Majorana Bound States in Two-Channel Time-Reversal-Symmetric Nanowire Systems

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We consider time-reversal-symmetric two-channel semiconducting quantum wires proximity coupled to a conventional s-wave superconductor. We analyze the requirements for a nontrivial topological phase and find that the necessary conditions are (1) the determinant of the pairing matrix in channel space must be negative, (2) inversion symmetry must be broken, and (3) the two channels must have different spin-orbit couplings. The first condition can be implemented in semiconducting nanowire systems where interactions suppress intra-channel pairing, while the inversion symmetry can be broken by tuning the chemical potentials of the channels. For the case of collinear spin-orbit directions, we find a general expression for the topological invariant by block diagonalization into two blocks with chiral symmetry only. By projection to the low-energy sector, we solve for the zero modes explicitly and study the details of the gap closing, which in the general case happens at finite momenta.

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Majorana fermionic bound states (MBS) are theoretically predicted to exist at the boundaries of topological superconducting states [1] and to have non-Abelian exchange statistics [2]. They are, therefore, promising proposals for realizations of elements of topological quantum computation [3], and currently there is an extensive search for candidate systems. Promising suggestions are hybrid condensed matter systems with s-wave superconductors proximity coupled to materials with strong spin-orbit coupling [4–7]. Recent theoretical proposals [8–10] for 1D and quasi-1D [11,12] topological superconducting systems and first experimental results [13–15] have received wide interest. Interestingly, the non-Abelian nature of the Majorana bound states can be explored also in 1D systems in a wire-network geometry [16,17].

All of the above refers to superconducting systems in the topological symmetry class D [7], where breaking of time-reversal symmetry (TRS) leads to a single localized MBS. With additional symmetry (BDI class) multiple nondisordered protected MBS are also possible in multichannel systems [18,19]. Recent papers have considered the possibility of realizing 1D topological superconductor systems with time-reversal symmetry (class DIII), supporting Majorana Kramers doublets in hybrid structures based either on superconductors with $d_{x^2-y^2}$-wave [20] or $s_{\pm}$-wave [21] pairing, noncentrosymmetric superconductors [22], bilayer 2D superconductors with spin-orbit coupling [23], or on 1D two-band models with a conventional s-wave supercoductor [24,25] under the assumption of a π phase difference between the pairing potentials in the two bands, mimicking the $s_{\pm}$ pairing considered in Ref. [21]. It is interesting to note that, even though two local MBS together form a usual fermion, the exchange of two Kramers pairs of MBS also constitutes a non-Abelian operation [26]. Moreover, just as for single MBS, the Kramers MBS can be detected either by tunneling spectroscopy or via unusual current-phase relations in a Josephson junction to an ordinary s-wave superconductor [27].

In this Letter, we investigate a model with two channels coupled to an s-wave superconductor; see Fig. 1. The two channels (wires) could be either two transverse modes in a single nanowire or separate nanowires, as illustrated in Fig. 1. We demonstrate that interwire pairing can give rise

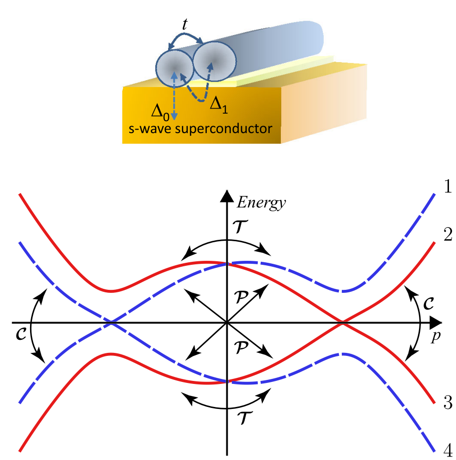

FIG. 1 (color online). Top: Sketch of the geometry of the two-channel (or two-wire) superconducting system. Bottom: General structure of the low-energy bands at the gapless transition point. Because the blocks in the block-diagonal Hamiltonian have chiral symmetry, the bands that cross at zero energy are related by $\mathcal{C}$, which means $E_2(p) = -E_3(p)$, while the bands that cross at $p = 0$ are related by TRS, $\mathcal{T}: E_1(p) = E_2(-p)$. Finally, particle-hole symmetry implies $\mathcal{P}: E_2(p) = -E_4(-p)$. This plot is generated using the model in Eq. (1), which assumes collinear spin-orbit coupling. However, it should be emphasized that the general structure is preserved even without this assumption.

