Dissipation in Ultrahigh Quality Factor SiN Membrane Resonators

S. Chakram, Y. S. Patil, L. Chang, and M. Vengalattore[*]
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853, USA
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We study the mechanical properties of stoichiometric SiN resonators through a combination of spectroscopic and interferometric imaging techniques. At room temperature, we demonstrate ultrahigh quality factors of 5×10^7 and a $f \times Q$ product of 1×10^{14} Hz. To our knowledge, these correspond to the largest values yet reported for mesoscopic flexural resonators. Through a comprehensive study of the limiting dissipation mechanisms as a function of resonator and substrate geometry, we identify radiation loss through the supporting substrate as the dominant loss process. In addition to pointing the way towards higher quality factors through optimized substrate designs, our work realizes an enabling platform for the observation and control of quantum behavior in a macroscopic mechanical system.

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Mesoscopic mechanical resonators with ultrahigh quality factors are ubiquitous ingredients in diverse applications of sensing, inertial navigation, and communications [1,2], as well as in foundational tests of quantum mechanics in macroscopic systems [3–6]. The quantum coherent control of these resonators and the realization of quantum-limited sensors require the cooling of these resonators to low phonon occupancies. However, to date, existing schemes of optomechanical cooling place stringent constraints on the mechanical and optical properties of these resonators. Crucially, overcoming the thermal coupling to the environment and the attainment of long coherence times require low mechanical resonance frequencies, a large "frequency–quality factor product" ($f \times Q > k_B T_{\mathrm{ambient}}/h$), and low optical absorption [7,8]. This combination has hitherto been difficult to achieve despite the exploration of a wide range of micro- and nanoscale systems. Thus, recent demonstrations of optomechanical cooling to the mechanical ground state [9,10] require cryogenic cooling of the mechanical system to reduce the thermal coupling to the environment.

Stoichiometric silicon nitride membrane resonators show great promise for optomechanics due to their high quality factors and low optical absorption [11–13]. Recent work has demonstrated $f \times Q$ products at room temperature as high as 2×10^{13} Hz in these resonators [14]. As such, the dissipation mechanisms of SiN resonators have been the focus of intense study, with several works shedding light on damping due to localized bulk or surface defects [15–17], thermoelastic damping (TED) [18], mechanism-independent intrinsic mechanisms of loss [19,20], as well as dissipation due to acoustic radiation into the supporting substrates [21–24]. Due to this wide range of loss processes, it has been challenging to clearly pinpoint the dominant source of dissipation in various parameter regimes as well as to identify potential routes to further enhance the quality factors of these resonators.

In this Letter, we demonstrate membrane resonators of stoichiometric silicon nitride with quality factors of 5×10^7 and a frequency-Q product $f \times Q \sim 1 \times 10^{14}$ Hz that is more than an order of magnitude greater than the requirement for ground state cooling and coherent quantum control of a room temperature optomechanical system. Further, we identify radiation loss through the supporting substrate as the dominant loss mechanism for a wide range of parameters. This finding points to further enhancements of the performance of such membrane resonators through appropriate material choice and design of the supporting substrate. In addition, our resonators are a promising platform for the observation and control of quantum behavior in a mesoscopic mechanical system.

The mechanical oscillators in our study are fabricated by NORCADA Inc., and consist of LPCVD silicon nitride square membranes under high tensile stress of around 0.8–0.9 GPa. The membranes range in thickness from $h \sim 30$–200 nm with lateral dimensions in the range $L \sim 0.5$–5 mm. The membranes are deposited on single crystal silicon wafers. The membranes constitute one arm of a Michelson interferometer while the other (reference) arm is actively stabilized against ambient vibrations. This realizes a precise measurement of the instantaneous position of the membrane with a sensitivity of 0.1 $\mathrm{pm/Hz}^{1/2}$ for typical powers of 200 μW incident on the membrane. The optical measurements are performed with a external-cavity diode laser operating at a wavelength of 795 nm. At this wavelength, real and imaginary parts of the refractive index of the membranes are measured to be $\mathrm{Re}(n) = 1.95 \pm 0.02$ and $\mathrm{Im}(n) < 10^{-5}$, respectively, leading to a peak reflectivity of 0.34 for the 100 nm thick resonators (see the Supplemental Material [25] for details).

