Quantum Error Correction for Metrology

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We propose and analyze a new approach based on quantum error correction (QEC) to improve quantum metrology in the presence of noise. We identify the conditions under which QEC allows one to improve the signal-to-noise ratio in quantum-limited measurements, and we demonstrate that it enables, in certain situations, Heisenberg-limited sensitivity. We discuss specific applications to nanoscale sensing using nitrogen-vacancy centers in diamond in which QEC can significantly improve the measurement sensitivity and bandwidth under realistic experimental conditions.

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The precise measurement of physical quantities is of great importance in science. Quantum metrology [1,2] provides an efficient framework for understanding the fundamental limits of the achievable accuracy in the determination of a parameter (e.g., a magnetic field or frequency), given a certain amount of resources (e.g., number of available atoms or time). In recent years, the exploration of these limits in the presence of realistic imperfections and noise have been actively pursued [3–7]. A typical quantum measurement (e.g., the Ramsey method [8]) involves a sequence of measurement cycles of duration T within the total available time τ. Since each cycle introduces measurement noise, it is beneficial to extend T to its maximum value $T \to \tau$. However, in the presence of qubit noise of rate γ, the interrogation time T is inherently limited, since for times $T \geq 1/\gamma$ the phase information acquired during the interrogation is lost [3]. One technique to counter environmental noise in metrology is dynamical decoupling (DD) [9–16]. Here, a series of control pulses (or continuous wave control fields) effectively achieves a cancellation of the coupling Hamiltonian between the system (i.e., qubit) and its environment to a certain order, thus, effectively reducing the value of γ [17]. However, in order to achieve sensitivity improvements, the pulse repetition rate of a DD protocol (which has to match the frequency of the measured signal) needs to be faster than the correlation time of the environmental bath τ_c. Therefore, for environments with fast internal dynamics, DD is not feasible.

In this Letter, we propose a complementary approach that employs quantum error correction (QEC) [18–20] for metrology. In contrast to DD, the QEC operations have to be implemented on time scales of the noise rate γ. Our approach is independent of the correlation time of the bath, and it is capable of correcting noise even in the Markovian limit ($\tau_c \to 0$). A direct application is to nanoscale measurements of magnetic and electric fields using nitrogen vacancy (NV) centers in diamond. We show that, in such measurements, significant improvements in sensitivity and detector bandwidth can be obtained. Our approach can be understood as a sequential feedback protocol. When applied to ensembles of N qubits, it can yield, in certain situations, Heisenberg-limited scaling ($\propto 1/N$) of the measurement uncertainty. In the example we consider, this allows us to surpass the precision bound derived in [7]. This illustrates that the recently developed methods to derive precision bounds in the presence of noise [5,6] (which were used in [7]) have to be applied with care, as for certain noise models, these bounds may be surpassed using sequential feedback [21].

QEC is based on the fact that any kind of noise, discrete or continuous, can be represented by a discrete set of error operation elements $\{E_0, \ldots, E_w\}$. It is then possible—through the use of redundant degrees of freedom (provided, e.g., by ancilla qubits)—to encode the logical information in a subspace $\mathcal{C}$ (the so called quantum code) of the Hilbert space $\mathcal{H}$ such that each of the errors E_i maps the code to a respective orthogonal and undeformed subspace $\mathcal{E}_i$, allowing us to detect and correct errors that have occurred. Consider a state that evolves within the code space $\mathcal{C}$ under the action of the Hamiltonian H generating the signal we aim to measure: $e^{-iHt}|\Psi\rangle = |\Psi_{\phi(t)}\rangle \in \mathcal{C}$ for $|\Psi\rangle \in \mathcal{C}$ [$\phi(t)$ denotes, e.g., the accumulated phase]. If an error E_i occurs, the state is mapped to $E_i|\Psi_{\phi(t)}\rangle = |\eta_{\phi(t)}\rangle \in \mathcal{E}_i$. In the simplest case, the spaces $\mathcal{E}_i$ and $\mathcal{C}$ are orthogonal, and we are able to reliably detect this error by measurement of the projector on $\mathcal{E}_i$ (the so-called syndrome operator). Evidently, this is not always possible, e.g., in the case where the generator of the signal is proportional to the error operation element $H \propto E_i$, and any conceivable QEC code will also "correct" the signal. In what follows, we derive a general set of conditions under which QEC can be employed to improve metrology.

