Entanglement Monotonicity and the Stability of Gauge Theories in Three Spacetime Dimensions

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We employ the recent results on the generalization of the central charge theorem to three spacetime dimensions to derive nonperturbative results for several strongly interacting quantum field theories, including quantum electrodynamics (QED-3), and the theory corresponding to certain quantum phase transitions in condensed matter systems. In particular, by demanding that the universal constant part of the entanglement entropy decreases along the renormalization group flow (F theorem), we find sufficient conditions for the stability of QED-3 against chiral symmetry breaking and confinement. Using similar ideas, we derive strong constraints on the nature of quantum critical points in condensed matter systems with topological order.

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Strongly interacting quantum field theories in three spacetime $(2+1)$D dimensions often arise as a low-energy description of condensed matter systems and also serve as a test bed for phenomena in $(3+1)$D. Since there are only a few interacting quantum systems that can be solved exactly, it is natural to ask whether there might exist general principles that constrain the set of possibilities for a given problem, even if they do not provide a direct solution, An example of such a principle is the "central charge theorem" [1] in $(1+1)$D and its four-dimensional version, the "a theorem" [2–4], which constrains the renormalization group (RG) flows of Lorentz invariant systems. In this Letter, we employ the recent generalization of the central charge theorem to $(2+1)$D, the "F theorem" [5–11], to derive the low-energy behavior of several strongly interacting $(2+1)$D theories of interest to condensed matter physics.

Our work is motivated by the continual theoretical and experimental interest in discovering and understanding quantum phases of matter that lie beyond Landau's order parameter description [12]. Two prime examples of such phases are fractional quantum Hall phases and quantum spin liquids (QSL) [12–14]. A unique feature of these phases is that their ground state is "topologically ordered": despite short-range correlations for all local operators, the ground state is not smoothly connected to a direct product state [12,15–17]. A related class of phases are *gapless* spin liquids, which again lack a local order parameter, and whose dynamics is often described by strongly coupled gauge theories such as the conformal phase of QED or QCD in $(2+1)$D. In this Letter, we show that the F theorem sheds light on the following two important questions that concern such phases.

(1) What is the nature of quantum phase transitions between conventional symmetry-broken phases and topologically ordered phases? We show that the F theorem implies that, on very general grounds, such phase transitions cannot be described by conventional Landau-Ginzburg order parameter theory.

(2) When are gapless spin liquids stable? We make progress on this question by determining sufficient conditions for the stability of the deconfined phase of interacting gauge theories in $(2+1)$D, which describe "algebraic quantum spin liquids" [12,18–24]. Specifically, we provide a nonperturbative upper bound on the amount of matter content required to deconfine QED in $(2+1)$D and also discuss generalization to non-Abelian gauge theories (QCD). We also address the related problem of the phase diagram of noncompact QED in $(2+1)$D, where one undergoes a quantum phase transition from a symmetry-breaking Goldstone mode phase to a symmetric phase as one tunes the number of flavors [25,26]. This is the $(2+1)$D analog of the chiral symmetry breaking (CSB) in the standard model of particle physics in $(3+1)$D [27].

Let us recall the "c theorem" [1]: the central charge c of a conformal field theory (CFT) decreases along the renormalization group flow, as one flows from a UV fixed point to an IR fixed point. As an example, a relevant perturbation to the tricritical Ising CFT ($c=7/10$) can result in only two unitary CFTs: the Ising critical point ($c=1/2$) or a fully gapped system ($c=0$). There is a similar theorem for four-dimensional CFTs, Cardy's a theorem [2,3] that has been placed on rigorous footing [4] in the recent past.

Recently, there has been progress in developing an analog of the c theorem for three spacetime dimensions [5–11]. Specifically, Casini and Huerta [7] have shown that, for Lorentz invariant theories, if one writes the entanglement entropy for a circular region of radius R as $S(R)=\alpha R-\gamma+O(1/R)$, the universal constant γ decreases along the RG flow: $\gamma_{\mathrm{UV}}\geq\gamma_{\mathrm{IR}}$. A related development has been the conjecture, accompanied by a perturbative proof, that the universal part of the free energy of a

CFT on a three-sphere S^3 (denoted as F) decreases along the RG flow and takes a stationary value at the fixed points [8,9]. Reference [11] showed that $F = \gamma$ at the conformally invariant fixed points. Therefore, the nonperturbative result of Ref. [7] $\gamma_{\mathrm{UV}} \geq \gamma_{\mathrm{IR}}$ automatically implies $F_{\mathrm{UV}} \geq F_{\mathrm{IR}}$ for flows between CFTs. In the rest of this Letter, we will consider consequences of this statement for a variety of condensed matter systems.