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to a topologically nontrivial phase with Kramers MBS at the ends. This relies only on having different spin-orbit coupling in the two wires and on the square of the interwire pairing being larger than the product of the intrawire pairings. In a conventional (i.e., constant phase) s-wave superconductor, the latter condition cannot be achieved without interactions [21]. However, intrawire pairing can be significantly suppressed by repulsive intrawire interactions [28–30], thus enhancing the role of the interwire pairing.

Our analysis is carried out under the simplifying assumption of a collinear spin-orbit coupling axis in the two wires. However, the structure of the low-lying energy bands (shown in Fig. 1) relies entirely on the TRS and is, therefore, preserved even when relaxing this assumption. In the case of collinearity, the Hamiltonian can be block diagonalized into two blocks related by time reversal $\mathcal{T}$ and particle-hole $\mathcal{P}$ transformation. Each individual block has merely chiral symmetry $\mathcal{C}$ and is simple enough that we can give an analytical expression for the corresponding (class AIII) topological invariant, which changes with the gap closings.

The Bogoliubov–de Gennes Hamiltonian for the TRS two-channel nanowire system is

$$H_{\mathrm{BdG}} = \frac{1}{2}\int_{-\infty}^{\infty} \Psi^{+}(x)\mathcal{H}\Psi(x)dx, \tag{1}$$

where the first-quantization Hamiltonian is ($\hbar = 1$)

$$\mathcal{H} = \left(\frac{p^2}{2m} - \mu + V\lambda_z + t\lambda_x + (\alpha + \gamma\lambda_x + \beta\lambda_z)p\sigma_z\right)\tau_z + (\Delta_0 + \Delta_3\lambda_z + \Delta_1\lambda_x)\tau_x, \tag{2}$$

where μ is the chemical potential, V is the difference in electrical potentials, α and β are the symmetric and antisymmetric parts of the spin-orbit coupling coefficients, γ is the interwire spin-orbit coupling, and $\Delta_0 \pm \Delta_3$ and Δ_1 are the intrawire and interwire pairing potentials, respectively. Pauli matrices σ, λ, and τ act on the three two-dimensional spaces: spin, wire index, and electron-hole, respectively. In writing Eq. (2), we have used the conventional Nambu basis: $\Psi(x) = (\Psi_\uparrow(x), \Psi_\downarrow(x), \Psi_\downarrow^\dagger(x), -\Psi_\uparrow^\dagger(x))$.

The Hamiltonian (2) belongs to the topological symmetry class DIII with both antiunitary particle-hole and time-reversal symmetries, and hence unitary chiral symmetry [7,31]. In our basis, $\mathcal{T} = i\sigma_y\mathcal{K}$, $\mathcal{P} = \sigma_y\tau_y\mathcal{K}$, and $\mathcal{C} = i\mathcal{T}\mathcal{P}$, where $\mathcal{K}$ is complex conjugation. Because the Hamiltonian is block diagonal in spin space, we can write it as

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_{p,\uparrow} & 0 \\ 0 & \mathcal{H}_{p,\downarrow} \end{pmatrix}, \qquad \mathcal{H}_{p,\sigma} = \begin{pmatrix} \mathcal{H}^0_{p,\sigma} & \Delta \\ \Delta & -\mathcal{H}^0_{p,\sigma} \end{pmatrix}, \tag{3}$$

where $\mathcal{H}^0_{p,\sigma}$ and Δ are 2×2 matrices in wire-index space. The two blocks in Eq. (3) are related by time-reversal and particle-hole symmetry, $\mathcal{T}\mathcal{H}_{p,\uparrow}\mathcal{T}^{-1} = \mathcal{H}_{-p,\downarrow}$ and $\mathcal{P}\mathcal{H}_{p,\uparrow}\mathcal{P}^{-1} = -\mathcal{H}_{-p,\downarrow}$, which means that each block only has chiral symmetry $\mathcal{C}\mathcal{H}_{p,\sigma}\mathcal{C}^{-1} = -\mathcal{H}_{p,\sigma}$. Considered separately, the Hamiltonian in each block belongs to symmetry class AIII [7]. The gap of the spectrum of $\mathcal{H}$ vanishes for certain parameters, indicating a potential topological transition. The gap closing happens at finite momenta, which distinguishes this system from the above-mentioned $s_\pm$-wave pairing models [32]. This is illustrated in Fig. 1, which shows the generic situation for the low-energy bands at the point where the gap closes.

