact on CMB because of their oscillatory nature [46].

In order for a string network to maintain scaling, long strings must chop off loops that subsequently radiate away. Pulsar timing measurements [58] and gravitational wave detectors [59] strongly constrain the amount of gravitation waves produced by loops of *local* cosmic strings [60–63], giving bounds much tighter than the current CMB constraints. However, the amount of the gravitational wave emission from string loops and kinks is not as established [36,64] as the effects of the large-scale dynamics of the string network on CMB. Also, the tight gravity wave bounds do not apply to *global* strings (we thank Alex Vilenkin for pointing this out), i.e., those formed as a result of spontaneously broken global (as opposed to local) gauge symmetries. We note that future *B*-mode experiments can come close to providing bounds [48,54,55] comparable to those from gravity wave probes.

In this Letter, we use the newly released BICEP2 and POLARBEAR data to constrain properties of a wide variety of cosmic string networks. We consider two cases: one in which there is no contribution to B modes from the inflationary gravity waves and one in which there is a mixture of the inflationary and string contributions. We provide quantitative answers to the following questions: (1) Can cosmic strings provide a good fit to the BICEP2 and POLARBEAR B-mode spectra without any contribution from inflationary tensor modes? (2) Is the fit improved by adding a cosmic string contribution to the inflationary B modes? (3) What are the implications of the new B-mode data for the properties of cosmic string networks?

In this Letter, we will refer to the tensor-to-scalar ratio r evaluated at the scale $k = 0.002\ \mathrm{Mpc}^{-1}$. To model the strings, we use the unconnected segment model (USM) [65–69], which offers the ability to mimic the CMB spectra from different types of strings. The USM model was introduced in Refs. [29,66], based on the approach suggested in Ref. [65], developed into its present form in Ref. [67], and implemented in a publicly available code CMBACT [68]. The string unequal time correlators of the USM model can be derived using analytical expressions developed in Ref. [69], which we use in this work.

In the USM, in addition to the dimensionless string tension $G\mu$, there are two important parameters, the scaling parameter ξ, which sets the effective interstring distance [$\xi \equiv La/\eta$, where L is the mean interstring distance (related to the string energy density via $\rho_s = \mu/L^2$) and η is the conformal time. We stress that ξ is an effective parameter in the USM model and $\xi > 1$ does not necessarily imply the presence of superhorizon correlations in the model whose spectra are reproduced by the USM], and the root-mean-square (rms) velocity v. On cosmological scales, probed by the CMB measurements, the fine details of the string evolution do not play a major role. It is the large-scale properties, such as the scaling distance and the rms velocity, that determine the shape of the string-induced spectra. The overall normalization of the spectrum depends on $G\mu$ as well as the string number density, controlled by ξ.

The advantage of working with the USM is that one can quickly scan over the spectra of many different types of cosmic defects to see if any of them happen to be favored by data. Of course, this requires that the USM is able to provide a satisfactory fit to the CMB spectra or, equivalently, to the stress-energy unequal time correlators (UETC), derived from available numerical simulations. For instance, it was shown in Ref. [36] that the USM can reproduce the CMB spectra derived from the simulations of local strings by Refs. [49,51,52]. Fits to UETC from simulations by other groups [70–72] can also be performed but are not available at this time.

The thin red short-dashed line in Fig. 1 shows a typical B-mode spectrum generated by local strings. It is primarily sourced by *vector* modes and has two peaks. The less prominent peak at $\ell \sim 10$ is due to the rescattering of photons during reionization, whereas the main peak, at higher ℓ, is the contribution from last scattering. Both peaks are quite broad because a string network seeds fluctuations over a wide range of scales at any given time. The position of the main peak is determined by the most dominant Fourier mode stimulated at last scattering, which is set by the values of ξ and v [50]. The power tends to move to lower multipoles (larger angular scales) when either v or ξ are increased. Increasing v also increases the width of the peak. In fact, because $v < 1$ sets a maximum scale, it takes a large increase in ξ to move the peak to the left (to lower ℓ) even by a small amount.