General Formalism.—Let us assume we have N detector qubits to measure the parameter ω (e.g., a magnetic or electric field) of a Hamiltonian

$$H_s = \frac{\omega}{2}\mathcal{G}, \tag{1}$$

where $\mathcal{G}$, in general, can be any sum of single- or multiqubit operators. During the evolution, the qubits are subject to some arbitrary form of noise which is described by the quantum operation [20]

$$\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger. \tag{2}$$

If we further denote $\mathcal{M}\rho = e^{-iHt}\rho e^{iHt}$, the goal of QEC for metrology is to design a recovery operation $\mathcal{R}$, such that

$$(\mathcal{R}\circ\mathcal{E}\circ\mathcal{M})(\rho) \propto \mathcal{M}\rho, \tag{3}$$

for all states within a certain quantum code $\rho \in \mathcal{C} \leq \mathcal{H}$. Note, that Eq. (3) has to be understood in the short-time limit where $\mathcal{E}\circ\mathcal{M} \approx \mathcal{M}\circ\mathcal{E}$; i.e., recovery operations have to be applied on time scales short compared to the noise rate γ. Defining P as the projector on the code space $\mathcal{C}$, the recovery operation $\mathcal{R}$ exists if and only if the two conditions (i) $[\mathcal{G}, P] = 0$, (ii) $PE_i^\dagger E_j P = A_{i,j}P$, are fulfilled, with $A = (A_{i,j})$ being a hermitian matrix. Condition (ii) guarantees that the error operation elements E_i map the code space onto orthogonal and undeformed subspaces. One can show in a constructive proof [20] that this implies the existence of an $\mathcal{R}$ fulfilling Eq. (3) $\forall\, \mathcal{M}\rho \in \mathcal{C}$. Because of condition (i), $\mathcal{C}$ is an invariant subspace of $\mathcal{G}$, such that $\mathcal{M}(\rho) \in \mathcal{C}$, $\forall \rho \in \mathcal{C}$, proving the existence of $\mathcal{R}$ as defined in Eq. (3).

However, these conditions alone also allow for solutions in which the generator $\mathcal{G}$ acts as the identity on the code. Obviously, such a code is useless for metrology, since the action of the Hamiltonian yields a global phase on the code states. To exclude these trivial solutions, we further require the maximum quantum Fisher information [22] within the code space to be larger than zero (iii) $\xi \equiv \max_{|\Psi\rangle\in C}\langle\Delta\mathcal{G}^2\rangle_\Psi > 0$, where $\langle\Delta\mathcal{G}^2\rangle_\Psi = \langle\Psi|G^2|\Psi\rangle - \langle\Psi|G|\Psi\rangle^2$. Since the achievable precision in a noise-free measurement of ω is $\delta\omega \propto 1/\sqrt{\xi}$ [2], ξ also serves as a figure of merit which quantifies how useful a particular code $\mathcal{C}$ is for metrology.

Example.—Consider the model system of a single qubit subject to phase noise (pure dephasing) sensing a signal in x direction described by the Hamiltonian

$$H_s = \frac{\omega}{2}X_1, \tag{4}$$

where X_1 is the x Pauli operator (Z_1 and Y_1 denote the remaining Pauli matrices). Phase noise is described by the operation elements $E_0 = \sqrt{1-p}\mathbb{1}$, $E_1 = \sqrt{p}Z_1$, where p is the error probability. Using standard Ramsey spectroscopy, the qubit interrogates the parameter for the Ramsey time T, and after n repetitions we can determine the value of ω with accuracy [23]

$$\delta\omega \approx \frac{1}{T\sqrt{n}} = \frac{1}{\sqrt{T\tau}}, \tag{5}$$

where we defined the total measurement time $\tau = nT$. In the presence of noise, the maximal Ramsey time is limited, $T \leq 1/\gamma$ ($\Leftrightarrow p = \gamma T \leq 1$), resulting in the suboptimal measurement accuracy

