Quantum-Limited Amplification via Reservoir Engineering

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(Received 8 November 2013; published 4 April 2014)

We describe a new kind of phase-preserving quantum amplifier which utilizes dissipative interactions in a parametrically coupled three-mode bosonic system. The use of dissipative interactions provides a fundamental advantage over standard cavity-based parametric amplifiers: large photon number gains are possible with quantum-limited added noise, with no limitation on the gain-bandwidth product. We show that the scheme is simple enough to be implemented both in optomechanical systems and in superconducting microwave circuits.

DOI: 10.1103/PhysRevLett.112.133904 PACS numbers: 42.65.Yj, 03.65.Ta, 07.10.Cm, 42.50.Wk

Introduction.—The past few years have seen a resurgence of interest in amplifiers working near the fundamental limits set by quantum mechanics [1], in contexts varying from quantum information processing in circuits [2–4], to radio astronomy [5], to ultrasensitive force detection (e.g., for gravity wave detection [6]). The standard paradigm for a quantum-limited, phase-preserving amplifier is the nondegenerate parametric amplifier (NDPA) [7–10], which is based on a coherent interaction involving three bosonic modes (pump, signal, and idler). This interaction simply converts a pump mode photon into two photons, one in the signal mode, the other in the idler mode. The result is that weak signals incident on the signal mode are amplified, with the minimum possible added noise. There has been remarkable progress in realizing such amplifiers using superconducting circuits [11–18]. This in turn has enabled a number of breakthroughs, from the measurement of mechanical motion near the quantum limit [19], to the measurement of quantum jumps of a superconducting qubit [20,21] and the implementation of quantum feedback schemes [22,23].

Despite their many advantages, standard cavity-based parametric amplifiers suffer from the limitation of having a fixed gain-bandwidth product: as one increases the gain of the amplifier, one also reduces the range of signal frequencies over which there is amplification. This is a fundamental consequence of the amplification mechanism, which involves introducing effective negative damping to the signal mode. The consequent reduced damping rate determines the amplification, but also sets the amplification bandwidth (see, e.g., [3]). This tradeoff between gain and bandwidth can severely limit the utility of cavity-based parametric amplifiers in many applications. Traveling-wave parametric amplifiers (TWPAs) [24,25] do not use cavities and are in principle not limited in the same way. In practice, however, good device performance and bandwidth of TWPAs is limited by the requirement of phase-matching (though see Ref. [26] for recent progress in the microwave domain).

In this work, we introduce a new approach for quantum-limited amplification based now on three localized bosonic modes. Unlike a NDPA, our scheme explicitly involves dissipative (i.e., non-Hamiltonian) interactions between the modes. We show that this approach allows a large gain with quantum limited noise, but crucially is not limited by a fixed gain-bandwidth product: the gain can be arbitrarily large without any corresponding loss of bandwidth. Note that non-Hamiltonian evolution is also utilized in a very different way in probabilistic amplifiers [27–30], which can stochastically amplify signals without adding noise.

Our approach is related to reservoir engineering [31], where one constructs a nontrivial dissipative reservoir that relaxes a system to a desired target state (e.g., an entangled state [32–36]). Here, we instead construct an engineered reservoir which mediates a dissipative amplification process. Our mechanism can also be interpreted as a kind of coherent feedback process [37–40], where the amplification is the result of an autonomous quantum nondemolition (QND) measurement combined with a feedback operation. Our scheme is simple enough to be realized using existing experimental capabilities, either with three-mode optomechanical systems (where a mechanical mode couples to two electromagnetic cavity modes) [41,42], or with superconducting circuits [16,43,44].