To establish that the closing and reopening of the gap is associated with a topological transition, a topological invariant is required. Since $\mathcal{H}_{p,\sigma}$ (in AIII) lacks particle-hole symmetry, a Pfaffian cannot be defined as for class D systems [1]. Nevertheless, we can still extract information about the sign of the gap from the square root of the determinant of the Hamiltonian. Transforming $\mathcal{H}_{p,\sigma}$ to

$$U\mathcal{H}_{p,\sigma}U^\dagger = \begin{pmatrix} 0 & \Delta - i\mathcal{H}^0_{p,\sigma} \\ \Delta + i\mathcal{H}^0_{p,\sigma} & 0 \end{pmatrix}, \tag{4}$$

using $U = \exp(i\tau_x\pi/4)$, the determinant reads

$$\det(\mathcal{H}) = |\det(\Delta + i\mathcal{H}^0_{p,\uparrow})|^2 |\det(\Delta + i\mathcal{H}^0_{p,\downarrow})|^2, \tag{5}$$

suggesting that the sign of the gap is encoded in the function $Z_p = \det(\Delta + i\mathcal{H}^0_{p,\uparrow}) = \det(\Delta + i\mathcal{H}^0_{-p,\downarrow})$. In fact, the winding number of $z_p = Z_p/|Z_p| = \exp(i\theta_p)$, defined as

$$W = \frac{1}{2\pi i}\int_{p=-\infty}^{p=\infty} \frac{dz}{z} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\frac{d\theta_p}{dp}, \tag{6}$$

takes only integer values since $z_{p=\infty} = z_{p=-\infty}$, and the topological invariant associated with the $\mathbb{Z}_2$ classification of the full class DIII Hamiltonian is given by $Q = (-1)^W$, similarly to the analysis by Tewari and Sau for BDI symmetry class models [33]. Nontrivial values of the winding number (topological invariant) are always related to the changes in the topology of the gapped system. In our model it corresponds to the number of Majorana bound states at each end of the nanowire.

Since, however, the winding number in Eq. (6) is not well suited for analytical evaluation, we shall instead determine the condition for a topological state directly from the determinant Z_p. This is done by first identifying the p values at which $\mathrm{Im}Z_p = 0$, giving two solutions:

$$p_{1(2)} = -m\alpha + \frac{m\gamma\Delta_1}{\Delta_0} + \frac{m\beta\Delta_3}{\Delta_0} \pm p_0,$$
$$p_0^2 = 2m\left(\mu + \frac{t\Delta_1}{\Delta_0} + \frac{V\Delta_3}{\Delta_0}\right) + m^2\left(\alpha + \gamma\frac{\Delta_1}{\Delta_0} - \beta\frac{\Delta_3}{\Delta_0}\right)^2. \tag{7}$$

Therefore, when p runs from $-\infty$ to $+\infty$, the complex number Z_p crosses the real axis exactly 2 times and encloses the origin if and only if

$$Q = \mathrm{sgn}[Z_{p_1} Z_{p_2}] = -1. \tag{8}$$

We can now draw some general conclusions. Firstly, it is straightforward (see Supplemental Material [34]) to show that the eigenvalues of the pairing matrix Δ must have different signs in order to have $Q = -1$. In other words, one must have $\det \Delta = \Delta_0^2 - \Delta_3^2 - \Delta_1^2 < 0$. Secondly, if we define an inversion symmetry by $\mathcal{I} = \lambda_x$, the Hamiltonian is inversion symmetric if $\mathcal{I}H(p)\mathcal{I} = H(-p)$. Setting the terms that break inversion symmetry to zero, i.e., $V = \alpha = \gamma = \Delta_3 = 0$, it can be seen that $Q = 1$. Therefore, inversion symmetry must be broken in order to have a topologically nontrivial phase. Finally, it follows that $Q = 1$ if $\gamma = \beta = 0$, which means that the spin-orbit matrix $\alpha + \gamma\lambda_x + \beta\lambda_z$ must have two different eigenvalues.

