Constrained Dynamics via the Zeno Effect in Quantum Simulation: Implementing Non-Abelian Lattice Gauge Theories with Cold Atoms

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We show how engineered classical noise can be used to generate constrained Hamiltonian dynamics in atomic quantum simulators of many-body systems, taking advantage of the continuous Zeno effect. After discussing the general theoretical framework, we focus on applications in the context of lattice gauge theories, where imposing exotic, quasilocal constraints is usually challenging. We demonstrate the effectiveness of the scheme for both Abelian and non-Abelian gauge theories, and discuss how engineering dissipative constraints substitutes complicated, nonlocal interaction patterns by global coupling to laser fields.

Laboratory experiments with atomic quantum-degenerate gases have established a synergetic link between atomic physics and condensed matter [1–3], and hold prospects for a similar connection to high-energy physics [4–12]. Loaded into optical lattices, cold atoms realize Hubbard models, which can be designed and controlled via external fields to mimic the dynamics of quantum many-body systems in equilibrium and nonequilibrium situations [3,13]. While a focus of research during the last decade has been the development of a toolbox for designing specific lattice Hamiltonians [1–3], we address below the problem of implementing desired Hubbard dynamics in the presence of constraints; i.e., we wish to keep the system dynamics within a certain subspace of the total Hilbert space. A familiar way of imposing such constraints is to add an energy penalty to the Hamiltonian [14]. Below, we describe an alternative scenario that is based on driving the system with engineered classical noise, exploiting the Zeno effect [15–20]. As we will see, "adding noise" provides a general tool to implement—in an experimentally efficient and accessible way—highly nontrivial constraints in quantum many-body systems.

The present work is motivated by the ongoing quest to build a quantum simulator for Abelian and non-Abelian lattice gauge theories (LGTs) with cold atoms in optical lattices [5–12]. LGTs play a prominent role in both particle and condensed matter physics: in the standard model, the interaction between constituents of matter are mediated by gauge bosons [21–24], and in frustrated magnetism, quantum spin liquids are suitably described in the language of gauge theories [14,25,26]. The key feature of a LGT is the presence of local (gauge) symmetries. The generators G_x^a of these local gauge transformations, with x denoting lattice sites and a a color index, commute with the lattice Hamiltonian, $[H_0, G_x^a] = 0$ for all x, a, and thus provide local conservation laws. They can be interpreted in analogy

to Gauss's law from electrodynamics, as they constrain the dynamics of the system to a physical subspace $\mathcal{H}_{\mathcal{P}}$ given by the constraints $G_x^a |\psi\rangle = 0$ [27]. In high-energy physics, gauge symmetries are from fundamental considerations exact, but in quantum simulations these symmetries will normally be approximate on some level in the microscopic model. Thus, the microscopic Hamiltonian will be of the form $H_{\text{micro}} = H_0 + H_1$, with $H_0 \sim J$ the desired gauge-invariant part, and $H_1 \sim \lambda$ a perturbation, which drives the system dynamics outside of the gauge-invariant subspace $\mathcal{H}_{\mathcal{P}}$, see Fig. 1(a). A central challenge of quantum simulating LGTs is to introduce mechanisms that suppress these errors.

Gauge constraints via classical noise.—A common strategy to restrict the dynamics to a certain subspace is by adding an energy penalty to the microscopic Hamiltonian: $H_{\text{micro}} = H_0 + H_1 + H_U$, with $H_U \sim U \gg \lambda$. In this case,
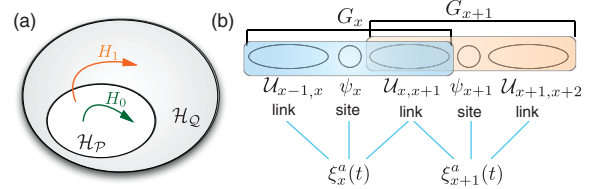

FIG. 1 (color online). Dissipative protection of gauge invariance in implementations of a lattice gauge theory (LGT). (a) The dynamics H_0 happens within the physically relevant subspace $\mathcal{H}_{\mathcal{P}}$, defined by $G_x^a |\psi\rangle = 0$, but gauge-variant perturbations H_1 may drive the system into the unphysical subspace $\mathcal{H}_{\mathcal{Q}}$ (where $G_x^a |\psi\rangle \neq 0$). (b) LGTs consist of fermions ψ_x living on sites, coupled to gauge fields $\mathcal{U}_{x,x+1}$ living on links. The dynamics can be constrained to the physical subspace by coupling independent noise sources linearly to each generator. The multisite structure of the generators implies that the noise has to be correlated quasilocally in space.

