Fault-Tolerant Measurement-Based Quantum Computing with Continuous-Variable Cluster States

Nicolas C. Menicucci*
School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia
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A long-standing open question about Gaussian continuous-variable cluster states is whether they enable fault-tolerant measurement-based quantum computation. The answer is yes. Initial squeezing in the cluster above a threshold value of 20.5 dB ensures that errors from finite squeezing acting on encoded qubits are below the fault-tolerance threshold of known qubit-based error-correcting codes. By concatenating with one of these codes and using ancilla-based error correction, fault-tolerant measurement-based quantum computation of theoretically indefinite length is possible with finitely squeezed cluster states.

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Gaussian cluster states.—Quantum computing (QC) harnesses inherently nonclassical features of quantum physics to perform computations that would be impractical for any ordinary (classical) computer [1]. This requires making quantum systems interact in a carefully controlled and coherent manner, which is often very difficult. On the other hand, measuring quantum systems is usually much easier. Measurement-based QC makes use of this fact, replacing the difficulty of coherently controlling interactions between quantum systems with the up-front challenge of creating an entangled resource known as a cluster state [2], whereafter local adaptive measurements alone enable the full power of QC [3].

Normally in a quantum computer, quantum information is stored in qubits [1], but continuous-variable (CV) approaches also exist [4] in which wave functions over a continuous quantum variable are the basic information carriers. When it comes to measurement-based QC, optical CV cluster states [5,6] offer a distinct advantage over their optical-qubit counterparts [7,8] because they are much easier to make experimentally [9–11]. In fact, highly scalable experimental designs exist for creating very large CV cluster states [12–16], and an experimentally demonstrated 10 000-mode CV cluster state [9] now holds the world record for the largest entangled state ever created in which each constituent quantum system (in this case, a temporal packet of light) is individually addressable. This shatters the previous record of 14 trapped ions [17] by 3 orders of magnitude. Even more recently, a frequency-encoded CV cluster state has claimed second place with 60 entangled frequency modes and the promise of thousands more available [11].

This ease of experimental generation and scalability comes at the price of inescapable noise when these states are used for quantum information processing [18,19]. Ideal CV cluster states are unphysical [20], so when discussing their physical realization, one always speaks of Gaussian states [21] for which certain linear combinations of quadrature variables have reduced variance (i.e., squeezing) [18,20]. As these variances tend to zero, or, equivalently, the squeezing tends to infinity, these states become better and better approximations to ideal CV cluster states [20], but the required energy diverges. Keeping the energy finite requires that the squeezing remain finite, which means that even with perfect experimental equipment, information degradation is inevitable when using CV cluster states for measurement-based QC.

When used in the real world, both qubit and CV cluster states will suffer from noise, but one might wonder whether the intrinsic noise of CV cluster states due to finite squeezing might be fundamentally different in some way. Previous results showed that there is no easy fix for this type of noise [22,23] and left hanging in the air the question of whether finitely squeezed (and thus physical) CV cluster states were at all useful for practical measurement-based QC of indefinite length. If not, it would mean there was a fundamental deficiency in CV cluster states not suffered by their qubit-based cousins. The possibility remained, however, that the noise might be handled using well-established methods of error correction and fault tolerance [24–31] applied to qubits encoded as CV wave functions (e.g., [32]), a possibility that the authors themselves point out [22].

Fault-tolerant QC (see Ref. [33] for a review) is the ability to reduce logical errors in a quantum computation to arbitrarily low levels if the physical error rate of the individual gates comprising the computation is below a fixed, positive value called the fault-tolerance threshold. In other words, if the probability of error in every physical gate can be guaranteed to be below the threshold, then these noisy gates can be used to implement quantum error correction of noisy quantum information in a way that can make the computation's overall error rate as low as one desires, no matter how long the computation.

Qubit cluster states admit a fault-tolerance threshold for measurement-based QC [34,35], which can be made strikingly high (1.4%) using topological methods [36] and which can be further refined to a few percent by postselection [37–39]. Fault-tolerance thresholds for more traditional codes (i.e., concatenated codes) vary, with typical thresholds being 10^{-6} [25–28], 10^{-4} [31], and 3×10^{-3} [40]—and up to a few percent with postselection [29].

