Broken SU(4) Symmetry and the Fractional Quantum Hall Effect in Graphene

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We describe a variational theory for incompressible ground states and charge gaps in the $N = 0$ Landau level of graphene that accounts for the fourfold Landau level degeneracy and the short-range interactions that break SU(4) spin-valley invariance. Our approach explains the experimental finding that gaps at odd numerators are weak for $1 < |\nu| < 2$ and strong for $0 < |\nu| < 1$. We find that in the SU(4) invariant case the incompressible ground state at $|\nu| = 1/3$ is a three-component incompressible state, not the Laughlin state, and discuss the competition between these two states in the presence of SU(4) spin-valley symmetry-breaking terms.

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Introduction.—The fractional quantum Hall effect (FQHE) is a transport anomaly that occurs whenever a two-dimensional electron system (2DES) in a strong perpendicular magnetic field has a gap for charged excitations at a fractional value of the Landau level (LL) filling factor. Gaps at fractional filling factors can only be produced by electron-electron interactions. The FQHE has, therefore, been a rich playground for the study of strongly correlated phases of the electron liquid, hosting a variety of exotic phenomena including fractional and non-Abelian quasiparticle statistics [1] and electron-hole pair superfluidity [2].

Since its discovery [3] more than three decades ago, the FQHE has been studied almost exclusively in the 2DESs formed near GaAs/AlGaAs heterojunctions. Because of their small Zeeman to cyclotron energy ratio [4], the electron spin degree of freedom in the $N = 0$ LL of the GaAs conduction band is often experimentally relevant, endowing the FQHE with ground and quasiparticle states that would not occur in the spinless fermion case [5].

The $N = 0$ LL of monolayer graphene is nearly fourfold degenerate because of the presence of spin and valley degrees of freedom and is partially occupied over the filling factor range from $\nu = -2$ to 2, opening the door to SU(4) manifestations of the FQHE. However, because graphene sheets on substrates generally have stronger disorder than modulation-doped GaAs/AlGaAs 2DESs, it has until recently not been possible to observe their fractional quantum Hall effects. Recent studies of high-quality graphene samples have started to clear the fog [6–11], however, and the view that has emerged is surprising. Experiments indicate that the graphene FQHE is stronger for $0 < |\nu| < 1$ than for $1 < |\nu| < 2$ and that phase transitions between distinct states at the same ν value occur as a function of magnetic field strength [10,11]. In this Letter, we shed light [12] on these trends by using a variational approach to account for weak SU(4) symmetry breaking and by constructing rules that allow SU(4) FQHE states in the range $0 < |\nu| < 1$ to be generated starting from well-known *seed* states in the range $1 < |\nu| < 2$. Surprisingly, we find that in the absence of symmetry-breaking terms the ground state at $|\nu| = 1/3$ is *not* of the simple Laughlin type.

Hard-core SU(4) states.—We begin by considering the SU(4) invariant Coulomb-interaction model in the $N = 0$ LL. It is convenient to define a filling factor measured from the empty $N = 0$ LL: $\tilde{\nu} \in [0, 4] = 2 + \nu$. Progress can be achieved by starting from $\tilde{\nu} \leq 1$ zero-energy eigenstates of the V_0 hard-core model, in which only the $m = 0$ Haldane pseudopotential is nonzero [13]. We will refer to these states as *seed states* in the remainder of this Letter. Note that the manifold of seed states is large and includes many states that are not relevant at low energies. However, our assumption is that the Coulomb interaction will select a ground state from among those states on the basis of $m > 0$ Haldane pseudopotentials. What is crucial for what follows is that seed states are not influenced by the short-range interactions that break the SU(4) symmetry because they have zero probability for the spatial coincidence of particles. They can be written as a product of the Vandermonde determinant and a SU(4) bosonic wave function, which forces them to have filling factors $\tilde{\nu} \leq 1$ [14,15]. Because of the Pauli exclusion principle, seed states include all the single-component incompressible states like the Laughlin states [16], single-component composite fermion states [17,18], and Moore-Read states [19]. Several multi-component states, like the spin-singlet Halperin state at $\tilde{\nu} = 2/5$ [4], also belong to this class.