to order λ^2/U, manifolds with different eigenvalues of H_U become decoupled. In the context of LGTs, one can use $H_U = U\sum_{x,a}(G_x^a)^2$ to impose the Gauss law constraints [5,7,9,14,28,29]. Since in a LGT the generators G_x^a are complicated expressions of the matter and gauge fields—especially in non-Abelian models—adding an interaction term involving the square of these poses a formidable challenge [30]. In contrast, we pursue here the strategy of enforcing the constraints $G_x^a|\psi\rangle = 0$ by adding classical noise terms [31,32] to the microscopic Hamiltonian

$$H_{\text{micro}}(t) = H_0 + H_1 + \sqrt{2\kappa}\sum_{x,a}\xi_x^a(t)G_x^a, \quad (1)$$

which are linear in G_x^a and involve independent white-noise processes $\xi_x^a(t)$ with $\langle\langle\xi_x^a(t)\xi_y^b(t')\rangle\rangle = \delta_{xy}\delta_{ab}\delta(t-t')$. Each realization of the noise will give rise to an evolution of the system described by a stochastic state vector $|\psi(t)\rangle$. Averaging over the noise fluctuations leads to the density operator $\rho = \langle\langle|\psi(t)\rangle\langle\psi(t)|\rangle\rangle$, obeying the master equation (see the Supplemental Material [33])

$$\dot{\rho} = -iH_{\text{eff}}\rho + i\rho H_{\text{eff}}^\dagger + 2\kappa\sum_{x,a}G_x^a\rho G_x^a, \quad (2)$$

with non-Hermitian Hamiltonian H_{eff},

$$H_{\text{eff}} = H_0 + H_1 - i\kappa\sum_{x,a}(G_x^a)^2.$$

The effective Hamiltonian H_{eff} contains a damping term involving the square of the generators, introduced by the noisy single-particle terms in Hamiltonian Eq. (1). For $\kappa/\lambda \gg 1$, this term constrains the evolution to $\mathcal{H}_{\mathcal{P}}$. In fact, this scale separation of a large rate vs a small energy scale provides the dissipative analogue of the energetic protection described above, which relies on the separation of two energy scales. In the present case, the protection term arises from a linear coupling of the generators to a classical noise source, which—in contrast to an energy penalty H_U—does not require complicated, nonlocal two-body interactions.

To demonstrate the dissipative protection explicitly, we integrate out the dynamics of the gauge-variant space $\mathcal{H}_{\mathcal{Q}}$, defined by $G_x^a|\psi\rangle \neq 0$, to leading order in λ/κ. Denoting by $\mathcal{P}$ the projector on $\mathcal{H}_{\mathcal{P}}$, we obtain the following master equation for the gauge-invariant part of the density operator $\rho_{\mathcal{PP}} = \mathcal{P}\rho\mathcal{P}$:

$$\dot{\rho}_{\mathcal{PP}} = -i\tilde{H}_{\text{eff}}\rho_{\mathcal{PP}} + i\rho_{\mathcal{PP}}\tilde{H}_{\text{eff}}^\dagger,$$

with effective non-Hermitian Hamiltonian

$$\tilde{H}_{\text{eff}} \approx \mathcal{P}(H_0 + H_1)\mathcal{P} - i\mathcal{P}H_1\mathcal{Q}\frac{1}{\kappa\sum_{x,a}(G_x^a)^2}\mathcal{Q}H_1\mathcal{P}, \quad (3)$$

where $\mathcal{Q} = \mathbb{1} - \mathcal{P}$ (see the Supplemental Material [33]). Starting from an initial state $|\psi_0\rangle \in \mathcal{H}_{\mathcal{P}}$, the evolution under this Hamiltonian is restricted to $\mathcal{H}_{\mathcal{P}}$, but reduces the norm of the state vector, signifying the transfer of population to the gauge-variant subspace $\mathcal{H}_{\mathcal{Q}}$. This population transfer sets a time scale $t \lesssim \kappa/\lambda^2$ below which the dissipative protection of local quantities is effective. Therefore, in the strong-noise limit considered in this work, gauge invariance is protected for long times compared to the time scales at which errors accumulate without the engineered noise, $t \sim 1/\lambda$.