Since one can, in principle, implement any unitary on CV-encoded quantum information using a CV cluster state (albeit noisily), our strategy will be to encode qubits as CV wave functions [32] in a way that maps the natural noise of a CV cluster state into noise on the gates processing the encoded qubits. Higher squeezing will produce a lower gate error rate. If the squeezing is high enough, this error rate will be below the threshold for some known error-correcting code as discussed above, and we can use the CV cluster state to implement fault-tolerant QC on the encoded qubits. Our goal, then, will be to prove the existence of a squeezing threshold: a constant, finite level of squeezing above which fault-tolerant measurement-based QC is possible using encoded qubits and a concatenated error-correcting code, assuming no other noise beyond that introduced by finite squeezing alone [18,19].

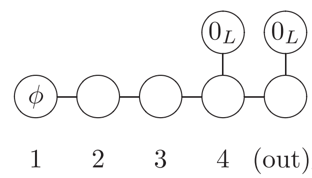
GKP-encoded qubits and Gaussian channels.—The qubit encoding of Gottesman, Kitaev, and Preskill (GKP) [32] in its simplest form encodes one qubit per oscillator. The position-space wave function for each of the logical computational basis states is an evenly spaced comb of δ functions separated by $2\sqrt{\pi}$, and the two states' combs are offset by $\sqrt{\pi}$ from each other—specifically, $|j_L\rangle \propto \sum_{s\in\mathbb{Z}} |(2s+j)\sqrt{\pi}\rangle_q$ ($j = 0,\ 1$), where $|s\rangle_q$ is an eigenstate of position for the oscillator. A physical realization of this encoding replaces the δ functions with sharp Gaussians and limits their heights according to a large Gaussian envelope. Although challenging to create, proposals exist to generate such states optically [41] or by a variety of other methods [32,42–45].

This encoding protects quantum information against random shifts in the quadrature variables $\hat{q}$ (position) and $\hat{p}$ (momentum) [32]. When Gaussian distributed, a random shift is called a Gaussian channel and can be modeled as Gaussian convolution of the input Wigner function. This is exactly the noise model of CV cluster-state QC [18,19], making GKP an appealing qubit encoding—as long as error correction can be performed with minimal deviation from the measurement-based paradigm (cf. Ref. [35]). Fortunately, GKP error correction [32] dovetails nicely with CV cluster states, with details found in Sec. I.A of the Supplemental Material [54].

Fault-tolerant Clifford gates.—The workhorse of (qubit-based) fault-tolerant quantum computation is the Clifford group [1,31], which can be generated by supplementing the Pauli group with the single-qubit gates of Hadamard and phase, as well as with a two-qubit gate such as the controlled-Z gate (sometimes called CPHASE). We need to be able to perform all of these gates with a below-threshold error rate.

The GKP-encoded Pauli group is just the CV Weyl-Heisenberg group restricted to shifts by integer multiples of $\sqrt{\pi}$ in position and/or momentum [32]. In CV measurement-based quantum computation, such displacements are ubiquitous and are therefore considered free to implement, and everything else is done with measurements [18]. GKP-encoded Hadamard and phase gates correspond to the Fourier transform $\hat{F} = e^{i(\pi/4)(\hat{q}^2+\hat{p}^2)}$ and shear $\hat{P} = e^{(i/2)\hat{q}^2}$, respectively, and the qubit controlled-Z gate is just a CV $\hat{C}_Z$ gate with weight ± 1 ($\hat{C}_Z[\pm 1] = e^{\pm i\hat{q}\otimes\hat{q}}$) [32]. All of these CV operations are Gaussian unitaries, which are easy to implement on a CV cluster state [18]. This is a huge advantage because it means the entirety of the GKP-encoded Clifford group inherits this ease of implementation.