From thin plate theory, the mechanical eigenfrequencies of the membranes are $\omega_{jk} = 2\pi\sqrt{\sigma/4\rho L^2}\sqrt{j^2 + k^2}$ where σ is the intrinsic tensile stress, $\rho = 2.7$ g/cm^3 is the mass density, and L is the lateral dimension of the membrane. We have confirmed that this relation is accurate at the 0.1% level by spectroscopy of the various modes and estimate the tension in the range of 0.8–0.9 GPa for the various samples studied.

The mechanical quality factors for various modes are measured through ringdown measurements of the membrane oscillation. For this, the membrane is piezoactuated at the various membrane resonances up to amplitudes of around 200 pm for durations of 10 ms before switching off the drive. The amplitude of membrane oscillation is then monitored through a lock-in amplifier to measure the $(1/e)$ decay time τ of the oscillation amplitude. The quality factor is then estimated as $Q_{jk} = \omega_{jk}\tau/2$, where ω_{jk} is the eigenfrequency of the mode under study.

Various checks of systematic effects on these measurements were made to ensure the validity of our interpretation. These include the negligible effect of radiation pressure or photothermal heating due to the laser field on the mechanical motion, the linearity of the drive, as well as the negligible influence of viscous damping at our operating background pressure of $p \sim 2 \times 10^{-7}$ Torr (see the Supplemental Material [25] for details). Changes in the peak amplitude by up to a factor of 5 in either direction do not change the measured value of the quality factor, indicating that we are operating far from any intrinsic nonlinearities of the mechanical resonator. For the typical amplitudes of mechanical motion during the ringdown measurements, self-stiffening nonlinearities were measured to be below the 1 ppm level. Finally, the mechanical linewidth inferred from thermal Brownian motion is consistent with that derived from the ringdown measurements.

We typically extend our measurements up to mode indices $(j^2 + k^2)^{1/2} \sim 40$. For higher mode indices, the rapidly increasing density of modes renders it challenging to accurately resolve individual modes. More importantly, we also observe substantial intermodal coupling between proximal modes, which complicates the interpretation of the measured quality factors.

We distinguish between two regimes of behavior. (i) For mode indices $(j^2 + k^2)^{1/2} < 4$, the measured Qs exhibit highly nonmonotonic behavior with increasing frequency. We also observe a large variation and a sensitive dependence of the quality factors on the clamping mechanism as well as the geometry of the modal structure. Our observations in this regime are quantitatively consistent with the dominant loss mechanism being anchor losses from the membrane into the supporting mount [21]. The sensitive dependence of the Qs on the clamping mechanism can be greatly reduced (filled diamond, Fig. 1) by ensuring minimal contact between the supporting silicon wafer and the in-vacuum mount, reinforcing the above interpretation. (ii) For mode indices $(j^2 + k^2)^{1/2} \gtrsim 4$, we observe a characteristic variation of Qs within any frequency band that we discuss in detail in a later section. In Fig. 1, we show the largest measured Qs (filled square) within each

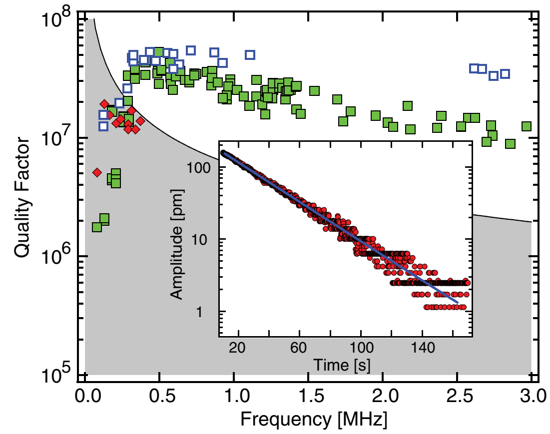

FIG. 1 (color online). Peak mechanical quality factors of a $L = 5$ mm, $h = 100$ nm SiN membrane versus frequency (filled square). The solid line corresponds to $f \times Q = k_B/h \times 300$ K. For low mode frequencies ($\nu_{jk} < 300$ kHz) , the quality factors can be improved by an order of magnitude (filled diamond) simply by reducing the contact region between the substrate and the in-vacuum mount. The weak frequency dependence of the measured Qs at high frequencies is further reduced for $h = 30$ nm (opened square) (see text for discussion). Inset: characteristic mechanical ringdown of the (5,5) mode at $\nu_{55} = 407$ kHz.