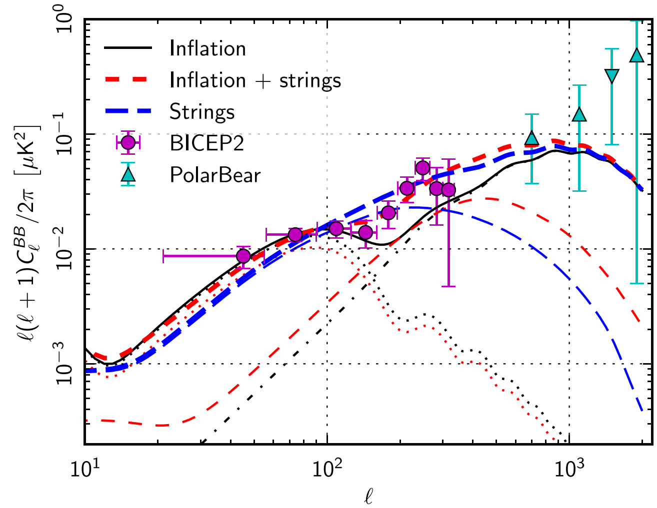

FIG. 1 (color online). The thick blue long-dashed line is the best-fit lensing + strings model ($r = 0$), with the thin blue long-dashed line showing the corresponding string contribution alone. The thick red short-dashed line is the best-fit lensing + strings + inflation model ($r = 0.15$), with the corresponding string contribution plotted as a thin red short-dashed line. The lensing contribution is shown separately with a thin black dot-dashed line. The BICEP2 best-fit inflationary model ($r = 0.2$) contribution is shown with a thin black dotted line, and the solid thin black line is the sum of $r = 0.2$ and lensing contributions. The circles show the band powers measured by BICEP2, and the triangles are the POLARBEAR data (the third band is negative with its absolute value plotted as an inverted triangle).

Let us briefly comment on how we quantity the string contribution to CMB. Bounds on cosmic strings are often quoted solely in terms of $G\mu$. Such bounds implicitly assume the scaling configuration of local strings in the Abelian Higgs model, where at any time there is roughly one Hubble length string per Hubble volume. More generally, the bound on strings depends on the combination of $G\mu$ and the string number density $N_s \propto \xi^{-2}$. [In the one-scale model, $N_s \propto \xi^{-3}$. However, the interstring distance, which can be very small in models with lower intercommutation probabilities, need not to be the same as the coherence scale along the string, which remains of $\mathcal{O}(H^{-1})$]. Typically, $\xi \lesssim 1$, but can be much smaller in models with lower intercommuting probabilities. Moreover, different types of observations probe different combinations of ξ and μ. As shown in Ref. [42], CMB power spectra (and other two-point correlation functions) constrain $\mu\sqrt{N_s} \sim \mu/\xi$, while gravity wave probes essentially constrain the string energy density given by μ/ξ^2. To avoid the model dependence when interpreting the CMB bounds in terms of $G\mu$, we follow the Planck Collaboration [73] and quantify the amount of the anisotropy contributed by strings in terms of f_{10}, which is the fractional contribution of strings to the CMB temperature spectrum at $\ell = 10$, $f_{10} \equiv C_{10}^{\rm str}/C_{10}^{\rm tot}$. The first year Planck data constrain it at $f_{10} \lesssim 0.03$ [73]. In this work, we do not fit to Planck data, instead focusing on the implications of the B-mode data alone. Values of f_{10} that exceed Planck bounds can be disregarded. We also ignore the small theoretical uncertainty involved in calculating the lensing contribution to the B-mode spectrum.