We now demonstrate that many important incompressible states with $\tilde{\nu} \in (1, 4]$ are simply related to $\tilde{\nu} \leq 1$ seed states. We first note that global particle-hole symmetry of the $N = 0$ LL maps eigenstates with $\tilde{\nu} \in [0, 2]$ to eigenstates at $4 - \tilde{\nu} \in [2, 4]$. This reduces our task to constructing states in $\tilde{\nu} \in (1, 2]$. In the following, we denote multicomponent states by a vector specifying the partial fillings of each nonempty component: $(\nu_1, \ldots, \nu_k)$, with $\tilde{\nu} = \sum_i \nu_i$. (We require $\nu_i \geq \nu_{i+1}$ to avoid double counting states that are related by a global SU(4) transformation.) Two simple mappings generate states in $\tilde{\nu} \in [1,2]$ from seed states in $\tilde{\nu} \in [0,1]$. The first is particle-hole conjugation restricted to two components, which maps (ν_1, ν_2) to $(1-\nu_2,\ 1-\nu_1)$ [12]. The second takes any seed wave function with three components or less, i.e., $(\nu_1, \ldots, \nu_k)$ with $k \leq 3$, and multiplies it by the Vandermonde determinant of one of the empty components, producing a state with flavor composition $(1, \nu_1, \ldots, \nu_k)$. Particle-hole conjugation involving three components does not yield states that cannot be obtained by combining these two rules.

We will focus on the states at $\tilde{\nu} = p/3$, with $p = \{1,2,4,5\}$. The $\tilde{\nu} \leq 1$ seed states are well known. The ground state for $\tilde{\nu} = 1/3$ is the Laughlin state, which is a SU(4) ferromagnet. At $\tilde{\nu} = 2/3$, the single-component particle-hole conjugate of the Laughlin state competes with the two-component singlet state with flavor composition (1/3, 1/3), which has lower Coulomb energy [20] and can be thought of as a composite fermion state with negative effective field [18]. At $\tilde{\nu} = 4/3$, we obtain two competing states with flavor compositions (1, 1/3) and (2/3, 2/3), obtained by the two-component particle-hole conjugation from $\tilde{\nu} = 2/3$. These two states are well known from work on the FQHE of spinful fermions. However, at $\tilde{\nu} = 5/3$ we obtain two states by acting on the seed states at $\tilde{\nu} = 2/3$ with the second mapping. These states have flavor compositions (1, 2/3) and (1, 1/3, 1/3) The appearance of a three-component state at $\tilde{\nu} = 5/3$ demonstrates that there is no reason to anticipate a simple relationship between $\tilde{\nu}$ and $2-\tilde{\nu}$ states in graphene. The (1, 1/3, 1/3) state has not previously been discussed as a possible $|\nu| = 1/3$ ground state.

The energy of any state constructed via these mapping rules can be calculated provided the energy of the seed state is known. If the Coulomb energy per flux quantum of the seed state is $E_{\tilde{\nu}}$, then, the energies of the states obtained are, respectively,

$$E_{2-\tilde{\nu}} = E_{\tilde{\nu}} + (1-\tilde{\nu})2E_1, \quad E_{1+\tilde{\nu}} = E_{\tilde{\nu}} + E_1, \tag{1}$$

where $E_1 = -\sqrt{\pi/2}e^2/2\epsilon l$ and l is the magnetic length. This allows us to predict the energetic ordering of the $\tilde{\nu} = p/3$ states. At $\tilde{\nu} = 4/3$, the two-component particle-hole conjugate of the singlet (2, 2/3, 2/3) has lower Coulomb energy than the (1, 1/3) state. At $\tilde{\nu} = 5/3$, Eqs. (1) predict that (1, 1/3, 1/3) has lower Coulomb energy than (1, 2/3). This observation is important, because the state that has been thought to be experimentally realized is (1, 2/3) [10–12], not (1, 1/3, 1/3). We note that, although our discussion has been centered around the incompressible ground states, the mappings and Eqs. (1) apply equally well to charged and neutral excited states generated from zero-energy eigenstates of the V_0 hard-core model.