Model.—While our scheme is amenable to many possible realizations, we focus here for concreteness on a three-mode optomechanical system. Two cavity modes (frequencies ω_1 and ω_2), are coupled to a single mechanical mode ω_M, cf. Fig. 1. The cavity photons interact with the mechanical mode via radiation pressure forces, and the system is described by the Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}_S + \hat{\mathcal{H}}_{\mathrm{diss}}$. Here, $\hat{\mathcal{H}}_S$ is the coherent system Hamiltonian ($\hbar = 1$),

$$\hat{\mathcal{H}}_S = \sum_{j=1,2} \{\omega_j + g_j(\hat{b} + \hat{b}^\dagger)\} \hat{a}_j^\dagger \hat{a}_j + \omega_M \hat{b}^\dagger \hat{b}, \qquad (1)$$

where $\hat{b}$ ($\hat{a}_j$) is the annihilation operator for the mechanical resonator (cavity j), and g_j is the optomechanical coupling strength for cavity j. $\hat{\mathcal{H}}_{\mathrm{diss}}$ describes the damping of all three modes (each by independent baths), and the laser drives on the two cavity modes; these are treated at the level

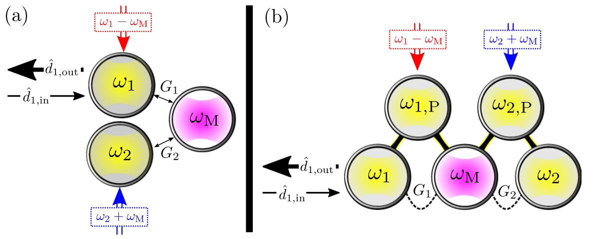

FIG. 1 (color online). (a) Schematic showing the optomechanical realization of the dissipative amplification scheme. Two driven cavities (1,2) are both coupled parametrically to a third auxiliary mechanical mode. The mechanics mediates a dissipative interaction between modes 1 and 2. Signals incident on either cavity are amplified in reflection. (b) Alternate realization, where two pump modes $\omega_{1,P}$, $\omega_{2,P}$ are used to generate the required interaction Hamiltonian, Eq. (2); this setup could be directly implemented using superconducting microwave circuits [46].

of standard input-output theory [3,45], resulting in cavity (mechanical) damping rates κ_j (γ).

In what follows, the two cavity modes will play roles similar to "signal" and "idler" modes in a NDPA, whereas the mechanics will be used to mediate an effective interaction between them. To achieve this, we assume a strong coherent drive on each cavity, detuned to the red (blue) mechanical sideband for cavity 1 (2), (i.e., drive frequencies $\omega_{L,1/2} = \omega_{1/2} \mp \omega_M$). We work in an interaction picture with respect to the free Hamiltonians, and perform displacement transformations: $\hat{a}_j \equiv \bar{a}_j e^{\pm i\omega_M t} + \hat{d}_j$, where $\bar{a}_j$ is the average classical amplitude of cavity j due to the laser drive; we take these to be real without loss of generality. Assuming the standard experimental situation where g_j are small and $\bar{a}_j$ are large, we linearize $\hat{\mathcal{H}}_S$, resulting in

$$\hat{\mathcal{H}}_S = G_1(\hat{d}_1 \hat{b}^\dagger + \hat{d}_1^\dagger \hat{b}) + G_2(\hat{d}_2 \hat{b} + \hat{d}_2^\dagger \hat{b}^\dagger) + \hat{\mathcal{H}}_{\mathrm{CR}}. \quad (2)$$

Here, $G_j = g_j \bar{a}_j$ are the many-photon optomechanical couplings, and $\hat{\mathcal{H}}_{\mathrm{CR}}$ describe nonresonant interaction processes. We focus on the good-cavity limit $\omega_M \gg \kappa_j, \gamma$, where the effects of $\hat{\mathcal{H}}_{\mathrm{CR}}$ will be negligible. We will thus start by dropping $\hat{\mathcal{H}}_{\mathrm{CR}}$ for transparency; i.e., we make the rotating wave approximation (RWA); full results beyond the RWA are presented in the figures and in the Supplemental Material [46].