We first discuss how the string- *only* model compares to inflation. The blue solid lines in Fig. 2 show the marginalized likelihoods of f_{10}, ξ, and v obtained by fitting the string B-mode spectra, combined with lensing, to the BICEP2 and POLARBEAR data. There is a well-defined peak at $f_{10} = 0.036 \pm 0.008$, with preference for larger ξ values. The overall χ^2 value is only slightly worse ($\Delta\chi^2 = 2.65$) compared to that of inflation, although the string model has two additional parameters. The corresponding string contribution to the B-mode spectrum is shown with a thin blue long-dash line in Fig. 1, and strings + lensing is shown with a thick blue long-dash line. Not surprisingly, the data, which has a bump at $\ell \sim 100$, favors a spectrum with a peak at a lower ℓ, which is at $\ell \sim 250$ for the best-fit model. A model with such large values of ξ corresponds to rare and heavy strings—the implied value of $G\mu$ in this model is 5×10^{-6}, but their number density is low, which allows it to remain consistent with Planck bounds. The peak position for this model is closer to that of global strings and textures [51,56] and certainly not representative of local strings [36]. This is also clear from the likelihood plot for ξ, which effectively *rules out* models with $\xi < 1.8$ (2σ) as the only primordial source of B modes. For reference, the B-mode spectra from the local string simulation of Ref. [49] correspond to the USM with $\xi \approx 0.4$ [36]. We can foresee that models with global strings, textures [51,56], or global phase transitions, of the kind discussed in Refs. [74,75], would provide a much better fit than local strings.

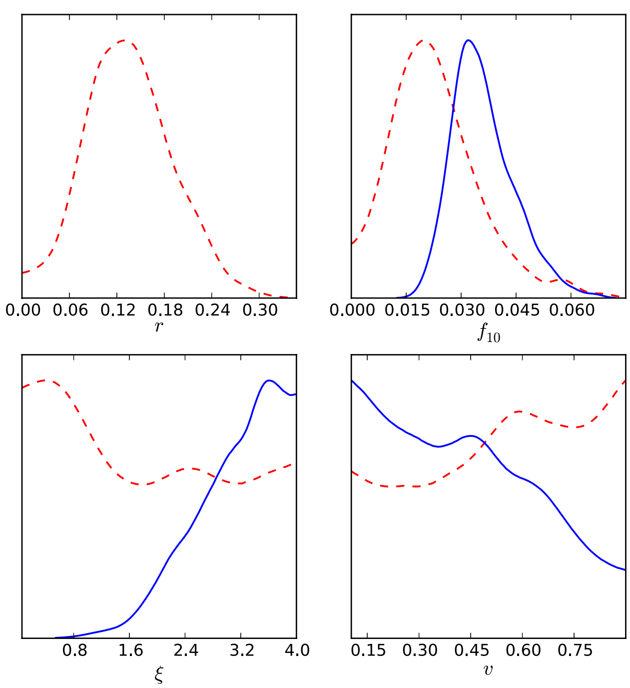

FIG. 2 (color online). Marginalized likelihoods derived from the BICEP2 and POLARBEAR data for the scalar-to-tensor ratio r, the strength of the string contribution f_{10}, the interstring distance ξ, and the rms velocity v. The red dotted lines are for the lensing + strings + inflation model, whereas the blue solid lines are for the lensing+strings fit only.

We now consider the model in which both strings *and* inflation generate B modes. The red dotted lines in Fig. 2 show the marginalized likelihoods of r and the string parameters in a model with an additional inflationary tensor mode contribution. In this case, the fit is improved relative to the model with no strings, with $\Delta\chi^2 = -6.06$ and three additional parameters. The marginalized string fraction is $f_{10} = 0.025 \pm 0.014$, which corresponds to $G\mu \approx 4 \times 10^{-7}$, with slight preference for lower values of ξ, characteristic for local strings, and $r = 0.14 \pm 0.05$.

Figure 3 shows the marginalized joint likelihood for r and the strength of the string contribution f_{10}. It clearly shows that a combination of the two contributions fits the data better than when either of them is zero. The thin vertical line indicates the approximate upper bound on f_{10} from Planck. (The Planck Collaboration did not scan over all values of ξ and v; instead it provided two separate bounds on f_{10} corresponding to two different string models. We quote the weaker of the two bounds because both models were included in the USM parameter space covered by our fit.) It should be noted that the improvement in the fit comes primarily from data points at higher ℓ, while the BICEP2 collaboration warns [24] that points at $\ell > 150$ should be considered as preliminary.