Broken SU(4) symmetry.—It has become clear from experimental [21,22] and theoretical [12,23–26] studies that short-range valley-dependent corrections to the long-range SU(4) symmetric Coulomb interactions play a significant role in determining the ground state of the quantum Hall ferromagnet state realized at neutrality ($\tilde{\nu} = 2$) in graphene. In this section, we describe their influence on the $N = 0$ fractional quantum Hall regime. The symmetry-breaking interactions can be modeled as *zero-range* valley-dependent pseudopotentials [26],

$$H_a = \sum_{i<j,\sigma} V_\sigma \tau_\sigma^i |0\rangle_{ij\,ij}\langle 0| \tau_\sigma^j \tag{2}$$

where τ_σ^i is a Pauli matrix that acts on the valley degree of freedom of particle i, $\sigma = \{x, y, z\}$, $|0\rangle_{ij\,ij}\langle 0|$ projects the pair state of particles i and j onto relative angular momentum 0, and V_σ is a valley-dependent Haldane pseudopotential. Because conservation of total crystal momentum implies that the number of electrons in each valley is conserved, we have $V_x = V_y \equiv V_\perp$. The system's weakly broken SU(4) symmetry is, therefore, characterized by three parameters V_z, $V_\perp$, and by the Zeeman field strength h. The values of V_z and $V_\perp$ are dependent on the component of magnetic field perpendicular to the graphene plane $B_\perp$, whereas the Zeeman strength is determined by the total magnetic field; therefore, their relative strengths can be controlled by tilting the magnetic field away from the 2DES normal.

We assume that the symmetry-breaking terms are not strong enough to alter the Coulomb correlations of the SU(4) model states. Much as in the case of standard magnetic systems, the role of the anisotropy terms is to select the four-component spinors assigned to wave function components. Since more than one incompressible state might enjoy good Coulomb correlations at a given $\tilde{\nu}$, symmetry-breaking terms will also alter the energy balance between these states. In order to compute the contribution to total energy arising from the symmetry-breaking terms, we separate the spinors into those that are *completely* filled whose orbital wave function is a Slater determinant and those that are *fractionally* filled whose orbital wave function is a hard-core model zero-energy eigenstate [27]. The total anisotropy energy per flux quantum is

$$\epsilon_a = \frac{1}{2}\mathrm{tr}(P_i H_i^{\mathrm{HF}}) + \mathrm{tr}(P_f H_i^{\mathrm{HF}}) - \frac{h}{2}\mathrm{tr}(P_i \sigma_z), \tag{3}$$

where $P_i = |\chi_1\rangle\langle\chi_1| + \cdots + |\chi_k\rangle\langle\chi_k|$ is the projector onto the completely filled spinors, $P_f = \nu_{k+1}|\chi_{k+1}\rangle\langle\chi_{k+1}| + \cdots + \nu_4|\chi_4\rangle\langle\chi_4|$ is a weighed projector onto fractionally filled spinors , σ_z is a Pauli matrix acting on spin, and $h = g\mu_B B/2$. In Eq. (3) H_i^{HF} is the anisotropy contribution to the Hartree-Fock quasiparticle Hamiltonian that one would obtain if there were no fractionally occupied components,

$$H_i^{\mathrm{HF}} = \sum_\sigma V_\sigma[\mathrm{tr}(P_i\tau_\sigma)\tau_\sigma - \tau_\sigma P_i \tau_\sigma] - h\sigma_z. \quad (4)$$

The spinors which appear in the projection operators are fixed by minimizing the anisotropy energy. Equation (3) follows from the hard-core assumption, and from the following property of completely filled spinors:

$$\hat{\rho}_m(r)|\Psi\rangle = \frac{1}{2\pi l^2}|\Psi\rangle, \quad (5)$$

where $\hat{\rho}_m(r) \equiv \hat{P}_{LLL}(\sum_i \delta(\hat{r}_i - r)|\chi_m\rangle_{ii}\langle\chi_m|)\hat{P}_{LLL}$ is the particle density projected to the mth completely filled spinor. These equations can be viewed as a generalization of the Hartree-Fock theory of integer quantum Hall ferromagnets. In particular, Eq. (3) reproduces the anisotropy energy expressions in Ref. [26] for the special case of neutral graphene, i.e., for $\nu_1 = \nu_2 = 1$ and $\nu_3 = \nu_4 = 0$. It also reproduces the expressions of Ref. [12] for the special case where the fractionally filled spinors are assumed to have canted antiferromagnetic order.