This suppression of transitions by fast classical fluctuations is related to motional narrowing [34,35], and to dynamical decoupling techniques [36] such as bang-bang control [16,37], which suppress unwanted couplings to an environment, e.g., in a quantum information context. More specifically, our scheme can be seen as a classical analogue of the quantum Zeno effect [17,38], which has been discussed in the context of quantum control [18–20], but also of quantum many-body systems [39–42]. In the standard quantum Zeno effect, the required dissipation originates from an interaction of the system with quantum fluctuations of the bath or frequent measurements, while here the dissipation is simulated by classical fluctuations of the perturbation field.

Dissipative protection in Abelian LGTs.—As a first illustrative and conceptually simple example, we demonstrate protection of gauge invariance in a one-dimensional U(1) lattice model, the Schwinger model, whose Hamiltonian takes the form [43]

$$H_0 = \sum_x[J(\psi_x^\dagger\mathcal{U}_{x,x+1}\psi_{x+1} + \text{H.c.})$$
$$+ m(-1)^x\psi_x^\dagger\psi_x + \frac{\tilde{g}^2}{2}E_{x,x+1}^2]. \quad (4)$$

Here, ψ_x are (staggered) fermionic matter fields defined on the vertices of the lattice, $\mathcal{U}_{x,x+1}$ are the gauge fields defined on the bonds between x and $x+1$ [see Fig. 1(b)], and $E_{x,x+1}$ is the corresponding electric field, satisfying $[E_{x,x+1}, \mathcal{U}_{x,x+1}] = \mathcal{U}_{x,x+1}$ [21–24]. The potential term $\sim m$ corresponds to a mass term for the fermionic fields, whose alternating sign stems from the use of "staggered fermions" [25]; J and $\tilde{g}$ are the tunneling and gauge-coupling coefficients, respectively. The generators of the $U(1)$ gauge transformations for this model are given by $G_x = \psi_x^\dagger\psi_x - E_{x,x+1} + E_{x-1,x} + [(-1)^x - 1]/2$. The corresponding Gauss law $G_x|\psi\rangle = 0$ is the lattice equivalent of the one of continuum quantum electrodynamics. In the Wilson formulation of LGTs [25,44], $\mathcal{U}_{x,x+1}$ are complex phase variables, but for our purposes the quantum link model (QLM) formalism [43,45–48] is more convenient, where the link variables are represented by spin degrees of freedom, i.e., $\mathcal{U}_{x,x+1} \equiv S_{x,x+1}^+$, $E_{x,x+1} \equiv S_{x,x+1}^z$. We choose here a representation using spin-1/2 degrees of freedom,
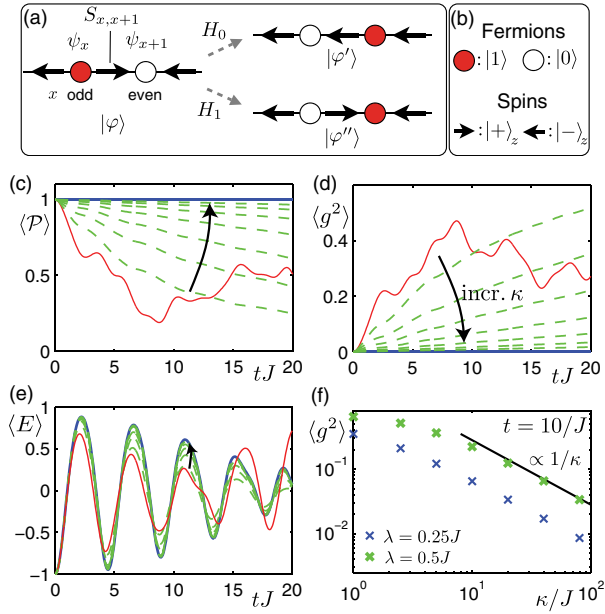
FIG. 2 (color online).   (a) System dynamics of a U(1) LGT in the ideal case (H_0, top) and under the effect of undesired single particle tunneling (H_1, bottom); (b) on-site Hilbert spaces. (c)–(f) Dissipative protection of quench dynamics in a $U(1)$ QLM with $N_s = 4$ sites connected by $N_l = 3$ links (open boundary conditions). (c) Population of gauge-invariant subspace. (d), (f) Average violation of gauge constraint, quantified by $g^2 = \sum_x G_x^2/N_s$. (e) Average electric field $E = \sum_x E_{x,x+1}/N_l$. In panels (c)–(e), blue curves indicate the ideal dynamics. Thin red curves show the detrimental influence of gauge-variant fermion tunneling ($\lambda/J = 0.25$). The arrows (from red to blue) show how increasing κ restores the ideal dynamics (green dashed curves; $\kappa/J = 1$, 2.5, 5, 10, 20, 40, 80). Panel (f) shows the scaling with κ of $\langle g^2 \rangle$ at a fixed time; the black line is a guide to the eye indicating a scaling $\propto 1/\kappa$. All results are obtained from the full master equation with Hamiltonian $H_0 + H_1$ (see text), starting from the eigenstate of H_0 for $m \to \infty$.