Our findings carry implications for models that predict defects. The inability of *local* strings to fit the B-mode spectrum *on their own* poses a problem for the simplest and most studied brane inflation models in which inflation ends with a production of cosmic superstrings [41–43]. Such models predict tiny values of r, and the only observable B modes could come from strings, which are effectively of local, Nambu-Goto type. The fact that local strings do not fit the BICEP2 data puts these scenarios under pressure. Generally, since r tends to be small in hybrid inflation-type models, it is not clear if inflationary models with such large

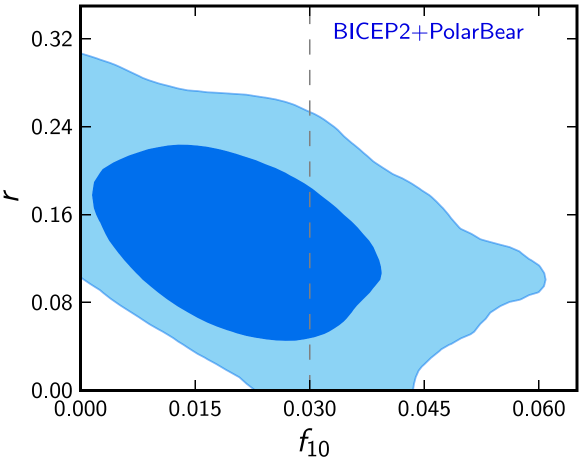

FIG. 3 (color online). The marginalized joint likelihood for the tensor-to-scalar ratio r and the strength of the string contribution f_{10}. The two different shades indicate the 68% and 95% confidence regions. The vertical dashed line indicates the approximate bound on f_{10} from Planck.

values of r can be consistent with the production of cosmic strings.

The main reason cosmic strings struggle to provide a good fit to the BICEP2 data is the presence of B-mode power on smaller angular scales. This power could be suppressed if strings were to form not after but during inflation [76,77]. Such strings could remain far separated and prevented from reaching a scaling solution until the onset of decoupling (this idea was pointed out to us by Alex Vilenkin). A related scenario was recently discussed in Ref. [78] as a way of eliminating the presence of loops during the radiation era and, thus, evading the tight pulsar bounds on cosmic strings. There may be an impetus for investigating such models further in the context of string-sourced B modes.

To summarize, we have shown that the B-mode spectra measured by BICEP2 and POLARBEAR are consistent with a contribution from vector modes sourced by cosmic strings. Working with the USM model allowed us to scan over a wide range of scaling defect models parametrized by the effective density parameter ξ and the rms velocity v. In order for strings to provide a satisfactory fit to the data on their own, the ξ parameter needs to be extremely large, well beyond values typical for local strings. The string spectra that fit the data best are more representative of global strings and textures.

When the string contribution is considered together with the inflationary B-mode spectrum, they improve the overall fit. This is primarily because the string contribution allows the model to pass through the data points at $\ell > 150$. The best-fit USM model in this case is consistent with B-mode spectra from simulations of local strings.

In both cases, with and without the inflationary contribution, the best fit for f_{10} is close to but still below the bound set by Planck based on fits to the CMB temperature spectra. Thus, we expect that a joint fit that included the Planck data would not significantly change the conclusions of this Letter. Such a fit must be performed in the future when more data become available.

We have argued that detectable B modes can be produced by cosmic defects. Other interesting possibilities include phase transitions [75] and primordial magnetic fields [79,80]. Thus, BICEP2 results are exciting not only because of the potential discovery of the signal from inflationary gravity waves but also because they have pioneered the era of precision B-mode science—a new frontier for testing fundamental physics with cosmology.

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Note added.—While this paper was in preparation, a related short paper was posted on arXiv.org [81] commenting on similar ideas. Our work provides quantitative answers to some of the questions posed in Ref. [81].