frequency band. These peak Qs reach a plateau around 5×10^7 and are a weak function of the resonant frequencies with a scaling estimated as $Q_{jk} \sim \nu_{jk}^{-(0.7\pm0.15)}$ for the 100 nm membranes. This scaling becomes much weaker as the thickness of the membrane is reduced (opened square, Fig. 1), resulting in Qs that are almost independent of frequency. In this latter regime, the measured Qs are more robust to variations in the clamp, making it less obvious that direct anchor loss is the dominant loss mechanism.

We have measured the peak Qs in the plateau regime for a range of membrane geometries ranging in width from $L = 0.5$–5 mm and in thickness from $h = 30$–200 nm (Fig. 2). For $L/h < 10^5$, we find the following scaling relations $Q \sim (L/h)^2$ and $f \times Q \sim (L/h)$. The noticeable discrepancy in this scaling for the thinnest membranes is, at present, unexplained [26].

We have performed a series of experiments to elucidate the limiting damping process of the mechanical excitations in the plateau regime. As mentioned earlier, the range of damping mechanisms in these membranes can range from a variety of intrinsic processes (TED, Akhiezer damping [27], defect induced dissipation, etc.) to extrinsic (anchor) loss. Based on models and experimental measurements described below, we find quantitative evidence that our current quality factors are limited by anchor losses from the membrane into the substrate.

First, we have developed a model of thermoelastic dissipation in our membrane resonators that takes into account their large intrinsic tensile stress of ~1 GPa. Within this model, the mechanical motion of the membrane couples to a local temperature field associated with microscopic changes in the volume of the resonator. This results in local irreversible heat flows and dissipation (see the Supplemental Material [25] for more details). Our model is built upon the formalism introduced in Refs. [28,29] and accurately reproduces the eigenfrequencies of our resonators for the entire range of geometries studied. Importantly, for established material parameters of stoichiometric SiN, our model predicts a room temperature limit of $Q_{\mathrm{TED}} \sim 10^{12}$ for the frequency ranges of our study. Further, our model also predicts that the quality factor scales as $Q \sim 1/\nu_{jk}^2$ in the frequency regime $\nu_{jk}/\nu_{11} > 10$ in distinct contrast to the weak dependence experimentally observed (Fig. 1). Based on these model predictions, we discount TED in the membrane as a limiting influence of our observed quality factors.

Another source of dissipation is the coupling between the mechanical motion and intrinsic, localized defects within the membrane. While the microscopic origins of these defects remain unknown, these losses are typically modeled as being due to two-level systems whose energy splitting is modulated by the oscillating strain field [30]. The subsequent re-equilibration of these two-level systems results in attenuation of the mechanical energy. At the elevated temperatures of our experiments, these two-level systems can be regarded as being thermally activated over a wide range of energy scales. For the mechanical frequencies in this work, the two-level system model [31] predicts a scaling of $Q_{jk} \sim 1/\nu_{jk}$ and a dissipation that scales very weakly with the modal structures, dimensions of the resonator, and details of the support structure. These predictions are inconsistent with our observations.

The quality factors for a given sample remain stable within 10% of the measured values even after exposure to air for several days. We have also annealed the membranes at temperatures up to 650 °C under vacuum to reduce surface contamination without a significant change in quality factors. These observations rule out surface-induced losses as a contributing influence.