If $G_1 = 0$, the Hamiltonian of Eq. (2) describes an optomechanical NDPA, with the mechanics acting as idler; this was recently realized by Massel *et al.* [47]. One might guess that turning on the beam-splitter interaction with cavity 1 by making $G_1 \neq 0$ would simply act to laser cool and optically damp the mechanical mode [48,49], but not fundamentally change the amplification physics. This is not the case: as we show below, the coherence between the control lasers leads to a completely new mechanism. For $G_1 \geq G_2$, the interactions in Eq. (2) have been discussed as a means to generate photonic entanglement [34,50–55]; amplification was not discussed. In contrast, we focus on the case $G_1 = G_2$; while this only leads to minimal intracavity entanglement [52], it is optimal in allowing the mechanics to mediate amplifying interactions between the two cavities.

Dissipative interactions.—If the mechanical resonator was strongly detuned (in the interaction picture) from the two cavity modes by a frequency Δ, then standard adiabatic elimination of the mechanics would yield the NDPA Hamiltonian, $\hat{\mathcal{H}}^{\mathrm{PA}} = \tilde{G}\hat{d}_1\hat{d}_2 + \mathrm{H.c.}$, with $\tilde{G} \sim G^2/\Delta$. In contrast, we are interested in the resonant case, where the induced interactions are more subtle. As the system is linear, one can exactly solve the Heisenberg-Langevin equations corresponding to Eq. (2), and use these to derive effective equations for the cavity modes with the mechanics eliminated [46]. We first consider the simple limit where $\gamma \gg \kappa, G$; this results in effectively instantaneous induced interactions. We also specialize to the ideal case where $\kappa_1 = \kappa_2 \equiv \kappa$ (see [46] for $\kappa_1 \neq \kappa_2$). Introducing the effective coupling rate $\Gamma = 4G^2/\gamma$, the resulting Langevin equations for the cavity modes are

$$\dot{\hat{d}}_1 = -\frac{(\kappa + \Gamma)}{2}\hat{d}_1 - \frac{\Gamma}{2}\hat{d}_2^\dagger - \sqrt{\kappa}\hat{d}_{1,\,\mathrm{in}} + i\sqrt{\Gamma}\hat{b}_{\mathrm{in}}, \quad (3a)$$

$$\dot{\hat{d}}_2 = -\frac{(\kappa - \Gamma)}{2}\hat{d}_2 + \frac{\Gamma}{2}\hat{d}_1^\dagger - \sqrt{\kappa}\hat{d}_{2,\,\mathrm{in}} + i\sqrt{\Gamma}\hat{b}_{\mathrm{in}}^\dagger. \quad (3b)$$

The operators $\hat{d}_{j,\mathrm{in}}$ ($\hat{b}_{\mathrm{in}}$) describe the quantum and thermal noise incident on the two cavities (the mechanics); they have zero mean and correlation functions $\langle \hat{o}_{\mathrm{in}}(t)\hat{o}_{\mathrm{in}}^\dagger(t')\rangle = \langle \hat{o}_{\mathrm{in}}^\dagger(t)\hat{o}_{\mathrm{in}}(t')\rangle + \delta(t-t') = \delta(t-t')(\bar{n}_o^T + 1)$, where $o = d_j$, b, and $\bar{n}_o^T$ is the thermal occupancy of each bath.

The mechanical resonator gives rise to two effects in Eqs. (3). First, it gives rise to an additional positive damping Γ of mode 1, and an additional negative damping $-\Gamma$ of mode 2; each effect corresponds simply to one of the two terms in Eq. (2). In contrast, the joint action of both interaction terms gives rise to terms in Eqs. (3) reminiscent of a NDPA, where $\hat{d}_1$ is driven by $\hat{d}_2^\dagger$ and vice versa. Note crucially the opposite sign of this term in Eq. (3a) versus Eq. (3b); this difference implies that these terms *cannot* be derived from an NDPA interaction Hamiltonian $\hat{\mathcal{H}}^{\mathrm{PA}}$. Instead, they correspond to an effective *dissipative* parametric interaction. Such terms can be obtained from Lindbladian dissipators in a quantum master equation [46]; they are also sometimes referred to as a phase-conjugating interaction [56]. On their own, such terms cause a coherent rotation between $\hat{d}_1$ and $\hat{d}_2^\dagger$, and as such no amplification. However, when combined with the mechanically induced damping and antidamping terms, one finds a striking result: the linear system described by Eqs. (3) always gives rise to exponential decay in the time domain at a rate $\kappa/2$, *irrespective* of the value of Γ [46]. Thus, unlike a standard paramp, the mechanically induced cavity-cavity interactions here do not give rise to a slow system decay rate, and do not cause any instability (i.e., the linear system is stable for all values of Γ). This conclusion holds even when γ/κ is finite: the system decay rates are independent of G [46].