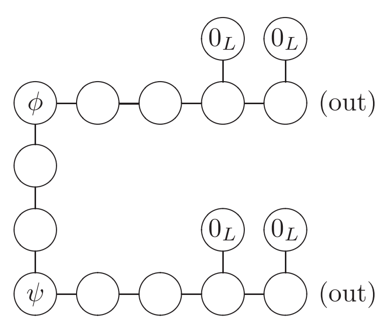
Any single-mode Gaussian unitary can be implemented using four quadrature measurements, $\{\hat{p} + m_j\hat{q}\}_{j=1}^{4}$, on a linear CV cluster state (also known as a CV quantum wire) [46]. We define the measurement vector $\mathbf{m} = (m_1, \ldots, m_4)$ to be the vector containing the four shearing parameters [19] associated with the quadrature measurements. $\mathbf{m}^{(I)} = (0,0,0,0)$, $\mathbf{m}^{(F)} = (1,1,1,0)$, and $\mathbf{m}^{(P)} = (1,0,0,0)$ implement the identity $\hat{I}$, Fourier transform $\hat{F}$, and shear $\hat{P}$, respectively. The following piece of an ancilla-supplemented CV cluster state allows these Gaussian unitaries to be implemented on the input state $|\phi\rangle$, followed by GKP error correction (blank nodes are $\hat{p}$-squeezed vacuum states; nodes with 0_L are GKP-encoded ancillas $|0_L\rangle$; links are $\hat{C}_Z$ gates with weight $+1$ [18]):



(1)

To implement gate $\hat{G}$, we must perform quadrature measurements associated with $\mathbf{m}^{(G)}$ on nodes 1–4 on the bottom row. Measuring the ancillas (and appropriately displacing the nodes below) performs the GKP error correction on both $\hat{q}$ and $\hat{p}$, as shown in Sec. I.A of the Supplemental Material [54]. To apply gates sequentially, one identifies the output node with node 1 of the next cluster.

Implementing a CV $\hat{C}_Z$ gate requires links in the second lattice dimension [18]. Here is an ancilla-supplemented cluster that implements a GKP error-corrected $\hat{C}_Z$ gate:



(2)

Measuring every mode in $\hat{p}$ except the two output modes implements a $\hat{C}_Z$ gate on the input state $|\phi\rangle \otimes |\psi\rangle$, followed by two GKP error-corrected identity gates (see Supplemental Material [54] for details). Single-mode gates [using Cluster (1)] and $\hat{C}_Z$ gates [using Cluster (2)] can be combined into an arbitrary Clifford circuit by identifying each output node with the input of the next gate and including additional identity gates where required. While this is undoubtedly not the most efficient implementation, it is the simplest for a proof-of-principle demonstration of fault tolerance, which is the goal of this work.

Concatenated codes.—GKP error correction projects the Gaussian noise into a particular shift error using (slightly noisy) ancillas. This shift error is then corrected by shifting back to the code space in the direction that corresponds to the shift being smallest. If the shift is too big, a qubit-level logical error results. The error rate is determined by the initial noise in the data register and in the ancilla [47]. After error correction in both quadratures, however, the original data-register noise has been completely replaced by independent, uncorrelated noise from the ancillas, thereby converting the Gaussian noise (from propagation through the cluster) into local, independent Pauli errors after each gate. Thus, noise correlations cannot build up between distant data registers.

By abstractly treating the GKP error-corrected gates as faulty qubit gates, we can concatenate the GKP error correction with a qubit-level error-correcting code [33] and completely forget about the fact that, at the physical level, we are using CV information processing. Then, if the error rate is low enough (discussed next), we can implement Clifford gates fault tolerantly by further concatenation. At that point, the only other ingredient required is the ability to distill a "magic state" for use in implementing a non-Clifford gate (discussed subsequently).

Squeezing threshold.—To determine the amount of squeezing required for fault-tolerant QC, we use a physically motivated model of encoded states in which the Wigner function for an ideal GKP-encoded state, which is a regular lattice of $\pm\,\delta$ functions [32], is replaced by a corresponding lattice of sharp $\pm$ Gaussian spikes, each of which has the same 2×2 covariance matrix $\boldsymbol{\eta}$, which we call the error matrix. For these states to have finite energy, we require that the height of a given Gaussian spike is itself distributed according to a (very large) Gaussian envelope in both quadratures. This is consistent with the original proposal by GKP but extended to the possibility of larger envelopes, which correspond to mixed states. Because $\boldsymbol{\eta}$ is the same for each spike, the height of each spike is irrelevant in measurements of $\hat{q} \bmod \sqrt{\pi}$, which are used for error correction, and we can focus on $\boldsymbol{\eta}$ alone.