which corresponds to a 1D version of the Schwinger model with a finite electric flux running through the system, also corresponding to a finite θ angle [5,43]. Despite its simplicity, this model displays various interesting features related to gauge theories, such as confinement and string-breaking phenomena [5,43].

The system dynamics between two sites as induced by Eq. (4) is sketched in Figs. 2(a) and 2(b). On the left-hand side, a typical gauge-invariant state $|\varphi\rangle$ is illustrated, where the Gauss law is satisfied at both vertices. Under the action of the correlated tunneling contained in H_0, ($\psi_{x+1}^\dagger S_{x,x+1}^- \psi_x + \text{H.c.}$), the fermion tunnels from x to $x + 1$, and the center spin $S_{x,x+1}$ flips, preserving gauge invariance in the final state $|\varphi'\rangle$. Processes of this kind describe the creation of a particle-antiparticle pair accompanied by an excitation of the gauge field. In typical implementations, additional gauge-variant imperfections appear, such as single-fermion tunneling

$H_1 = \lambda \sum_x (\psi_{x+1}^\dagger \psi_x + \text{H.c.})$. Once such processes are allowed, the system dynamics involves states of the form $|\varphi'\rangle$, where the condition $G_x|\varphi'\rangle = 0$ is not satisfied for all x, thus leading to leakage into $\mathcal{H}_Q$. Imperfections of this kind correspond to a creation of a particle-antiparticle pair without any effect on the gauge fields, destroying gauge invariance.

To illustrate the dissipative protection in this model, we study quantum-quench dynamics as illustrated in Figs. 2(c)–(f), where we prepare the system in the ground state of $m = \infty$ and quench to $m = 0$ at time $t = 0$. The system evolves under H_0 given by Eq. (4) plus a gauge-variant fermion hopping H_1 [49]. For $\lambda \neq 0$, gauge invariance is clearly violated and the evolution of observables deviates from the ideal case [thin red curves in panels (c)–(e)]. Increasing the strength κ of the dissipation (green dashed curves) gradually restores the ideal dynamics (blue curves). The expected scaling $\propto 1/\kappa$ of the protection mechanism is confirmed in panel (f). While this example illustrates that the dissipative protection works in principle, we now apply the same mechanism to a more complicated non-Abelian LGT, where enforcing gauge invariance via noise may prove a considerable advantage in the design of an atomic quantum simulator.

Dissipative protection in non-Abelian LGTs.—We now illustrate the dissipative protection of gauge invariance for a non-Abelian LGT, namely a U(2) QLM that may be realistically realized in cold-atom experiments (see below). The presence of color (gauge) degrees of freedom allows us to investigate, in this simplified model, physical phenomena related to general non-Abelian gauge theories like QCD, such as chiral symmetry breaking and confinement, and its phase diagram may support exotic condensate phases [9]. Its Hamiltonian, which belongs to a class of more general QLMs including $U(N)$ and $SU(N)$ symmetries, reads $H_0 = H_J + H_m$ [50], where $H_m = m\sum_x(-1)^x \psi_x^{\alpha\dagger} \psi_x^\alpha$ describes staggered fermions and H_J is the interaction between matter and gauge field

$$H_J = J\sum_x \psi_x^{\alpha\dagger} \mathcal{U}_{x,x+1}^{\alpha\beta} \psi_{x+1}^\beta + \text{H.c.}$$

$$\equiv J\sum_x \psi_x^{\alpha\dagger} r_x^\alpha l_{x+1}^{\beta\dagger} \psi_{x+1}^\beta + \text{H.c.} \quad (5)$$