Non-Landau quantum phase transitions and the F theorem.—Gapped QSLs are interacting spin systems that have intriguing properties such as anyonic excitations in $(2+1)$D, and more generally, a robust set of degenerate ground states on a torus (topological order) [12,15,16]. Another hallmark of these phases is the presence of topological entanglement entropy F_{topo}: the entanglement entropy of the ground state for a disk-shaped subregion of size L scales as $S(L) = \alpha L - F_{\mathrm{topo}} + O(1/L)$, where F_{topo} is a universal number that depends only on the phase of matter under consideration, analogous to a gapless CFT in $(2+1)$D [28,29], and α is nonuniversal. Indeed, a different way to understand this result is that the low-energy effective theory of gapped QSLs is a topological quantum field theory, which is a very special kind of CFT with zero (rather than infinite) correlation length.

Locally, QSLs are indistinguishable from a gapped featureless paramagnet (FP) which, unlike a QSL, has a unique ground state on a torus, and is smoothly connected to a direct product state by local unitary operators [17]. The local indistinguishability follows from the fact that neither a gapped QSL nor a FP has a local order parameter, and consequently, the correlation functions of all local operators decay exponentially in either of these phases. Do QSLs differ from FPs when one undergoes a phase transition to an ordered phase which has a local order parameter? Case-by-case evidence suggests that such an expectation is indeed correct [30,31]. We now show that this result follows on general grounds from the F theorem and reflects the nonlocal nature of entanglement in a QSL.

Let us remind ourselves that the phase transition between a FP and a symmetry-breaking order parameter phase can be understood within the conventional Landau-Ginzburg-Wilson paradigm. As an example, consider a bilayer spin-1/2 Heisenberg model in $(2+1)$D with interlayer coupling $J_\perp \gg J_\parallel$ [30]. As $J_\perp$ is reduced, at a particular critical value of the ratio $J_\perp/J_\parallel$, the system undergoes a quantum phase transition in the O(3) Wilson-Fisher universality to an ordered antiferromagnet.

We now show that despite local indistinguishability between a FP and a QSL, a quantum phase transition out of a gapped SU(2) symmetric QSL to a symmetry broken phase can never be in the O(3) Wilson-Fisher universality class. We provide a proof by contradiction (Fig. 1). Let us assume that such a transition were indeed in the O(3) Wilson-Fisher universality class. Since in

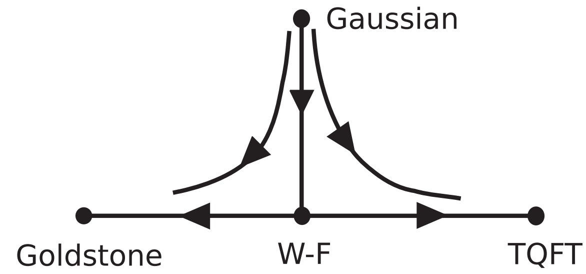

FIG. 1. A RG flow that is prohibited due to the F theorem. Gaussian, W-F, Goldstone, and TQFT refer, respectively, to the Gaussian fixed point, the Wilson-Fisher fixed point, a Goldstone mode phase, and an arbitrary topologically order phase of an SU(2) symmetric spin system in $(2+1)$D.

$(2+1)$D the Gaussian fixed point is unstable towards the Wilson-Fisher fixed point, the F theorem implies that

$$F_{\mathrm{Gaussian}} \geq F_{\mathrm{O(3)Wilson\text{-}Fisher}} \geq F_{\mathrm{topo}}, \tag{1}$$

where $F_{\mathrm{Gaussian}} = 3F_{\mathrm{scalar}} \approx 0.18$, $F_{\mathrm{scalar}} \approx 0.06$ being the F for a free real scalar [9,32]. F_{topo}, on the other hand, takes only a discrete set of values, since it is related to the quantum numbers of anyonic excitations [28,29]. The smallest possible value of F_{topo} is attained by the Laughlin $\nu = 1/2$ state with $F_{\mathrm{topo}} = \log(\sqrt{2}) \approx 0.35$. Therefore, Eq. (1) can never be satisfied and, therefore, our initial assumption about the existence of an O(3) Wilson-Fisher transition must be wrong, even though the global symmetry is just SU(2). Indeed, all known transitions between an ordered phase and a topologically ordered phase satisfy the equation $F_{\mathrm{critical}} > F_{\mathrm{topo}}$ [10,33,34] whenever F_{critical} is calculable (e.g., via $1/N$ expansion). We note that there already exist several realistic models of frustrated magnets [35–37], which seemingly exhibit a direct phase transition between a topologically ordered paramagnet and an SU(2) symmetry-broken state, and the value F_{crit} for these transitions must satisfy $F_{\mathrm{crit}} \geq F_{\mathrm{topo}}$ by the above argument.