The full expression for the topological quantum number Q can be found algebraically, but is in general rather involved. Therefore, we present some special cases in the following. First, we write the result for the case $\Delta_3 = \gamma = 0$:

$$\begin{aligned} Q_{\Delta_3=0} &= \mathrm{sign}[A^2 - B^2], \\ A &= \Delta_0^2(V^2 + \delta^2 - 2m\alpha\beta V + \beta^2(p_0^2 + m^2\alpha^2)) \\ &\quad + t^2\delta^2, \\ B &= 2\Delta_0^2\beta p_0(m\alpha\beta - V), \end{aligned} \tag{9}$$

where $\delta^2 = \Delta_0^2 - \Delta_1^2$. From this it is evident that $\beta \neq 0$ is a necessary condition for a nontrivial phase, in agreement with the above general conclusion. If we further take $\alpha = t = 0$, the condition becomes

$$K_- < \Delta_1 < K_+, \tag{10}$$

with $K_\pm = \sqrt{(V \pm \sqrt{2\beta^2\mu m})^2 + \Delta_0^2}$. Clearly, this expression requires $\Delta_1 > \Delta_0$, which (as discussed in the introduction) could be realized due to repulsive interactions. Below, we look at the more general case of different intrawire pairings, in which case only $\Delta_1^2 > \Delta_0^2 - \Delta_3^3$ is required. In Fig. 2 the structure of the transition is shown for the $\Delta_3 = 0$ case. The top panels show the spectrum before, at, and after a transition point. In the middle panels, the corresponding Z_p trajectories in the complex plane are shown. Only when the parameter V is between the two

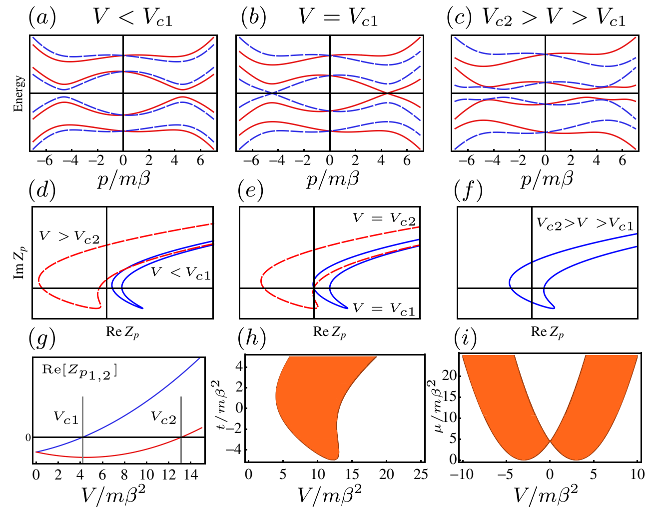

FIG. 2 (color online). (a)–(c) Spectrum of the two-wire model for three values of the potential difference V. In (a), the system is in the trivial state, $V < V_{c1}$, (b) is at the transition point, $V = V_{c1}$, while (c) shows the gapped spectrum in the topological phase. (d)–(f) The contour followed by the complex determinants Z_p from negative to positive values of p. In (d) the system is in the trivial state, which means V smaller than the lowest critical value V_{c1} or larger than the largest critical value V_{c2}. In (e) the contours go through the origin, which means that the gap closes at both $V = V_{c1}$ and $V = V_{c2}$, while in (f) the contour encircles the origin, signifying the topological state. The constant parameters in the plots are $\alpha = \gamma = 0$, $\Delta_1 = 10m\beta^2$, $\Delta_0 = 5m\beta^2$, $t = 0$, and $\mu = 10m\beta^2$, which gives $V_{c1} = 4.19m\beta^2$ and $V_{c2} = 13.13m\beta^2$. (g)–(i) Examples of the topological phase space. In (g) the real part of the Z_p determinant at $p = p_1$ and $p = p_2$ is shown as a function of V. For the topological criterion to be fulfilled, the product of the two function must be negative, which occurs for V between V_{c1} and V_{c2}. (h) Phase diagram when varying the potential V and the interwire tunneling t (same constant parameters as above), and (i) a phase diagram in the V-μ space (same as above except $\Delta_1 = 5m\beta^2$ and $\Delta_0 = 4m\beta^2$).