In addition to the above observations that rule out specific intrinsic processes, a more mechanism-agnostic argument can be formulated for a large class of intrinsic dissipation mechanisms. For any such process that couples mechanical motion to a source of dissipation, the leading symmetry-allowed term in the equation of motion must be proportional to the local curvature of the displacement field (see, for example, Ref. [19]). Thus, the measured quality factors for the various modes should correlate with the local modal curvature or higher powers thereof. In order to better quantify this reasoning, we have developed an interferometric imaging technique capable of spatially resolving the modal structure of the resonator [32]. These images (Fig. 3) yield a wealth of spatial information complementing our spectroscopic measurements. In the plateau regime, we observe that the quality factors can vary by almost 2 orders of magnitude for a corresponding variation in the integrated curvature of less than 20%, pointing to an extremely weak correlation between the two quantities.

We also note that we have measured Qs up to 2.7×10^7 in low stress SiN membranes ($\sigma = 0.25$ GPa) of similar geometry, i.e., within a factor of 2 of those measured in the high stress membranes. Further, we observe a substantial influence of the substrate on the membrane modes (see Fig. 3) with nominally degenerate eigenmodes hybridizing into more symmetric structures. At larger drive amplitudes than those used in this study, we observe multimode bistability as a result of such coupling to the substrate [33]. Based on the preceding arguments, we conclude that

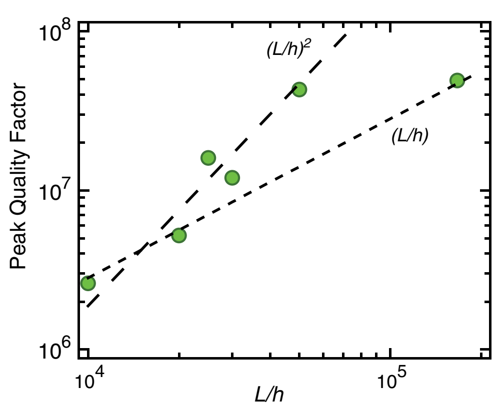

FIG. 2 (color online). Peak mechanical quality factors versus film geometry parametrized by the ratio of membrane width (L) to membrane thickness (h). For $L/h < 10^5$, we observe a scaling consistent with $Q \sim (L/h)^2$. A linear scaling is also shown for reference.

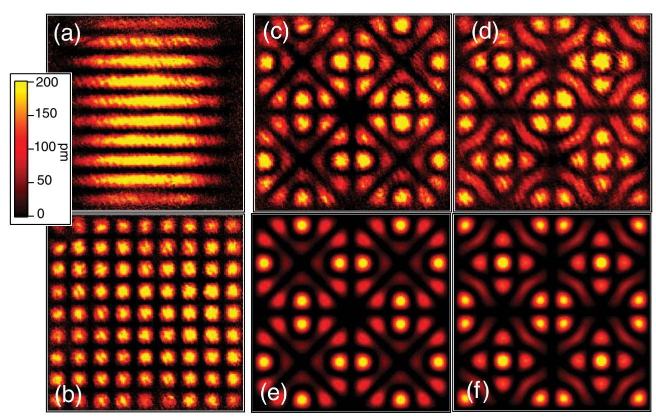

FIG. 3 (color online). Interferometric imaging of the mechanical modes: *in situ* images of the (a) (1,10) mode and (b) (9,9) mode (color scale for displacement shown). Substrate-induced coupling between proximal asymmetric eigenmodes results in hybridization into more symmetric structures. (c),(d) The modal structures corresponding to $\phi_{10,6} \pm \phi_{6,10}$ hybridized modes. These can be compared to the calculated mode profiles for this hybridization (e),(f).

there is little or no influence of intrinsic material processes on our measured Qs.

Finally, we discuss the role of radiation loss from the membrane into the supporting substrate. This loss of mechanical energy arising from the coupling to the external supporting substrate represents one of the fundamental restrictions to a high-Q device that is only weakly dependent on the material parameters of the resonator. Accordingly, various treatments and models of this radiation loss [34–36] have been developed to address design methodologies that can alleviate this loss. A particularly intuitive picture of "phonon tunneling" [21,22] has recently emerged, wherein the resonator can be regarded as a phononic cavity coupled to the external substrate through a weak coupling parameter.