The above analysis readily generalizes to systems with a global U(1) symmetry. For example, the transition between a superfluid and a $\nu = 1/2$ fractional quantum Hall state of bosons cannot be a conventional O(2) transition, which is indeed consistent with the proposed theory for this transition [38]. On the other hand, the transition between an integer quantum Hall state of bosons [39,40] and a superfluid is allowed to be in the O(2) universality class, since the integer quantum Hall state has $F = 0$. Indeed, consistent with this observation, the critical theory for such a transition has been argued to be just Wilson-Fisher O(2) [41].

Strongly interacting gauge theories and the F theorem.—Strongly interacting gauge theories often display rich phase diagrams as a function of the field content (e.g., number of flavors) and other tuning parameters (e.g., temperature). In this section, we will explore the phase diagram of $(2+1)$D gauge theories and show that the F theorem can be used to deduce a number of general results in this direction. Apart from being of general interest [25,26,42], these theories describe a large class of gapless quantum spin liquids [12,18–24], which are phases of interacting quantum spins without an order parameter description, and without any well-defined quasiparticles. Our results will have crucial implications for the stability of these phases.

We focus mainly on quantum electrodynamics in $(2+1)$D (QED-3), which is the theory of fermions coupled to a U(1) gauge field, and ask the following question: when do fermions survive as gapless excitations in the low-energy theory? There are at least two known mechanisms that can lead to spontaneous mass generation for fermions—chiral symmetry breaking [25,26] and confinement [43]. Of course, confinement may be accompanied by CSB, but it is important to make this distinction because CSB can occur even in the absence of monopoles (noncompact QED-3), while confinement is relevant only to the compact QED-3. We briefly discuss generalization to non-Abelian gauge theories later.

CSB in noncompact QED-3.—The Lagrangian for the noncompact QED-3 is

$$\mathcal{L}_{\text{QED-3}} = \sum_{a=1}^{N_f} \bar{\psi}_a[-i\gamma_\mu(\partial_\mu + ia_\mu)]\psi_a + \frac{1}{2e^2}F_{\mu\nu}F_{\mu\nu}, \quad (2)$$

where ψ is a two-component Dirac fermion with an even number $(= N_f)$ of flavors, and a_μ is a noncompact U(1) gauge field. Large-N_f arguments indicate that at $N_f \gg 1$, the theory is a conformal field theory with unbroken $U(N_f)$ symmetry, while below a critical $N_f = N_{fc}$, the fermions acquire a mass gap due to the spontaneously symmetry breaking $U(N_f) \to U(N_f/2) \times U(N_f/2)$, where the pattern of symmetry breaking is uniquely dictated by the Vafa-Witten theorem [26,42]. In particular, N_f flavors of the fermions spontaneously acquire a positive mass, while the rest of the N_f flavors acquire a negative mass, thus preserving the time-reversal symmetry.

A rough estimate on N_{fc} follows by simply comparing the F of interacting QED-3 at large N_f with that of the Goldstone mode phase. At large N_f, $F_{\text{QED-3}} \approx N_f F_{\text{Dirac}} + \frac{1}{2}\log(\pi N_f/8)$ [10], where $F_{\text{Dirac}} \approx 0.22$ is the F for a free Dirac fermion [32]. On the other hand, $F_{\text{Goldstone}} = ((N_f^2/2)+1)F_{\text{scalar}}$, where the factor of $(N_f^2/2)F_{\text{scalar}}$ comes from the Goldstone modes while the photon in the IR contributes an additional F_{scalar} [44,45] (recall that the photon in a noncompact QED remains gapless in the IR). Because of the different scaling of F with respect to N_f in the two phases, one finds that a symmetric phase is expected above $N_{fc} \gtrsim 10$.