transition points does the trajectory encircle the origin, which is topologically different from the situation with $V < V_{c1}$ or $V > V_{c2}$. To illustrate the sign change of Q, the lower-left panel shows the real parts of Z_{p1} and Z_{p2}, which have different signs only in the topologically nontrivial regime, in accordance with the criterion in Eq. (8). Finally, the two lower-right panels show phase diagrams in two cuts of the parameter space, illustrating the robustness of the topological phase.

Now consider a different geometry with different intrawire pairings, i.e., $\Delta_3 \neq 0$. As an illustrative case, we can choose the parameters as $\Delta_0 = \Delta_3 = \Delta_1$, which means that the intrawire pairing in wire 2 is zero, while the interwire pairing is half of the intrawire pairing in wire 1. Further, taking $\alpha = \beta$ and $\gamma = 0$, meaning that only wire 1 has spin-orbit coupling, the topological condition becomes $(4tV + \Delta_0^2)\Delta_0^2 + 4t^2V^2 < 8m\beta^2t^2(\mu + t + V)$. This could, for example, be a good approximation, if one wire is badly

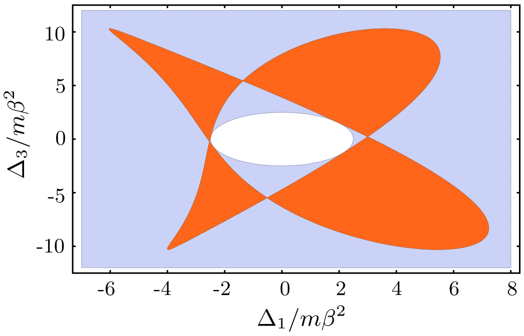

FIG. 3 (color online). Topological phase diagram in the Δ_1-Δ_3 plane for $\Delta_0 = 2.5m\beta^2$, $\alpha = 2\beta$, $\gamma = 0$, $V = 3m\beta^2$, and $\mu = t = 10m\beta^2$. The light gray region fulfills the condition that $\det\Delta < 0$, while the orange (dark gray) region corresponds the nontrivial topological phase.

connected to the superconductor. The more general situation, when the intrawire pairing is finite in both wires, is shown in Fig. 3.

A key feature of the topological phase is the existence of localized states at the boundaries. In the following, we find the general form of these modes using an effective model containing the low-energy bands shown in Fig. 1 at the transition point. The general form of the effective 1D Hamiltonian follows by projection onto the low-energy bands (see the Supplemental Material [34]):

$$H_{\rm low} = \left(\frac{p^2}{2m} - \tilde{\mu}\right)\tau_z + v(p\sigma_z - p_c)\tau_x, \tag{11}$$

where p_c is the momentum at which the gap closes and v and $\tilde{\mu}$ are effective parameters. This model describes a noncentrosymmetric superconductor because it contains both s- and p-wave components of the superconducting pairing potential and it is gapless when $\tilde{\mu} = \mu_c = p_c^2/2m$. If we consider a hard boundary and that the wires exist for $x > 0$, it is easy to show from the secular equation that solutions exist for $\tau\sigma < 0$ and $\tilde{\mu} > p_c^2/2m$. The two solutions then take the form $\psi_{1(2)}(x) = \chi_{1(2)} f_{1(2)}(x)$, in terms of the spinors $\chi_1 = (0,1,0,i)^T$ and $\chi_2 = (1,0,-i,0)^T$, and with $f_1 = f_2^*$ given by

$$f_1(x) = Ae^{-xmv}\sinh\left(x\sqrt{m^2v^2 - 2\tilde{\mu}m - 2imvp_c}\right), \tag{12}$$

where $A^2 = 8mv(\tilde{\mu} - \mu_c)/\sqrt{(2vp_c)^2 + (mv^2 - 2\tilde{\mu})^2}$.