Specializing the method of Ref. [47] to Gaussian-distributed shifts, we establish a minimum squeezing threshold as follows. Consider that the GKP encoding can perfectly correct a shift error when the magnitude of the shift error, plus the magnitude of the error in the ancilla used to measure the shift, is less than $\sqrt{\pi}/2$ [32,47]. When this bound is exceeded, a qubit-level logical Pauli error occurs because the state is "shifted back" in the wrong direction. Also note that there are two corrections ($\hat{q}$ and $\hat{p}$) per mode, per gate. For the gate to be free of error, *all* of these corrections must succeed.

The calculation proceeds, then, by identifying which gate has the largest probability of logical error as the error matrix evolves through Cluster (1) using measurement vectors $\mathbf{m}^{(I)}$, $\mathbf{m}^{(F)}$, and $\mathbf{m}^{(P)}$ and through Cluster (2) using just $\hat{p}$ measurements. Section I of the Supplemental Material [54] contains the details of the calculation; here we simply present the results.

The noisiest gate is the $\hat{C}_Z$ gate, so it sets the noise threshold. There are four corrections in this case. Assuming that the initial variance σ^2 in the Gaussian spikes of the encoded ancillas is the same as that of the initial momentum-squeezed vacuum states used to make the CV cluster state, two of the Gaussian-distributed shift errors (including ancilla noise) have variance $7\sigma^2$, and two others have variance $5\sigma^2$. Therefore, the probability that at least one of those corrections fails is

$$p_{\text{err}} = 1 - \left[\operatorname{erf}\left(\frac{\sqrt{\pi}}{2\sqrt{14}\sigma}\right)\right]^2 \left[\operatorname{erf}\left(\frac{\sqrt{\pi}}{2\sqrt{10}\sigma}\right)\right]^2. \quad (3)$$

When $p_{\text{err}} < p_{\text{FT}}$ for the fault-tolerance threshold p_{FT} for some qubit error-correcting code [33], we can concatenate the GKP code with that code and perform fault-tolerant measurement-based quantum computation. The variance σ^2 identified by this condition corresponds to a squeezing threshold of

$$s > -10 \log_{10}\left(\frac{\sigma^2}{1/2}\right). \quad (4)$$

For $p_{\text{FT}} = 10^{-6}$, which is a typical (and rather conservative) threshold for concatenated codes [25–27], this means that $\sigma^2 < 4.44 \times 10^{-3}$, which corresponds to $s > 20.5$ dB. Figure 1 shows a plot of this curve for intermediate squeezing levels, while Table II in the Supplemental Material [54] lists the squeezing corresponding to several other typical threshold values.

Magic-state distillation.—With nearly perfect Clifford gates in hand, computational universality is achieved by guaranteeing the ability to distill a so-called magic state from many noisy copies [50]. The procedure doesn't have to work every time, but when it does work, it has to produce a noisy state with sufficient fidelity to the state of interest. Fortunately, the noise thresholds for magic-state distillation are as high as 14%–17% in some cases [51], significantly less stringent than the Clifford-gate requirements of $\sim 10^{-6}$. As such, we can effectively ignore the errors introduced by the Clifford operations entirely [50,52,53].

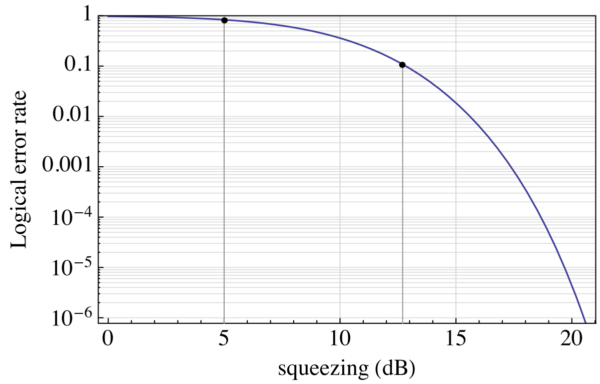


FIG. 1 (color online). Qubit-level logical error rate induced by GKP error correction with CV cluster states. The indicated level of squeezing is assumed to apply both to the initial momentum-squeezed states used to create the cluster state and in the Gaussian spikes that comprise the encoded GKP states. Also shown: maximum single-mode squeezing achieved to date (12.7 dB) [48,49] and squeezing achieved in a large CV cluster state (5 dB) [9].