Next, we obtain a nonperturbative upper bound on N_{fc}, under assumptions detailed below, by considering a deformation of superconformal QED-3, whose F is known exactly. The supersymmetric theory we consider is maximally chiral $\mathcal{N}=2$ superconformal QED-3 (SQED-3), whose field content consists of N_f Dirac fermions ψ, N_f complex scalars ϕ, a gauge field $\vec{a}$, (fermionic) gaugino λ, and a real scalar θ [10]. For completeness, we provide the action for this theory in a component form in the Supplemental Material [46]. We deform this theory by a mass term for ϕ ($\propto |\phi|^2$), as well as a mass for θ ($\propto \theta^2$). Note that these mass terms retain all the symmetries, in particular, the $U(N_f)$ flavor symmetry as well as the time-reversal symmetry. This ensures that no explicit mass is generated for the fields ψ and λ. We can now integrate out θ and ϕ, which generates new interactions for the leftover fields λ, $\vec{a}$, ψ:

$$\mathcal{L} = \mathcal{L}_{\text{QED-3}} - \frac{i}{4}\bar{\lambda}\gamma_\mu\partial_\mu\lambda + \Delta\mathcal{L}_1, \quad (3)$$

where $\Delta\mathcal{L}_1 \propto \bar{\lambda}\lambda\bar{\psi}\psi$. For $N_f \gg 1$, $\Delta\mathcal{L}_1$ is *irrelevant* at the QED-3 fixed point, and, therefore, the SQED-3 flows to QED-3. We now assume that this continues to be true at a finite N_f until QED-3 itself becomes unstable to a CSB phase at N_{fc}. This expectation is based on the fact that SQED-3 has more matter content compared to QED-3, and is therefore not expected to undergo a CSB instability before QED-3 does, as N_f is decreased from infinity. Additonally, the scaling dimension of the operator $\bar{\psi}\psi$, in a large-N_f expansion, is given by $\Delta_{\bar{\psi}\psi} = 2 + (128/(3\pi^2 N_f))$ [49]. Thus, it is reasonable to assume that $\Delta\mathcal{L}_1$ continues to be irrelevant at the QED-3 fixed point for the values of N_f we encounter below ($N_f \approx 10$). Assuming these assumptions hold, the F theorem implies $F_{\text{SQED-3}} \geq F_{\text{QED-3}} + F_{\text{Dirac}}$ and $F_{\text{QED-3}} \geq F_{\text{Goldstone}}$. One can now eliminate $F_{\text{QED-3}}$ to find the inequality $F_{\text{SQED-3}} \geq F_{\text{Goldstone}} + F_{\text{Dirac}}$. Both $F_{\text{SQED-3}}$ and $F_{\text{Goldstone}}$ are known exactly. From Ref. [8], $F_{\text{SQED-3}} = \frac{N_f}{2}\log(2) + \frac{1}{2}\log(N_f\pi/4) + (((-1)/2) + (20/3\pi^2))(1/N_f) + O(N_f^{-2})$, where we only quote the result at large N_f, which suffices for our purpose (i.e., using the exact result does not change the bounds on N_{fc}). Solving the aforementioned inequality, one finds that CSB is impossible when $N_f \geq 14$.

We note that our arguments have some similarity with those presented in Ref. [50], where it was suggested that the thermodynamic free energy density in flat space decreases under RG. However, there are known counterexamples to the monotonicity of free energy [8,51] and, therefore, it cannot be used to constrain RG flows.

Deconfinement in compact QED-3.—The Lagrangian for compact QED-3 is exactly the same as Eq. (2) except that now the monopoles in the gauge field are allowed. We are again interested in the phase diagram of this theory as a function of the number of flavors N_f. At $N_f = 0$, monopoles proliferate and the gauge field confines leading to mass gap [43]. In the opposite limit, $N_f \gg 1$, the theory is expected to be in the deconfined phase [19]. Therefore, there exists a critical N_{fc} below which compact QED-3 confines. More generally, one can imagine four distinct possibilities for the fate of this theory in the IR. (I) The theory confines with a mass gap to all excitations. (II) The theory confines while breaking the flavor symmetry $U(N_f)$ down to some smaller subgroup resulting in massless Goldstone bosons. (III) The theory deconfines with massless fermions in the IR. (IV) The theory deconfines while maintaining gap to the fermions and gauge fields in the IR [52].

In a remarkable paper, Vafa and Witten [26] argued that in $(2+1)$D, whenever there exist $N_f \geq 6$ massless fermions coupled to massless gauge bosons in the UV, then there necessarily exist massless particles in the IR. What is the nature of the massless phase when $N_f \geq 6$? The low-lying excitations in this massless phase may be the Goldstone modes associated with CSB, or they could also correspond to excitations of a (conformal) deconfined phase of QED-3 where the $U(N_f)$ symmetry is unbroken. We now show that the F theorem allows one to deduce a bound on N_f above which the latter possibility is necessarily realized.