Scattering properties.—While the mechanically induced interactions do not yield any net antidamping, they do nonetheless enable amplification. We use standard input-output theory to calculate the scattering matrix $\mathbf{S}[\omega]$ which relates output and input fields. For simplicity, we first neglect internal cavity losses. Introducing the cooperativity $\mathcal{C} = 4G^2/(\kappa\gamma)$, and defining the input and output vectors $\hat{\mathbf{D}}_l \equiv (\hat{d}_{1,l}, \hat{d}^\dagger_{2,l}, \hat{b}_l)^{\mathrm{T}}$ $(l \in \{\mathrm{in}, \mathrm{out}\})$, we find in the limit $\gamma \gg \kappa$, ω,

$$\hat{\mathbf{D}}_{\mathrm{out}}[\omega] = \mathbf{S}[\omega]\hat{\mathbf{D}}_{\mathrm{in}}[\omega], \tag{4}$$

$$\mathbf{S}[\omega] = \begin{pmatrix} \frac{2\mathcal{C}-1-\tilde{\omega}^2}{(1-i\tilde{\omega})^2} & \frac{2\mathcal{C}}{(1-i\tilde{\omega})^2} & \frac{2i\sqrt{\mathcal{C}}}{1-i\tilde{\omega}} \\ \frac{-2\mathcal{C}}{(1-i\tilde{\omega})^2} & -\frac{2\mathcal{C}+1+\tilde{\omega}^2}{(1-i\tilde{\omega})^2} & \frac{-2i\sqrt{\mathcal{C}}}{1-i\tilde{\omega}} \\ \frac{2i\sqrt{\mathcal{C}}}{1-i\tilde{\omega}} & \frac{2i\sqrt{\mathcal{C}}}{1-i\tilde{\omega}} & -1 \end{pmatrix}, \tag{5}$$

where $\tilde{\omega} = 2\omega/\kappa$. Note that at $\omega = 0$, the above result holds for any value of γ; the full expression of $\mathbf{S}[\omega]$ for arbitrary γ is given in the Supplemental Material [46]. For $\mathcal{C} > 1$, $\mathbf{S}[\omega]$ implies that signals incident on either cavity in a bandwidth $\sim\kappa$ around resonance will be amplified and reflected. For concreteness, we focus on signals incident on cavity 1 (see Supplemental Material [46] for the similar case of signals incident on cavity 2). The amplitude gain for such a signal at resonance is simply the reflection coefficient $S_{11}[0] = 2\mathcal{C} - 1 \equiv \sqrt{\mathcal{G}_1[0]}$. Clearly, the gain can be made arbitrarily large by increasing $\mathcal{C}$ with no corresponding reduction of bandwidth (which remains $\sim\kappa$). This is in stark contrast to a standard NDPA, and is a direct consequence of the behavior discussed above: the mechanically induced interactions do not induce any net negative damping of the system.

While for simplicity we have focused on the case where the mechanical damping γ is large, the same physics holds for an arbitrary κ/γ ratio. In the limit of large $\mathcal{C}$, the photon number gain is well approximated as

$$\mathcal{G}_1[\omega] \equiv |S_{11}[\omega]|^2 \simeq \frac{\mathcal{C}^2}{[1 + (2\omega/\gamma)^2][1 + (2\omega/\kappa)^2]^2}. \tag{6}$$

The effective bandwidth of the gain interpolates between κ for $\gamma/\kappa \gg 1$, and γ for $\gamma/\kappa \ll 1$. Our general conclusions still hold: the gain can be arbitrarily large by increasing $\mathcal{C}$, and there is no fundamental limitation on the gain-bandwidth product in this system.