[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[5] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[6] A. Vilenkin, Phys. Rev. D **27**, 2848 (1983).
[7] A. A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 719 (1979) [JETP Lett. **30**, 682 (1979)].
[8] V. F. Mukhanov and G. V. Chibisov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 549 (1981) [JETP Lett. **33**, 532 (1981)]
[9] S. W. Hawking, Phys. Lett. **115B**, 295 (1982).
[10] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).
[11] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
[12] A. H. Guth and S.-Y. Pi, Phys. Rev. D **32**, 1899 (1985).
[13] V. F. Mukhanov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 402 (1985) [JETP Lett. **41**, 493 (1985)].
[14] C. L. Bennett, A. Banday, K. M. Gorski, G. Hinshaw, P. Jackson, P. Keegstra, A. Kogut, G. F. Smoot, D. T. Wilkinson, and E. L. Wright *et al.*, Astrophys. J. **464**, L1 (1996).
[15] C. L. Bennett *et al.* [WMAP Collaboration], Astrophys. J. Suppl. Ser. **208**, 20 (2013).
[16] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5082.
[17] R. Crittenden, R. L. Davis, and P. J. Steinhardt, Astrophys. J. **417**, L13 (1993).
[18] R. A. Frewin, A. G. Polnarev, and P. Coles, Mon. Not. R. Astron. Soc. **266**, L21 (1994).
[19] D. D. Harari and M. Zaldarriaga, Phys. Lett. B **319**, 96 (1993).
[20] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997).