To begin with, when $N_f \geq 6$, the Vafa-Witten theorem immediately rules out possibilities (I) and (IV). Furthermore, if there exists a Goldstone mode phase for $N_f \geq 6$, the pattern of flavor symmetry breaking is again constrained to be $U(N_f) \to U(N_f/2) \times U(N_f/2)$ [26]. Entanglement monotonicity implies $F_{\text{QED-3}} \geq (N_f^2/2) F_{\text{scalar}}$. One can solve this inequality approximately by employing the large N_f result for $F_{\text{QED-3}}$, and one finds that above $N_f \approx 10$, the theory must be in a deconfined gapless phase. To obtain a strict bound on N_{fc}, we again deform maximally chiral $\mathcal{N}=2$ SQED-3 as in the case of noncompact QED. This procedure, under the same assumptions as before, leads to the conclusion that the theory deconfines when $N_f \geq 14$.

Non-Abelian Gauge theories.—The above arguments generalize to non-Abelian gauge theories. Consider QCD-3 with an arbitrary gauge group (N_c colors) coupled to N_f fermions in the fundamental representation. To the leading order in N_f, F for the deconfined phase is given by $N_f N_c F_{\text{Dirac}}$, while that for the confined Goldstone phase is given by $(N_f^2/2) F_{\text{scalar}}$. Entanglement monotonicity along with Vafa-Witten theorems imply that QCD-3 is stable against confinement when $N_f \gtrsim 2N_c (F_{\text{Dirac}}/F_{\text{scalar}}) \approx 8N_c$. This is consistent with the general intuition that as N_c increases, so does the critical number of fermions required for deconfinement [53,54]. One can systematically improve upon this estimate by considering $1/N_f$ corrections to the leading result for the entanglement of non-Abelian gauge theories [10], or by following the route of sandwiching the RG fixed point corresponding to QCD-3 between a SUSY QCD-3 and a Goldstone mode phase.

Discussion.—In this Letter, we employed the recent results on the monotonic behavior of the universal part of the entanglement under RG [5–11] to constrain the phase diagram of topologically ordered phases and gauge-matter theories in $(2+1)$D. In particular, we showed that the transitions to topologically ordered paramagnets in an SU(2) symmetric spin system can never lie in an O(3) universality class. In the context of gauge theories, we obtained nonperturbative bounds on the matter content sufficient to stabilize the conformal, deconfined phase of compact QED-3, and the analogous symmetric phase of noncompact QED-3. We also discussed generalizations to non-Abelian gauge theories. One might wonder if one could improve the bounds for confinement or CSB by a more clever choice of RG flow. Let us consider the RG flow from a theory consisting of free fermions and a free photon to QED-3, as the coupling (electric charge) is turned on. However, $F = \infty$ for a free photon [10], which seemingly does not provide a useful bound. As discussed in the Supplemental Material [46], under certain assumptions, it is conceivable that one can regularize the infinite F for a free photon in the UV, while considering the flow to interacting QED-3. This analysis suggests that the critical number of flavors for deconfinement, as well as CSB, satisfy a stronger constraint $N_{fc} < 8$.

Before concluding, we briefly mention a straightforward application of the F theorem to classical statistical mechanics. Consider an $O(n) \oplus O(m)$ vector model in $d=3$. Perturbative RG suggests that when $n+m \leq 3$, the most stable fixed point has an enlarged $O(n+m)$ symmetry, while when $n, m \gg 1$, the most stable fixed point corresponds to decoupled $O(n)$, $O(m)$ Wilson-Fisher fixed points [55]. The F theorem provides a nonperturbative insight into this problem. When $n, m \gg 1$, the decoupled fixed point has $F = F_{O(n)} + F_{O(m)} \approx (n+m) F_{\text{scalar}} - 2c$, while the fixed point with enlarged symmetry has $F = F_{O(n+m)} \approx (n+m) F_{\text{scalar}} - c$, where $c = (\zeta(3)/16\pi^2)$ is the universal subleading correction to F of the Wilson-Fisher fixed point [9]. Clearly, the decoupled fixed point is comparatively more stable. On this note, it will be interesting to consider further applications of the F theorem to multicomponent Landau-Ginzburg models.

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