Added noise.—Our scheme can also achieve a quantum-limited added noise. This follows immediately from the $\mathbf{S}$ matrix in Eq. (5). As usual, we define the added number of noise quanta of the amplifier by first calculating the noise spectral density of the amplifier output (i.e., $\hat{d}_{1,\mathrm{out}}[\omega]$). The contributions to this noise from the mechanical and cavity 2 input noises constitute the amplifier added noise. Expressing this as an equivalent amount of incident noise in the signal defines the number of added noise quanta $\bar{n}_{\mathrm{add}}[\omega]$; the quantum limit on this quantity in the large-gain limit is $\bar{n}_{\mathrm{add}}[\omega] \geq 1/2$ [3]. We find at zero frequency

$$\begin{aligned} \bar{n}_{\mathrm{add}}[0] &= \frac{(\sqrt{\mathcal{G}_1[0]}+1)^2}{\mathcal{G}_1[0]}\left(\frac{1}{2}+\bar{n}^T_{d_2}\right) + \frac{1+\sqrt{\mathcal{G}_1[0]}}{\mathcal{G}_1[0]}(1+2\bar{n}^T_b) \\ &= \frac{1}{2}+\bar{n}^T_{d_2} + \frac{2+2\bar{n}^T_{d_2}+2\bar{n}^T_b}{\sqrt{\mathcal{G}_1[0]}} + \mathcal{O}\left[\frac{1}{\mathcal{G}_1[0]}\right]. \end{aligned} \tag{7}$$

Thus, if cavity 2 is driven purely by vacuum noise, then in the large-gain limit our amplifier approaches the standard quantum limit on a phase-preserving linear amplifier. On some level, this is surprising. The ideal performance of a NDPA can be attributed to the fact that it has only a single additional degree of freedom beyond the signal mode [3]. In contrast, our system has two additional degrees of freedom (i.e., idler mode and mechanical mode); one might have expected that the presence of an extra mode would imply extra noise beyond the quantum limit. That this is not the case highlights the fact that the mechanical mode acts only as a means to mediate an effective dissipative coupling.

It is also worth stressing that in the large $\mathcal{G}_1$ limit, the contribution of mechanical thermal noise is suppressed by a factor $1/\sqrt{\mathcal{G}_1[0]}$. This is in stark contrast to the optomechanical NDPA of Ref. [47]. In that system, the mechanical mode acts as the idler; as such, quantum-limited performance is only possible if the mechanical resonator is at zero temperature, irrespective of the amplifier gain.

To illustrate the effectiveness of our scheme, we show in Fig. 2 expected results for the gain and added noise for a realization based on a microwave-cavity optomechanical system, similar to those in Refs. [57,58]. While such experiments typically have a small mechanical damping rate γ (and, hence, small bandwidth), one could use a third auxiliary mode to both laser cool the mechanical mode and enhance its linewidth [52]; we have assumed this situation. One could also use a GHz-frequency, low-Q mechanical resonator (similar to, e.g., Ref. [59]) to achieve bandwidths $\sim$10–100 MHz [46].

Connection to QND measurement.—To provide further intuition on the mechanism underlying our scheme, it is useful to consider the dynamics in terms of canonically conjugate quadrature operators. We introduce these operators in our interaction picture in the standard way: $\hat{d}_j \equiv (\hat{X}_j + i\hat{P}_j)/\sqrt{2}$ and $\hat{b} = (\hat{U} + i\hat{V})/\sqrt{2}$. The interaction Hamiltonian in Eq. (2) (with $G_1 = G_2 = G$) then becomes

$$\hat{\mathcal{H}}_{\mathrm{int}} = \sqrt{2}G(\hat{U}\hat{X}_+ + \hat{V}\hat{P}_-), \tag{8}$$

where we have introduced joint cavity quadrature operators $\hat{X}_\pm = (\hat{X}_1 \pm \hat{X}_2)/\sqrt{2}$, $\hat{P}_\pm = (\hat{P}_1 \pm \hat{P}_2)/\sqrt{2}$.