Here, α, $\beta = 1$, 2 represents the U(2) color degree of freedom (repeated indices are contracted). As before, the fermionic matter fields ψ_x^α live on the vertices of a lattice, while for non-Abelian gauge fields it is convenient to represent link variables by rishon fermionic fields l_x^α and r_x^α living on the links to the left and right of a given site, $\mathcal{U}_{x,x+1}^{\alpha\beta} \equiv r_x^\alpha l_{x+1}^{\beta\dagger}$ [43,48] [see Fig. 3(a)]. The U(2) gauge symmetry is split into a U(1) part, with generator

$$G_x = \psi_x^{\alpha\dagger} \psi_x^\alpha - (l_{x+1}^{\alpha\dagger} l_{x+1}^\alpha - r_x^{\alpha\dagger} r_x^\alpha)/2$$

$$+ (l_x^{\alpha\dagger} l_x^\alpha - r_{x-1}^{\alpha\dagger} r_{x-1}^\alpha)/2 - 1,$$
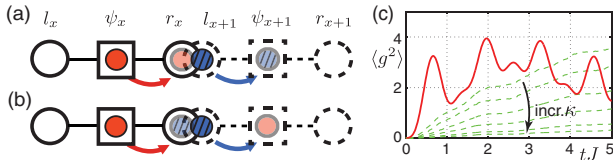
and a SU(2) part, with generators

FIG. 3 (color online). U(2) LGT. (a) Basic dynamics of Eq. (5). The matter/gauge-field coupling corresponds to a simultaneous color-conserving tunneling of one fermion at site x to the link x, $x + 1$ (red) and a rishon at $x, x + 1$ to the site $x + 1$ (blue, dashed). Matter and rishon sites are denoted by squares and circles, respectively; the blocks x and $x + 1$ are indicated by continuous and dashed contours. (b) An example for a gauge-variant process: correlated tunneling similar to panel (a), but accompanied by a change of color. (c) Numerical analysis of two building blocks, evolving under H_0 plus various error terms (calculations for the full master equation (2); for details see the Supplemental Material [33]). Thin red line: $\kappa = 0$, green dashed lines: $\kappa/J = 5, 10, 20, 40, 80, 160$ (increasing along the arrow).

$$G_x^a = \psi_x^{\alpha\dagger}\sigma_{\alpha\beta}^a\psi_x^\beta + r_x^{\alpha\dagger}\sigma_{\alpha\beta}^a r_x^\beta + l_x^{\alpha\dagger}\sigma_{\alpha\beta}^a l_x^\beta, \qquad a = 1, 2, 3.$$

The G_x^a commute with all G_x and satisfy $[G_x^a, G_y^b] = 2i\delta_{xy}\epsilon_{abc}G_x^c$, where ϵ_{abc} is the Levi-Città tensor and σ^a are Pauli matrices.

The basic system dynamics described by H_J is illustrated in Fig. 3(a): it corresponds to a simultaneous hopping of two particles, namely, ψ_x to r_x and simultaneously l_{x+1} to ψ_{x+1}. The color of both particles is preserved during the process ($[H_0, G_x^a] = 0$). In a typical microscopic implementation, one may obtain additional, undesired color-changing terms of the form $H_1^{(1)} = \lambda\sum_x(\psi_{x+1}^{2\dagger}l_{x+1}^1 r_x^{1\dagger}\psi_x^2 + \text{H.c.})$, as illustrated in Fig. 3(b). These do not commute with all generators, and therefore violate gauge invariance. To estimate the effect of terms such as this one, we analyzed the exact time evolution for two building blocks of the U(2) model. We included various realistic errors similar to $H_1^{(1)}$ that are specific for the cold-atom implementation described in the Supplemental Material [33], comprising a large class of generic errors. As Fig. 3(c) shows, without protection the mean value of the sum of all generators, $g^2 = \sum_x[G_x^2 + \sum_a(G_x^a)^2]/N_s$, quickly acquires large values, indicating the loss of gauge invariance (red solid line). However, under the noise protection generated by Eq. (1), gauge invariance may be retained on the time scale of several tunneling events (green dashed lines). This example demonstrates that the proposed protection mechanism works also for more complicated non-Abelian models including several noncommuting generators.