$$\delta\omega \geq \sqrt{\frac{\gamma}{\tau}}. \tag{6}$$

Now, let us assume we have an ancilla at our disposal which neither interacts with the parameter nor is subject to noise. By defining the simple code spanned by the two states $|1\rangle \equiv |++\rangle$ and $|0\rangle \equiv |--\rangle$ [where $\pm$ in the first (second) slot represents X eigenstates of the single detector (ancilla) qubit], one readily checks that, $[H_s, P] = 0$, $\xi = 2$, and $A = \mathrm{diag}(1-p, p)$; i.e., the requirements for QEC are met. To perform the measurement, we initialize the system in the state $|\Psi\rangle = (|++\rangle + |--\rangle)/\sqrt{2} \in \mathcal{C}$. Under the action of the Hamiltonian, the state accumulates a phase $\phi = \omega t$: $|\Psi(t)\rangle \propto (|++\rangle + e^{-i\phi}|--\rangle)/\sqrt{2}$. If a Z error occurs the state is mapped to $|\Psi(t)\rangle \propto (|-+\rangle + e^{-i\phi}|+-\rangle)/\sqrt{2}$, such that the subsequent evolution reduces the phase, rather than increasing it, resulting in a randomized signal for $T \geq \gamma^{-1}$.

To implement QEC, we divide the Ramsey time T into r intervals of equal duration $\alpha = T/r$, and perform a QEC step $\mathcal{R}$ after each segment ($\mathcal{R}$ is assumed to be instantaneous on time scales of the evolution), as illustrated in Fig. 1. The QEC operation $\mathcal{R}$ consists of two steps: (1) Measuring the syndrome operator X_1X_2 (with X_2 acting on the ancilla spin). (2) For outcome -1: Application of a Z_1 gate. For outcome $+1$: No action is required. Although

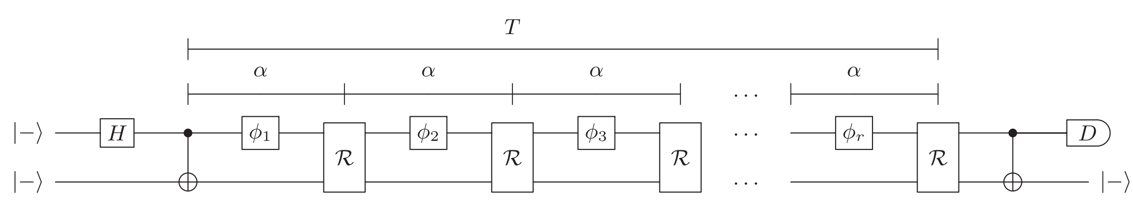

FIG. 1. Circuit model of the QEC for the model described by Eq. (4). The code state is prepared by application of a Hadamard (H) and CNOT gate. After each segment of free evolution of duration $\alpha = T/r$, the QEC operation $\mathcal{R}$ is applied. After the final decoding and measurement (D), the effective error rate has been reduced by a factor T/α.

single errors within a segment α can be corrected with the operation $\mathcal{R}$ (assuming perfect gates), they introduce a small phase uncertainty, due to the fact that the exact time of the error within the interval α is unknown. Despite this small residual uncertainty, we demonstrate in [24] that, by performing r QEC steps, we can extend the Ramsey time linearly to a value $T \to r\gamma^{-1}$, if $\alpha \ll \gamma^{-1}$. Consequently, after $r \approx \gamma\tau^{-1}$ repetitions, we can extend the interrogation time to its maximum value $T \to \tau$, and achieve the best sensitivity allowed by quantum mechanics

$$\delta\omega \approx 1/\tau. \tag{7}$$

This result is confirmed by numerical simulations displayed in Fig. 2. Even for relatively low repetition rates of the recovery operations, $\alpha\gamma = 1$, the linear, noise-free scaling is recovered. For imperfect recovery operations (failing with probability $p_{\mathrm{error}} = 10^{-3}$), and residual parallel noise components ($\gamma_\parallel = 10^{-3}\gamma$), a significant constant improvement is found.

Quantum metrology in the presence of perpendicular noise as described by Eq. (4) has been investigated in [7] for the case of multiparticle measurements. There, using the methods introduced in [5,6], a general precision bound is derived, yielding an optimal asymptotic scaling of the sensitivity $\delta\omega \propto 1/(N^{5/6}\sqrt{\tau})$. While this result represents a scaling better than the standard quantum limit (i.e., $\propto 1/\sqrt{N}$), it can further be improved by allowing for sequential feedback protocols, as represented by the QEC-based method we now suggest. Being provided with N detector spins we define the code $|1\rangle \equiv |+\rangle^{\otimes N}$ and $|0\rangle \equiv |-\rangle^{\otimes N}$ (note that here no ancilla is needed). Assuming