Equation (3) can also be used to compute anisotropy energy contributions to the charge gaps. Assuming that quasiparticle states in the broken symmetry case evolve *adiabatically* from SU(4) states, we label them by SU(4) quantum numbers. Quasielectron-quasihole pair states can be labeled by integers that specify changes in the occupation numbers for each flavor relative to the incompressible ground state. Assuming that flavor flips involve only the fractionally filled and empty spinors, the integers satisfy $\delta N_{k+1} + \cdots + \delta N_4 = 0$ [28]. We find that the gap of an incompressible state is the SU(4) Coulomb gap plus the following correction:

$$\Delta_a = \sum_{j=k+1}^{4} \delta N_j \langle\chi_j|H_i^{\mathrm{HF}}|\chi_j\rangle. \quad (6)$$

Ground states and gaps at $\tilde{\nu} = p/3$.—The hard-core seed states at $\tilde{\nu} = 1/3$ and $\tilde{\nu} = 2/3$ do not experience the short-range valley-dependent interactions [11,12]. At $\tilde{\nu} = 1/3$ we, therefore, expect a fully spin polarized Laughlin state, with a remnant valley SU(2) symmetry. The quasiparticles are, therefore, expected to be large valley Skyrmions [5,29]. The gap is expected to be reduced by a factor of approximately 5, relative to the single-component case, to $\Delta_{1/3}^{\mathrm{sky}} \approx 0.023e^2/\epsilon l$ [5,29,30], possibly explaining why it is unobservable in suspended graphene samples [9–12,31]. At $\tilde{\nu} = 2/3$, we expect a fully spin polarized valley-singlet state. Two types of quasiparticles might be relevant at this filling fraction. In the absence of Zeeman terms, quasiparticles could lower their Coulomb energy by making flavor flips into the completely empty spinors. This is the behavior found for composite fermion wave functions at $\tilde{\nu} = 2/5$ [32]. A numerical study of SU(4) flavor-reversed quasiparticles would be needed to quantitatively assess this scenario at $\tilde{\nu} = 2/3$. At higher fields, one would recover the picture of fully spin polarized quasiparticles in the SU(2) valley space. The gap would then be [20] $\Delta_{2/3} = 0.0784e^2/\epsilon l$ [33].

Anisotropy has a greater impact for $\tilde{\nu} = \{4/3, 5/3\}$. At $\tilde{\nu} = 4/3$, we have two candidate incompressible states, namely, (1, 1/3) and (2/3, 2/3). To discuss their competition, it is convenient to perform a global particle-hole transformation to the states (1, 1, 2/3) and (1, 1, 1/3, 1/3), respectively. An analysis of the possible ordered phases leads to the phase diagram in Fig. 1 (Supplemental Material [34]). Experiments suggest canted antiferromagnetic order at $\tilde{\nu} = 2$ [22] and are [12] consistent with $V_\perp/h \sim -10$. According to the phase diagrams in Fig. 1, this would imply that the (1, 1, 2/3) state is a collinear antiferromagnet (CoAFM) in perpendicular field measurements, whereas (1, 1, 1/3, 1/3) is a canted antiferromagnet (CaAFM). We estimate that the critical field for the transition between (1, 1, 2/3) and (1, 1, 1/3, 1/3) states is

$$B_c = \frac{1}{(1 - h/|V_\perp|)^2}\left(\frac{\delta\epsilon_{2/3}^c}{h}\right)^2, \quad (7)$$

where $\delta\epsilon_{2/3}^c$ is the Coulomb energy difference per particle between the single-component state and the singlet at $\tilde{\nu} = 2/3$ and all the quantities on the right-hand side of