The two zero modes $\psi_{1(2)}$ are *not* Majorana bound states, because they are not eigenstates of $\mathcal{P}$, but only of $\mathcal{C}$. These solutions are the chiral symmetry-protected Jackiw-Rebbi-type topological solitons [35–37]. We can, however, make linear combinations that are Majorana bound states. One example of a linear combinations that gives MBS (i.e., which fulfills $P\psi_M = \psi_M$) is

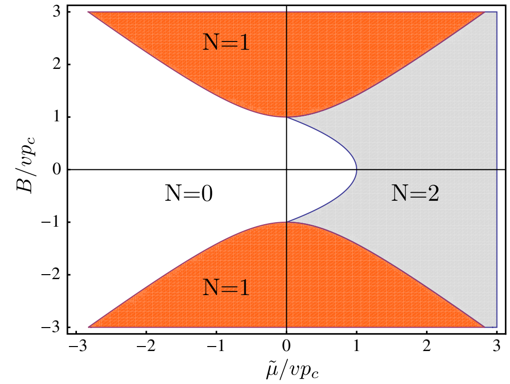

FIG. 4 (color online). Topological phase diagram for the low-energy model [Eq. (11)] in the presence of a magnetic field, assumed to be orthogonal to the z axis, and for $p_c = 2vm$. Orange (dark gray) regions have single localized MBS, while the light gray region has MBS doublets.

$$\psi_{M,1} = \frac{i\psi_1 + \psi_2}{\sqrt{2}}, \qquad \psi_{M,2} = \frac{\psi_1 + i\psi_2}{\sqrt{2}}. \tag{13}$$

These are MBS and transform to each other under TRS: $\mathcal{T}\psi_{M,2} = \psi_{M,1}$ and $\mathcal{T}\psi_{M,1} = -\psi_{M,2}$, which means that we have a Kramers pair of MBS.

Finally, we consider the effect of a Zeeman term that can split the two zero modes. The Hamiltonian (11) gets an additional time-reversal symmetry breaking term:

$$H_Z = \mathbf{B}\cdot\boldsymbol{\sigma}. \tag{14}$$

If the field points along the spin-orbit direction, the chiral symmetric states ψ_1 and ψ_2 are still eigenstates, but the degeneracy is lifted by $2B_z$. A more interesting case is when the magnetic field is perpendicular to the spin-orbit direction, for example, pointing in the x direction. Figure 4 represents the topological phase diagram in this case. Three distinct phases correspond to different numbers N of MBS in each end of the effective 1D system. At zero magnetic field the nanowire belongs to the DIII topological symmetry class and at a finite magnetic field to the BDI class (with effective time-reversal symmetry $\mathcal{T} = \sigma_x\mathcal{K}$). Topological phase transitions to the phase with $N = 1$ are associated with the gap closing at zero momentum and can be described by the equation $|B| = \sqrt{(vp_c)^2 + \tilde{\mu}^2}$. The transition between phases $N = 0$ and $N = 2$ is related to the gap closing at the Fermi momentum ($p = \sqrt{2m\tilde{\mu}}$) and can be described by the equation $vp_c = \sqrt{B^2 + 2v^2m\tilde{\mu}}$. Note that disorder that breaks the effective TRS splits the $N = 2$ MBSs, except at $B = 0$ (which is the DIII situation studied above), while the $N = 1$ regions are stable and merely reduce to class D.

To conclude, we have shown that a pair of time-reversal-symmetric nanowires proximity coupled to a superconductor can be driven into a nontrivial topological phase which supports a Kramers pair of Majorana bound states in each end. The key ingredients are interwire pairing and different spin-orbit interaction in the two wires. In the absence of interwire pairing, one needs intrawire pairing with different signs. With the assumption of parallel spin-orbit directions in the wires, the topological structure of the model could be determined from the AIII symmetric block diagonal parts of the full Bogoliubov–de Gennes Hamiltonian. However, we emphasize that the assumption of collinearity is not crucial for the existence of the topologically nontrivial phase. We have presented an analytical approach to find the topological invariant, which allows a general examination of the conditions for topological phases in systems using only ordinary s-wave superconductors, proximity coupled to wires with spin-orbit coupling.

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Note added in proof.—A recent paper [38] investigates the conditions for interaction-induced negative determinant of the pairing matrix in a two wire setup.

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