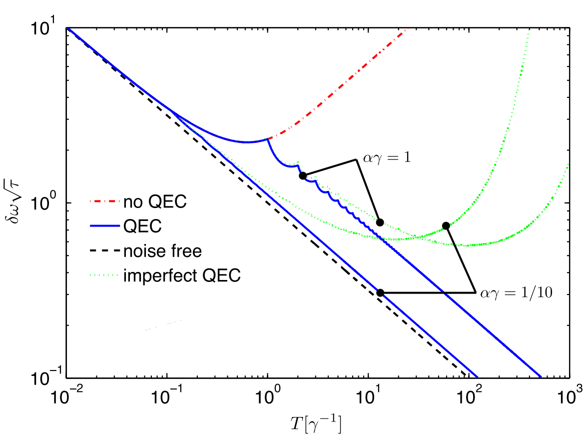

FIG. 2 (color online). Normalized estimation error $\delta\omega\sqrt{\tau}$ for the model of transversal noise (see text), for sufficiently large τ. In the standard approach (dashed-dotted line) the interrogation time has an optimal value $T \approx \gamma^{-1}$, limiting the achievable sensitivity. Ideally, QEC (solid line) can restore the noise-free scaling (dashed line) ($\propto 1/\sqrt{T}$) even for relatively small QEC repetition rates $1/\alpha = \gamma$. The dotted lines show the achievable sensitivity in the presence of a small parallel noise component $\gamma_\parallel = 10^{-3}\gamma$, and a probability $p_{\mathrm{error}} = 10^{-3}$ that the QEC operation fails.

independent Z noise acting on the individual detector spins, the error operation elements are given as $E_0 = \sqrt{1 - Np}\mathbb{1}$, and $E_i = pZ_i$ $(i = 1\ldots N)$, where we neglect operation elements of order $O(p^2)$ or higher. Again, one readily checks that all requirements for QEC are fulfilled with $\xi = (2N)^2$, indicating the potential for Heisenberg-limited spectroscopy. We prepare the system in $|\Psi\rangle = (|+\rangle^{\otimes N} + |-\rangle^{\otimes N})/\sqrt{2} \in \mathcal{C}$, which accumulates the phase Φ N times faster than uncorrelated qubits. In this situation, a single error Z_i can be detected by measuring the syndrome operators $X_{i-1}X_i$ and X_iX_{i+1}, and corrected by an appropriate π rotation. A single QEC operation $\mathcal{R}$ consequently involves $N - 1$ syndrome measurements of the operators $X_iX_i + 1$ $(i = 1\ldots N - 1)$. As above, repetitive application of $\mathcal{R}$ allows us to extend the Ramsey time to the maximum value $T \to \tau$, achieving, in principle, the Heisenberg limit of metrology [2] $\delta\omega \approx 1/(N\tau)$ with an optimal scaling in both resources time τ and particle number N [25].

These considerations demonstrate that, under certain conditions, QEC (and possibly other sequential feedback protocols) provides a way to surpass sensitivity bounds derived using the methods introduced in [5,6]. While these works include the possibility of feedback at the measurement stage they do not allow for feedback in a sequential fashion as suggested here. In contrast, the effects of sequential feedback have been considered in [4], where it was concluded that no improvement beyond the standard quantum limit can be found if the channel associated with the system evolution between two feedback operations is of full rank. However, this result does not contradict our findings, as, in our protocol, the QEC operation is employed explicitly in the short-time limit, where the channel associated with the particular noise model we consider is of less than full rank.