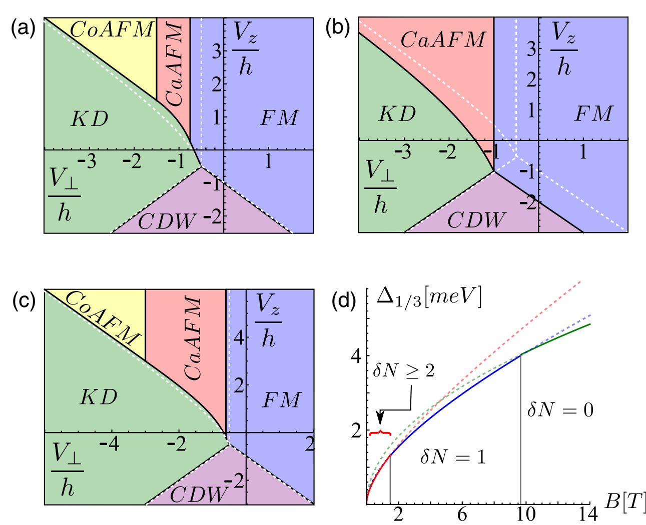

FIG. 1 (color online). Phase diagrams for (a) (1, 1, 2/3), (b) (1, 1, 1/3,1/3), (c) (1, 1, 1/3). The dashed lines are the boundaries of the integer quantum Hall ferromagnet states at neutrality [26]. The valley-dependent interaction parameters are believed to place graphene in the CoAFM region for panels (a) and (c) and in the CaAFM region for (b). FM, KD, and CDW correspond to ferromagnet, Kekule-distortion, and charge-density-wave phases, respectively (see the Supplemental Material [34]). (d) Field dependence of the gap for the Laughlin-type state (1, 1, 1/3), δN indicates the number of spin flips of the corresponding quasielectron-quasihole pair.

this equation are understood to be evaluated at 1 T. Exact diagonalization studies find that $\delta\epsilon^c_{2/3} \approx 0.009e^2/\epsilon l$ [20,35–37]. (Composite fermion trial wave functions significantly underestimate this difference, although they correctly predict the ground state to be a singlet [38].) In a SU(2) system like GaAs with symmetry broken only by the Zeeman term, the transition at $\tilde{\nu} = 2/3$ occurs at $B_c = (\delta\epsilon^c_{2/3}/h)^2$. Equation (7) reduces to this expression for $h \ll |V_\perp|$ because the anisotropy energy difference between the CoAFM and CaAFM states is dominated by Zeeman energies in this limit (Supplemental Material [34]). We, therefore, obtain that $B_c = 4.7$ T for $|V_\perp|/h = 10$ and $B_c = 6$ T for $|V_\perp|/h = 5$ [39], in agreement with experiment [11]. An analysis of the gaps for the states at $\tilde{\nu} = 4/3$ indicates that the quasiparticles involve a few flavor flips [41], in analogy with GaAs [36].

We will now discuss the competition at $\tilde{\nu} = 5/3$ between the three-component state (1, 1/3, 1/3) and the two-component state (1, 2/3). To the best of our knowledge, previous theoretical studies have assumed that the incompressible state at $\tilde{\nu} = 5/3$ is the (1, 2/3) Laughlin-type state, although the exact diagonalization study of Ref. [42], in which a finite Zeeman field was needed to stabilize the Laughlin state at $\tilde{\nu} = 7/3$, did provide a contrary hint. Note that $\tilde{\nu} = 7/3$ is the global particle-hole conjugate of $\tilde{\nu} = 5/3$. As previously shown, the singlet-type state (1, 1/3, 1/3) has lower Coulomb energy. When anisotropy is included, we find that the fully filled spinor of (1, 1/3, 1/3) is $|K,\uparrow\rangle$, while the fractionally filled spinors are $|K',\downarrow\rangle$ and $|K',\uparrow\rangle$ (Supplemental Material [34]). The anisotropy energy per flux quantum of this state exceeds that of the (1, 2/3) CoAFM state by $2(|V_\perp| - h)/3$. We, therefore, predict a transition from (1, 1/3, 1/3) to (1, 2/3) at the critical field,

$$B_c = \left(\frac{\delta\epsilon^c_{2/3}}{|V_\perp| - h}\right)^2, \tag{8}$$

where all quantities in the right-hand side are evaluated at 1 T. For $|V_\perp|/h = 10$, we obtain $B_c = 0.045$ T. (For $|V_\perp|/h = 5$ we obtain $B_c = 0.23$ T.) The transition field is small because $|V_\perp| \gg h$. This is consistent with the absence of an experimental transition in the field range where the FQHE is clearly observable [11]. Our estimates indicate, however, that this critical field increases with tilted magnetic field, making the realization of the three-component $\nu = 1/3$ state an experimental possibility [41].