Guided by our Q-factor measurements and modal images, we have extended this model to the higher mode indices of the plateau regime and obtain excellent agreement (see the Supplemental Material [25] for details). In order to decouple the geometrical dependence of the quality factor from other frequency-dependent factors, we compare the model to Qs measured over a narrow range of frequency corresponding to an arc of radius $\sqrt{j^2+k^2} \sim 12$ in the (j, k) space. In particular,

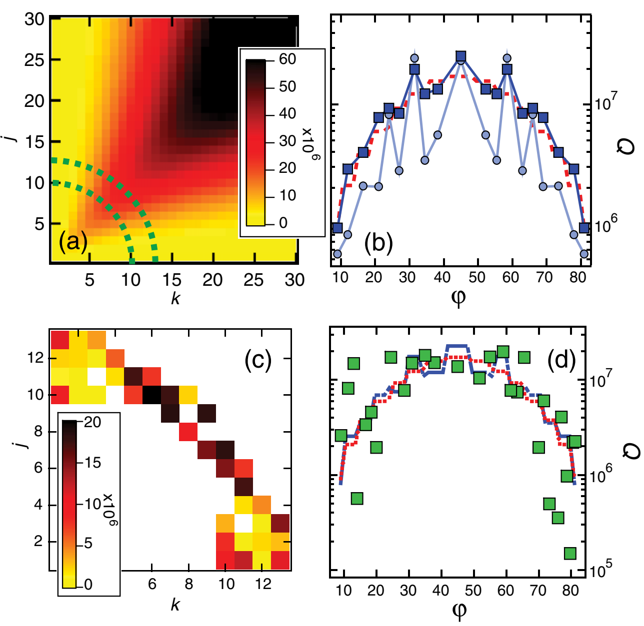

FIG. 4 (color online). (a) Predicted quality factors versus mode indices based on asymptotic limits of our anchor loss model (see the Supplemental Material [25] for details), (b) Predicted quality factors versus $\phi \equiv \arctan(j/k)$ for mode indices $(j^2+k^2)^{1/2} = 12$ with (without) substrate-mediated hybridization are shown as filled square (filled circle). Also shown is the asymptotic expression for the quality factors from our model (dashed line). (c) Measured Qs for mode indices indicated by the green arc in (a). (d) The angle-averaged measurements from (c) are compared to our predictions from (b).

we see that the substrate-induced mode coupling and ensuing hybridization (see Fig. 3) suppresses the large Q variation between modes of even and odd parity that is seen at lower frequencies. Instead, we observe a more gradual variation with lower Qs measured for modes with either $j \ll k$ or $j \gg k$, and higher Qs measured for $j \sim k$ (see Fig. 4). The close agreement between our observations and the tunneling model (suitably modified to include interference effects arising from substrate-induced hybridization) further clarifies the dominant role of anchor losses in determining our peak quality factors.

Our modified treatment of the phonon tunneling model accurately captures the increased robustness and enhanced quality factors arising from the interference between nominally degenerate resonator modes. One can extend this logic to the complementary scenario, i.e., where interference between distinct substrate modes leads to a similar enhancement of the quality factors. The extreme limit of this latter scenario is when the substrate exhibits an acoustic band gap in the vicinity of the relevant resonator modes. This should lead to a vastly enhanced quality factor and $f \times Q$ product that is limited, in principle, only by the material properties of the membrane.

In summary, we demonstrate stoichiometric SiN membrane resonators with ultrahigh quality factors up to 5×10^7 and $f \times Q \sim 10^{14}$ Hz that is, to our knowledge, the largest reported in a room temperature mechanical system. We have developed models of our system that identify anchor loss as the dominant decay mechanism that limits the current quality factors. Remarkably, the material properties of stoichiometric silicon nitride do not limit the performance of these membrane resonators even at such high quality factors. The true intrinsic material limitations of stoichiometric SiN on the quality factor and $f \times Q$ products of our resonators remain a topic for further study.

Our current system realizes an enabling platform for the optomechanical cooling and quantum control of a mesoscopic resonator. Further, the low mechanical frequencies and ultrahigh quality factors demonstrated in this work are ideally suited to various schemes to interface atomic gases or solid state spin systems to the mechanical degree of freedom, thereby realizing hybrid quantum devices for sensor and transduction applications as well as for fundamental studies [37].

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*mukundv@cornell.edu

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