Optical-lattice implementation.—In ultracold-atom implementations where the color index is represented by different internal atomic states, the standard strategy to suppress gauge-variant terms via quadratic energy penalties $U(G_x^a)^2$ amounts to engineering numerous local and non-local interactions with fine-tuned coefficients. From this regard, our dissipative approach is advantageous, since the preservation of gauge invariance requires driving the system with terms that are only linear in the generators. In the ultracold-atoms setting, the on-site single-particle noise terms $\xi_x^a(t)G_x^a$ can be realized by coupling internal atomic states to laser fields with suitable amplitude or phase noise, where noisy ac-Stark shifts and Raman processes allow us to impose the constraints on G_x and G_x^3 as well as $G_x^{1,2}$ (see the Supplemental Material [33]). Using high-resolution objectives [51–54], it is possible to engineer an independent noise source for each generator, as required by Eq. (1). However, in the common case where the dominant gauge-variant perturbations couple only nearest neighbors, one can simplify the experimental setup by using a noise pattern that is repeated periodically. In this way, one can enforce local gauge invariance by using global addressing together with a superlattice structure [33].

The last ingredient to quantum simulate the SU(2) LGT is then a natural realization of H_0 that does not interfere with the dissipative protection and thus does not lead to undesired heating of the system by the noise. In the Supplemental Material [33], we illustrate how models with $U(N)$ interactions (in particular focusing on the conceptually simpler U(2) case) can be engineered in spinor gases, where spin-changing collisions combined with state-dependent optical potentials provide a natural realization of the two-body interaction terms constituting H_J. Ideas along these lines for Abelian theories have also been discussed in Ref. [12].

The scheme outlined above could also be combined with energetic protection in cases where, e.g., the interactions only protect an Abelian symmetry, while the more challenging non-Abelian contributions are imposed via noise. This would facilitate the realization of previous proposals [9], extending their regime of applicability and providing additional means to improve the accuracy of gauge invariance in microscopic realizations.

Scaling and imperfections.—In contrast to quantum-computing purposes, we are interested here in many-body properties, such as the expectation value of low-order correlations and order parameters [55]. This ensures, in general, better scalability properties: while the leakage out of the $\mathcal{P}$ subspace is expected to increase with the system size, order parameters that quantify gauge invariance, such as g^2, are not severely affected by the system size itself. While checking these expectations for sufficiently large system sizes with LGTs is outside of computational capabilities, we have tested these scalings in the context of a simplified model where local conservation laws are imposed in the same dissipative manner. The results are described in the Supplemental Material [33] and clearly support these claims.

To further address the feasibility of our proposal, we have performed a numerical analysis of typical error

sources in realistic setups, such as particle loss and imperfect noise addressing. In particular, we found that the effects of the latter commonly scale as $\epsilon^2 \kappa$ for short time scales, where ϵ is the strength of the imperfections, and not as $\epsilon\kappa$ as naively expected. This is due to the particular characteristics of the most common addressing errors, which do not directly affect the gauge invariant subspace (see Ref. [33] for details).

Conclusions and outlook.—We have shown how classical noise can serve as a resource to engineer constrained Hamiltonian dynamics in quantum simulators, and in particular how Abelian and non-Abelian gauge invariance can be protected in atomic lattice implementations. The dissipative scheme has advantages with respect to the more conventional energy punishment, as coupling to generators is linear, local, and introduced by a physical resource which is independent of the engineered Hamiltonian dynamics. For gauge-variant perturbations that do not couple distant sites, the noise protection can be realized by global beams in a superlattice configuration. The mechanism is universal, as it can be extended to any symmetry and dimensionality, and can be applied to different microscopic systems beyond cold atom gases, such as superconducting qubits and trapped ions [28,29].

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