[21] U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997).
[22] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997).
[23] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. D **55**, 7368 (1997).
[24] P. A. R. Ade *et al.* [BICEP2 Collaboration], arXiv:1403.3985.
[25] P. A. R. Ade *et al.* [POLARBEAR Collaboration], arXiv:1403.2369.
[26] M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. **58**, 477 (1995).
[27] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 2000).
[28] A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); A. Vilenkin, Phys. Rev. Lett. **46**, 1496(E) (1981).
[29] A. Albrecht, R. A. Battye, and J. Robinson, Phys. Rev. Lett. **79**, 4736 (1997).
[30] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **76**, 1011 (1996).
[31] I. Tkachev, S. Khlebnikov, L. Kofman, and A. D. Linde, Phys. Lett. B **440**, 262 (1998).
[32] F. R. Bouchet, P. Peter, A. Riazuelo, and M. Sakellariadou, Phys. Rev. D **65**, 21301 (2001).
[33] M. Wyman, L. Pogosian, and I. Wasserman, Phys. Rev. D **72**, 023513 (2005); **73**089905(E) (2006)].
[34] N. Bevis, M. Hindmarsh, M. Kunz, and J. Urrestilla, Phys. Rev. Lett. **100**, 021301 (2008).
[35] R. Battye, B. Garbrecht, and A. Moss, Phys. Rev. D **81**, 123512 (2010).
[36] R. Battye and A. Moss, Phys. Rev. D **82**, 023521 (2010).
[37] R. Jeannerot, Phys. Rev. D **53**, 5426 (1996).
[38] R. Jeannerot, J. Rocher, and M. Sakellariadou, Phys. Rev. D **68**, 103514 (2003).
[39] J. Rocher and M. Sakellariadou, Phys. Rev. Lett. **94**, 011303 (2005).
[40] N. T. Jones, H. Stoica and S. H. H. Tye, J. High Energy Phys. 07 (2002) 051.
[41] S. Sarangi and S. H. H. Tye, Phys. Lett. B **536**, 185 (2002).
[42] L. Pogosian, S. H. Henry Tye, I. Wasserman, and M. Wyman, Phys. Rev. D **68**, 023506 (2003); **73**089904(E) (2006)].
[43] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister, and S. P. Trivedi, J. Cosmol. Astropart. Phys. 10 (2003) 013.
[44] G. Dvali and A. Vilenkin, J. Cosmol. Astropart. Phys. 03 (2004) 010.
[45] U. Seljak, U. L. Pen, and N. Turok, Phys. Rev. Lett. **79**, 1615 (1997).
[46] W. Hu and M. J. White, Phys. Rev. D **56**, 596 (1997).
[47] R. A. Battye, arXiv:hep-th/9806115.
[48] U. Seljak and A. Slosar, Phys. Rev. D **74**, 063523 (2006).
[49] N. Bevis, M. Hindmarsh, M. Kunz, and J. Urrestilla, Phys. Rev. D **76**, 043005 (2007).
[50] L. Pogosian and M. Wyman, Phys. Rev. D **77**, 083509 (2008).
[51] J. Urrestilla, N. Bevis, M. Hindmarsh, M. Kunz, and A. R. Liddle, J. Cosmol. Astropart. Phys. 07 (2008) 010.
[52] J. Urrestilla, P. Mukherjee, A. R. Liddle, N. Bevis, M. Hindmarsh, and M. Kunz, Phys. Rev. D **77**, 123005 (2008).
[53] P. Mukherjee, J. Urrestilla, M. Kunz, A. R. Liddle, N. Bevis, and M. Hindmarsh, Phys. Rev. D **83**, 043003 (2011).
[54] Y.-Z. Ma, W. Zhao, and M. L. Brown, J. Cosmol. Astropart. Phys. 10 (2010) 007.
[55] A. Avgoustidis, E. J. Copeland, A. Moss, L. Pogosian, A. Pourtsidou, and D. A. Steer, Phys. Rev. Lett. **107**, 121301 (2011).
[56] U.-L. Pen, U. Seljak, and N. Turok, Phys. Rev. Lett. **79**, 1611 (1997).
[57] N. Turok, U.-L. Pen, and U. Seljak, Phys. Rev. D **58**, 023506 (1998).
[58] F. A. Jenet, G. B. Hobbs, W. van Straten, R. N. Manchester, M. Bailes, J. P. W. Verbiest, R. T. Edwards, A. W. Hotan, J. M. Sarkissian, and S. M. Ord, Astrophys. J. **653**, 1571 (2006).
[59] B. P. Abbott *et al.* (LIGO Scientific Collaboration and VIRGO Collaboration), Nature (London) **460**, 990 (2009).
[60] T. Damour and A. Vilenkin, Phys. Rev. D **64**, 064008 (2001).
[61] T. Damour and A. Vilenkin, Phys. Rev. D **71**, 063510 (2005).
[62] X. Siemens, J. Creighton, I. Maor, S. R. Majumder, K. Cannon, and J. Read, Phys. Rev. D **73**, 105001 (2006).
[63] X. Siemens, V. Mandic, and J. Creighton, Phys. Rev. Lett. **98**, 111101 (2007).
[64] E. O'Callaghan, S. Chadburn, G. Geshnizjani, R. Gregory, and I. Zavala, Phys. Rev. Lett. **105**, 081602 (2010).
[65] G. R. Vincent, M. Hindmarsh, and M. Sakellariadou, Phys. Rev. D **55**, 573 (1997).
[66] A. Albrecht, R. A. Battye, and J. Robinson, Phys. Rev. D **59**, 023508 (1998).
[67] L. Pogosian and T. Vachaspati, Phys. Rev. D **60**, 083504 (1999).
[68] http://www.sfu.ca/~levon/cmbact.html.
[69] A. Avgoustidis, E. J. Copeland, A. Moss, and D. Skliros, Phys. Rev. D **86**, 123513 (2012).
[70] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. **64**, 119 (1990).
[71] A. A. Fraisse, C. Ringeval, D. N. Spergel, and F. R. Bouchet, Phys. Rev. D **78**, 043535 (2008).
[72] J. J. Blanco-Pillado, K. D. Olum, and B. Shlaer, Phys. Rev. D **83**, 083514 (2011).
[73] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5085.
[74] K. Jones-Smith, L. M. Krauss, and H. Mathur, Phys. Rev. Lett. **100**, 131302 (2008).
[75] J. B. Dent, L. M. Krauss, and H. Mathur, arXiv:1403.5166.
[76] G. Lazarides and Q. Shafi, Phys. Lett. **148B**, 35 (1984).
[77] Q. Shafi and A. Vilenkin, Phys. Rev. D **29**, 1870 (1984).
[78] K. Kamada, Y. Miyamoto, and J. 'i. Yokoyama, J. Cosmol. Astropart. Phys. 10 (2012) 023.
[79] A. Mack, T. Kahniashvili, and A. Kosowsky, Phys. Rev. D **65**, 123004 (2002).
[80] A. Lewis, Phys. Rev. D **70**, 043011 (2004).
[81] J. Lizarraga, J. Urrestilla, D. Daverio, M. Hindmarsh, M. Kunz, and A. R. Liddle, Phys. Rev. Lett. **112**, 171301 (2014).