Previous work has focused on the cubic phase state [18,32], but distilling this state requires an asymmetric noise model [32]. The natural noise model of CV cluster-state QC is symmetric in $\hat{q}$ and $\hat{p}$ on average [18,19], which is preferred when distilling an encoded Hadamard eigenstate $|\pm H_L\rangle$ [32]. Either state can be used to implement an encoded $\pi/8$ gate [32].

Since $\hat{F}|\pm H_L\rangle = \pm|\pm H_L\rangle$, a Hadamard eigenstate can be constructed by counting photons on one half of an encoded Bell pair [32], which can be created by applying a $\hat{C}_Z$ gate to $|+_L\rangle \otimes |+_L\rangle$, using measurements as discussed above. Then, we count photons on one side and obtain an outcome n. In the ideal case, if $n \bmod 4 = \{0, 2\}$, then the remotely prepared state is $|\pm H_L\rangle$, respectively (and an odd n is impossible). In the physical case, of course, errors in the encoded Bell pair will reduce this fidelity of identification and corrupt the average output state. As such, if we get an odd n, we know an error has occurred, so we discard the state and start over. If n is even, then ε is the probability that it reveals the wrong state at the output.

Reference [51] identifies $\varepsilon < 0.146$ as a tight threshold for being able to distill the resulting state [50], and this threshold holds even when distilling using noisy Clifford gates [53]. Assuming we begin with pure ancillas, the error probability ε is between 12.5% and 12.6% for squeezing between 12.8 and 20.5 dB (Clifford-gate error rate of 10^{-1} to 10^{-6}, according to Fig. 1), with a success probability (i.e., probability of obtaining an even outcome) of $2/3$. Since $\varepsilon < 14.6\%$, distillation is possible, thus completing the proof of fault tolerance for measurement-based QC using CV cluster states. Section II of the Supplemental Material [54] contains the details of the calculation, as well as some possible ways to optimize this method.

Universal resources.—Since the clusters used to perform Clifford gates and distill magic states all fit within a regular square lattice, we can create a universal resource by starting with an ordinary square-lattice CV cluster state of sufficient size and attaching GKP ancillas at regular intervals, like flowers growing in a regular pattern in the "garden" of the original lattice. One can even measure the ancillas directly after attachment. Either way, attaching the ancillas early means we are using a non-Gaussian resource state, evading known no-go results [22,23].

Alternatively, one can think of the act of attaching ancillas and measuring $\hat{p}$ as a single operation of non-destructively measuring $\hat{q} \bmod \sqrt{\pi}$ (with some noise). Thus, we can simply add to our toolbox of measurements a nondestructive measurement of $\hat{q} \bmod \sqrt{\pi}$ and view the original Gaussian cluster states as universal for fault-tolerant quantum computation using this augmented suite of measurements. This evades the no-go results of Refs. [22,23] because active error correction and concatenation are being used, which mean that the required size of the encoding will grow (albeit slowly) with the length of the computation [25].

Extensions.—While this analysis focuses exclusively on finite-squeezing noise, it can be straightforwardly generalized to include additional local Gaussian noise, photon loss, and detector inefficiency. While these extensions will generalize the threshold to also be a function of the additional noise parameters, they are not expected to change the fundamental result, which is the existence of some finite threshold.

Conclusion.—This is a theoretical breakthrough in our understanding of what is possible using measurement-based quantum computation with continuous-variable cluster states. With an appropriate qubit encoding, active error correction, and initial squeezing above a constant finite threshold, continuous-variable cluster states are universal for fault-tolerant measurement-based quantum computation of indefinite length.

While the encoding scheme presented here may be nonoptimal due to the prohibitive nature of the required states, it has at least the flavor of practicality since multi-qubit Clifford operations require only Gaussian unitaries. Furthermore, the existence of a finite squeezing threshold for continuous-variable cluster states when using this encoding may well spur new experimental developments in implementing these challenging states.

Regardless of the scheme's feasibility, a finite squeezing threshold is now known to exist for continuous-variable cluster states. This means that work can continue with confidence toward designing better schemes, improving the threshold, and achieving higher levels of squeezing.

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*ncmenicucci@gmail.com

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