Finally, we apply our formalism to determine the charge gaps of the particle-hole equivalent Laughlin-like states at $\tilde{\nu} = 5/3$ and $\tilde{\nu} = 7/3$, namely, (1, 2/3) and (1, 1, 1/3). In the perpendicular field configuration, these states are expected to be in the CoAFM phase (Supplemental Material [34]). For this state, there are two kinds of quasiparticles involving flavor flips. The first involves flips from the completely filled spinors. These quasiparticles have lower Coulomb energy but considerably larger anisotropy energy and are, thus, likely irrelevant in experiment (Supplemental Material [34]). We will focus on the second kind, which involve flips between the fractionally filled and the empty spinors. For the CoAFM state (1, 1, 1/3), we can choose the completely filled spinors to be $|K,\uparrow\rangle$, $|K',\downarrow\rangle$, and the 1/3 filled spinor to be $|K',\uparrow\rangle$. The quasiparticles can lower their energy by flavor flips from the spinor $|K',\uparrow\rangle$ into the unoccupied spinor $|K,\downarrow\rangle$. The anisotropy contribution to the gap from Eq. (6) per flavor flip is simply $2h$, the conventional single spin-flip Zeeman gap. This is analogous to the situation of GaAs at $\tilde{\nu} = 1/3$, where one expects the quasiparticles of the Laughlin state to involve a few spin flips up to magnetic fields $\sim$10 T [30,43–46]. Hence, it is likely that the quasiparticles of the $\tilde{\nu} = \{5/3, 7/3\}$ states in graphene involve a few spin flips as well.

Let us assess this scenario quantitatively. The conventional Coulomb gap of the Laughlin state without flavor flips is $\Delta^0_{1/3} \approx 0.1036e^2/\epsilon l$ [47]. The gap for a single flip corresponds to a spin-flipped quasielectron and a no-flip quasihole pair, and it is about $\Delta^1_{1/3} \approx 0.075e^2/\epsilon l$ [30,43,44,46]. The gap for two flavor flips, $\Delta^2_{1/3}$, is known with less accuracy but can be estimated to be lower than $\Delta^1_{1/3}$ by about $0.01e^2/\epsilon l$ [30,45,46], and it is expected to correspond to a single spin-flipped quasielectron and single spin-flipped quasihole pair. The predicted gap behavior is depicted in Fig. 1 and is in good agreement with experiment [10,11]. Figure 1 indicates that for most of the range probed in Refs. [10,11] the relevant quasiparticles involve a single spin flip.

In summary, we have developed a method to construct multicomponent incompressible and quasiparticles states for the $N = 0$ LL of graphene, starting from V_0 hard-core model seed states. We have provided simple variational formulas to determine how the short-range valley-dependent interactions select the broken-symmetry ground states and influence their gaps. We have applied this formalism to study the ground states and quasiparticles at $\tilde{\nu} = p/3$, revealing a previously unnoticed state with lower Coulomb energy than the Laughlin state at $\tilde{\nu} = 5/3$.

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[1] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008).
[2] J. Eisenstein and A. H. MacDonald, Nature (London) **432**, 691 (2004).