Equation (8) lets us understand the importance of having $G_1 = G_2$: for this choice, $\hat{X}_+$ and $\hat{P}_-$ are QND observables. They commute with the Hamiltonian and are thus conserved quantities. The QND interaction allows the mechanical resonator to "measure" both of these joint cavity quadratures: the $\hat{V}$ $(\hat{U})$ quadrature of the mechanical output field will contain information on $\hat{X}_+$ $(\hat{P}_-)$.

A QND measurement on its own will not generate amplification. The interaction in Eq. (8) does more: it also performs a kind of coherent feedback operation, where

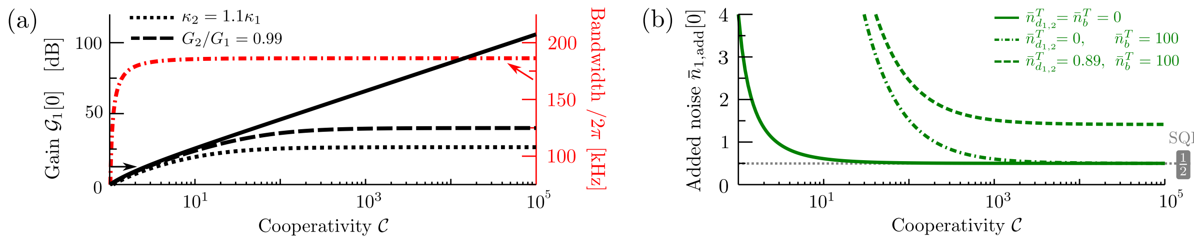

FIG. 2 (color online). (a) Black curves: photon number gain versus cooperativity $\mathcal{C}$ for parameters corresponding to a microwave-cavity optomechanical realization of the dissipative amplification scheme. We take $\omega_M/(2\pi) = 20$ MHz and $\kappa/(2\pi) = 1$ MHz (solid curve); the latter includes internal loss $\kappa^{\mathrm{int}}/(2\pi) = 10$ kHz. The dotted line includes the effect of asymmetric cavity damping (see legend); the dashed line shows instead the effects of $G_1 \neq G_2$ (see legend). We assume that the mechanical resonator is coupled to a third auxiliary cavity which is used to both cool and optically damp it, leading to a total mechanical damping rate of $\gamma/(2\pi) = 200$ kHz. Red dashed-dotted curve: bandwidth (defined as the full-width at half-maximum of $\mathcal{G}_1[\omega]$) versus $\mathcal{C}$, same parameters as the solid curve. (b) Amplifier added noise $\bar{n}_{\mathrm{add}}$ versus $\mathcal{C}$. Solid curve: mechanics and cavity 2 driven by vacuum noise only. Dashed-dotted curve: mechanics now driven by thermal noise. Dashed curve: both mechanics and cavity 2 driven by thermal noise. Other parameters identical to solid black curves in (a) and with $\bar{n}^T_{b,d_{1,2}}$ as denoted in the graph. All curves are produced without making the RWA (though they are well described by the RWA theory).

the results of the "measurement" are used to displace the unmeasured quadratures $\hat{X}_-$ and $\hat{P}_+$. For example, via the first term in Eq. (8), the mechanical $\hat{V}$ quadrature measures $\hat{X}_+$: at zero frequency (and ignoring noise), the Heisenberg equations of motion (EOM) yield $\hat{V} = -(2\sqrt{2}G/\gamma)\hat{X}_+$. But via the second term in Eq. (8), we see that $\hat{V}$ is a force on the $\hat{X}_-$ quadrature. Again, the EOMs at zero frequency yield $\hat{X}_- = (2\sqrt{2}G/\kappa)\hat{V} = -2\mathcal{C}\hat{X}_+$. This directly translates into the ($\omega = 0$) input-output relations