Applications.—QEC has recently been demonstrated experimentally in various different physical systems, such as trapped ions [26], superconducting qubits [27], and NV defect centers in diamond [28,29]. In the following, we consider an example from solid state nanosensing using NV centers in which our approach can be applied under realistic experimental conditions. Recent work [30] has suggested and experimentally demonstrated the use of NV centers for the sensing of electric fields with high sensitivity and spatial resolution, e.g., for the biological imaging of neural activity [31–35]. NV centers are optically addressable diamond lattice defects with a stable paramagnetic ground state of spin $S = 1$ [36]. In zero magnetic field, the spin state $|0\rangle$ is separated from the degenerate states $|1\rangle$ and $|-1\rangle$ by a splitting of $\omega_0 \sim 2\pi \times 3$ GHz. Electric fields perpendicular to the NV symmetry axis lift the remaining degeneracy by coupling the states $|1\rangle$ and $|-1\rangle$ at a strength $d_\perp = 17$ Hz cm V^{-1}. Identifying $|1\rangle$ and $|-1\rangle$ as the qubit states $(X_1 = |-1\rangle\langle 1| + |1\rangle\langle -1|)$, this enables the measurement of a dc or ac electric field using standard Ramsey spectroscopy. In the case of ac measurements, a constant magnetic field has to be applied to bring the $|1\rangle \leftrightarrow |-1\rangle$ transition in resonance with the electric field frequency. Because of the large zero-field splitting, x and y magnetic noise is highly suppressed [by a factor $(\omega_0\tau_c)^2$], and the dominant noise contribution limiting the sensitivity is provided by magnetic field fluctuations in z direction, accounting for pure dephasing of the qubit states. Furthermore, generically, the NV electron spin is hyperfine-coupled to the nuclear spin of the constituting nitrogen atom (whose coherence times are well beyond those of the NV center [11]), enabling coherent two-qubit operations [37]. Other nearby nuclear species (^{13}C) have recently been used to implement a QEC protocol [28,29].

For the simple QEC code we consider, the QEC operation $\mathcal{R}$ using the ^{15}N nuclear spin as the ancilla can be done on the time scale of a few microseconds, without performing a full measurement and feedback loop, as described in [24]. This, in principle, allows extending the Ramsey time to the NV center population relaxation time $T \to T_1$. Specifically, let us consider the case of dc or low frequency field sensing, relevant, e.g., in the biological imaging of neural activity [31–33]. In this case, DD cannot be used to improve the spin coherence time, and, generically, the interrogation time is limited by $T_2^* \approx 1$–$100\ \mu$s. Since, depending on the operational conditions, T_1 ranges from 10 ms up to 1 s, our QEC approach could potentially improve the sensitivity by a factor of $\sqrt{T_1/T_2^*} = 10$–10^3. In the case of ac metrology, standard sensing experiments that use DD techniques such as Hahn echo or Carr-Purcell-Meiboom-Gill sequence can achieve a suppression of the noise by a factor $(\Delta t/\tau_c)^2$ [17], where Δt denotes the duration of a single decoupling sequence. Under typical experimental conditions, this results in an effective coherence time of the order of $10\ \mu$s–1 ms $\ll T_1$ (for shallow NV centers). In such experiments [30], QEC can still improve sensitivity by a factor of 3 to 300, reaching values of the order of 1–10 $\mathrm{V\,cm^{-1}\,Hz^{-1/2}}$ for a single NV.

A second application of the QEC protocol involves ac magnetometry with NV centers. In contrast to the conventional approach employing decoupling or double resonance techniques [38], we consider a scheme in which we tune the transition frequency between the $|0\rangle$ and $|1\rangle$ sublevels of the NV center ground state into resonance with the target ac field by applying an external magnetic field. As before, the use of a simple QEC protocol enhances the qubit coherence ideally to a value $\sim T_1$. As shown in [24], similar to the above case, this approach can improve the sensitivity by a factor of 10 to 10^3, and allows us to expand the operational frequency range to several GHz. For applications requiring the use of diamond nanocrystals, the improvement could, in principle, be markedly higher due to the lower initial spin coherence times. The above considerations include the possibility of bulk magnetic and electric sensing with a macroscopic number of uncorrelated NV center spin detectors in a sample, since the QEC operation does not require individual addressing or measurement of different detector spins.

In summary, we have presented a QEC-based approach to enhance the sensing accuracy in quantum metrology in the presence of noise. We demonstrated that our technique can improve the sensitivity of nanoscale magnetic and electric field sensors under realistic experimental conditions. Identifying further relevant physical situations in which QEC can be employed to improve sensing—possibly by using more involved codes based on multiple qubits or multilevel systems—remains an interesting task. In particular, the combination of the complementary techniques of QEC and DD in sensing protocols appears to be a promising path with potential applications in a large variety of fields [9,12,13,38]. From a theoretical perspective, our approach demonstrates that sequential feedback protocols can, in certain situations (e.g., [7]), surpass the sensitivity bounds derived with the methods of [5,6]. While the conditions we derived for perfect noise cancellation with QEC are restrictive, and applicable only to specific models, it remains an interesting question whether more general feedback protocols can be applied to more generic scenarios, possibly at the cost of imperfect noise suppression.

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