[3] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[4] B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983).
[5] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16419 (1993).
[6] X. Du, I. Skachko, F. Duerr, A. Luican, and E. Y. Andrei, Nature (London) **462**, 192 (2009).
[7] K. I. Bolotin, F. Ghahari, M. D. Shulman, H. L. Stormer, and P. Kim, Nature (London) **462**, 196 (2009).
[8] F. Ghahari, Y. Zhao, P. Cadden-Zimansky, K. Bolotin, and P. Kim, Phys. Rev. Lett. **106**, 046801 (2011).
[9] C. R. Dean, A. F. Young, P. Cadden-Zimansky, L. Wang, H. Ren, K. Watanabe, T. Taniguchi, P. Kim, J. Hone, and K. L. Shepard, Nat. Phys. **7**, 693 (2011).
[10] B. E. Feldman, B. Krauss, J. H. Smet, and A. Yacoby, Science **337**, 1196 (2012).
[11] B. E. Feldman, A. J. Levin, B. Krauss, D. A. Abanin, B. I. Halperin, J. H. Smet, and A. Yacoby, Phys. Rev. Lett. **111**, 076802 (2013).
[12] See D. A. Abanin, B. E. Feldman, A. Yacoby, and B. I. Halperin, Phys. Rev. B **88**, 115407 (2013) for a related analysis of the SU(4) fractional quantum Hall effect in graphene.
[13] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); F. D. M. Haldane, in *The Quantum Hall Effect*, edited by R. Prange and S. Girvin (Springer-Verlag, New York, 1987), Vol. 1, Chap. 8.
[14] A. H. MacDonald, H. A. Fertig, and L. Brey, Phys. Rev. Lett. **76**, 2153 (1996).
[15] K. Yang, S. Das Sarma, and A. H. MacDonald, Phys. Rev. B **74**, 075423 (2006).
[16] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[17] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).
[18] J. K. Jain, *Composite Fermions* (Cambridge University Press, Cambridge, England, 2007).
[19] G. Moore and N. Read, Nucl. Phys. **B360**, 362 (1991).
[20] X. C. Xie, Y. Guo, and F. C. Zhang, Phys. Rev. B **40**, 3487 (1989).
[21] A. F. Young, C. R. Dean, L. Wang, H. Ren, P. Cadden-Zimansky, K. Watanabe, T. Taniguchi, J. Hone, K. L. Shepard, and P. Kim, Nat. Phys. **8**, 550 (2012).
[22] A. F. Young, J. D. Sanchez-Yamagishi, B. Hunt, S. H. Choi, K. Watanabe, T. Taniguchi, R. C. Ashoori, and P. Jarillo-Herrero, Nature (London) **505**, 528 (2013).
[23] J. Alicea and M. P. A. Fisher, Phys. Rev. B **74**, 075422 (2006).
[24] I. F. Herbut, Phys. Rev. B **75**, 165411 (2007).
[25] J. Jung and A. H. MacDonald, Phys. Rev. B **80**, 235417 (2009).
[26] M. Kharitonov, Phys. Rev. B **85**, 155439 (2012); **86**075450 (2012).
[27] For some states, like (2/3,2/3), a global particle-hole conjugation might be needed in order to conceptualize its fractionally filled spinors as having a hard-core wave function.
[28] Quasiparticles might involve flavor flips from the completely filled spinors. In the cases of interest in this Letter, we bring back the problem into the current formulation by performing a global particle-hole transformation.
[29] K. Moon, H. Mori, K. Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka, and S.-C. Zhang, Phys. Rev. B **51**, 5138 (1995).
[30] A. H. MacDonald and J. J. Palacios, Phys. Rev. B **58**, R10171 (1998).
[31] See B. Hunt, J. D. Sanchez-Yamagishi, A. F. Young, M. Yankowitz, B. J. LeRoy, K. Watanabe, T. Taniguchi, P. Moon, M. Koshino, P. Jarillo-Herrero, and R. C. Ashoori, Science **340**, 1427 (2013) for evidence that additional valley symmetry-breaking terms are relevant in graphene-hexagonal boron nitride heterostructures.
[32] C. Tőke and J. K. Jain, J. Phys. Condens. Matter **24**, 235601 (2012).
[33] These quasiparticles do not involve valley flips, since additional flips in the background of a singlet state tend to increase the Coulomb energy of the quasiparticles, contrary to the situation for polarized states [36].
[34] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.126804 for additional details on the broken symmetry phases and the flavor reversed charged quasiparticles.
[35] K. Niemel, P. Pietilinen, and T. Chakraborty, Physica (Amsterdam) **284B–288B**, 1716 (2000).
[36] K. Výborný, A. F. Dethlefsen, R. J. Haug, and A. Wójs, Phys. Rev. B **80**, 045407 (2009).
[37] S. C. Davenport, Ph. D. thesis, University of Oxford, 2013; S. C. Davenport and S. H. Simon, Phys. Rev. B **85**, 245303 (2012).
[38] X. G. Wu, G. Dev, and J. K. Jain, Phys. Rev. Lett. **71**, 153 (1993).
[39] In graphene $h/(e^2/\epsilon l) \sim 0.001\epsilon\sqrt{B[\mathrm{T}]}$. We qualitatively include the impact of screening by using the RPA value of the dielectric constant of suspended graphene $\epsilon = 1 + \pi\alpha/2 \approx 4.6$ [40].
[40] I. Sodemann and M. M. Fogler, Phys. Rev. B **86**, 115408 (2012).
[41] I. Sodemann, and A. H. MacDonald (to be published). A forthcoming publication will discuss additional fractions and the influence of tilting the magnetic field away from normal.
[42] Z. Papic, M. O. Goerbig, and N. Regnault, Phys. Rev. Lett. **105**, 176802 (2010).
[43] T. Chakraborty, P. Pietiläinen, and F. C. Zhang, Phys. Rev. Lett. **57**, 130 (1986).
[44] E. H. Rezayi, Phys. Rev. B **36**, 5454 (1987).
[45] A. Wójs and J. J. Quinn, Phys. Rev. B **66**, 045323 (2002).
[46] A. F. Dethlefsen, R. J. Haug, K. Výborný, O. Čertík and A. Wójs, Phys. Rev. B **74**, 195324 (2006).
[47] G. Fano, F. Ortolani, and E. Colombo, Phys. Rev. B **34**, 2670 (1986).