$$\hat{X}_{+,\,\mathrm{out}} = -\hat{X}_{+,\,\mathrm{in}}, \tag{9a}$$

$$\hat{X}_{-,\mathrm{out}} = 4\mathcal{C}\hat{X}_{+,\,\mathrm{in}} - \hat{X}_{-,\mathrm{in}}, \tag{9b}$$

where we neglect mechanical noise contributions. Thus, the joint measurement plus feedback operation has made $\hat{X}_-$ an amplified copy of $\hat{X}_+$, while leaving the QND observable $\hat{X}_+$ unperturbed. In an analogous fashion, $\hat{P}_+$ becomes an amplified copy of $\hat{P}_-$. If we now express $\hat{d}_{1,\mathrm{out}}$ in terms of joint quadratures, we can immediately understand how we obtain amplification. For large $\mathcal{C}$, we have

$$\begin{aligned}\hat{d}_{1,\mathrm{out}} &= \frac{1}{2}\sum_{\sigma=\pm}(\hat{X}_\sigma + i\hat{P}_\sigma)_{\mathrm{out}} \simeq \frac{1}{2}(\hat{X}_- + i\hat{P}_+)_{\mathrm{out}} \\ &\simeq \frac{1}{2}(4\mathcal{C})(\hat{X}_+ + i\hat{P}_-)_{\mathrm{in}} = 2\mathcal{C}(\hat{d}_1 + \hat{d}_2^\dagger)_{\mathrm{in}}. \end{aligned} \tag{10}$$

Thus, the QND measurement-plus-feedback operations on the joint quadrature operators directly let us understand the structure of the scattering matrix, and the observed amplification. Note that somewhat analogous QND interactions play a crucial role in the construction of continuous variable cluster states [60].

The QND form of Eq. (8) also explains the absence of any induced damping of the cavities by the mechanics, see [46]. When $G_1 \neq G_2$, the QND nature of the interaction is lost (i.e., X_+, P_- are no longer conserved), and thus for fixed G_2/G_1, $\mathcal{G}_1[0]$ saturates as a function of $\mathcal{C}_1$. The same is true when $\kappa_1 \neq \kappa_2$. One finds that in this case, the gain $\mathcal{G}_1[0]$ saturates at a value $[(\kappa_1 + \kappa_2)/(\kappa_2 - \kappa_1)]^2$ in the large $\mathcal{C}$ limit [46]. We stress that even with small coupling or damping rate asymmetries, one can achieve very large gains [see Fig. 2(a)] with no loss of bandwidth. One can even significantly increase the amplification bandwidth over the symmetric case, yielding amplitude-gain bandwidth products which far exceed κ (see Supplemental Material [46]).

Superconducting circuit realization.—Our scheme could also be realized in a superconducting circuit, where the required interactions in Eq. (2) are realized using Josephson junctions. Here, the role of the mechanical mode would now also be played by a microwave cavity mode, allowing γ to be large. Further details on such realizations are presented in the Supplemental Material [46], where we show that they offer advantages over conventional Josephson paramps, such as Ref. [61]. Using similar parameters to that work, our scheme can achieve quantum-limited amplification with a bandwidth of ~47 MHz and a amplitude gain-bandwidth product of ~1900 MHz, a factor of 3.8 larger than the device reported in Ref. [61]; unlike Ref. [61], this performance does not require a low-Q signal cavity. An optomechanical system using a high-frequency, low-Q mechanical resonator (like in the experiment of Ref. [59]) could also attain similar performance.

Conclusion.—We have described a new method for quantum-limited phase-preserving amplification which utilizes dissipative interactions; unlike standard cavity-based parametric amplifiers, it does not suffer from any fundamental limitation on the gain-bandwidth product. The scheme can be implemented both with optomechanics and with superconducting circuits.

This work was supported by the DARPA ORCHID program through a grant from AFOSR.

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