

## Objectivity in a Noisy Photonic Environment through Quantum State Information Broadcasting

J. K. Korbicz,<sup>1,2\*</sup> P. Horodecki,<sup>3,2</sup> and R. Horodecki<sup>1,2</sup>

<sup>1</sup>*Institute of Theoretical Physics and Astrophysics, University of Gdańsk, 80-952 Gdańsk, Poland*

<sup>2</sup>*National Quantum Information Centre in Gdańsk, 81-824 Sopot, Poland*

<sup>3</sup>*Faculty of Applied Physics and Mathematics, Gdańsk University of Technology, 80-233 Gdańsk, Poland*

(Received 31 December 2013; published 26 March 2014)

Recently, the emergence of classical objectivity as a property of a quantum state has been explicitly derived for a small object embedded in a photonic environment in terms of a spectrum broadcast form—a specific classically correlated state, redundantly encoding information about the preferred states of the object in the environment. However, the environment was in a pure state and the fundamental problem was how generic and robust is the conclusion. Here, we prove that despite the initial environmental noise, the emergence of the broadcast structure still holds, leading to the perceived objectivity of the state of the object. We also show how this leads to a quantum Darwinism-type condition, reflecting the classicality of proliferated information in terms of a limit behavior of the mutual information. Quite surprisingly, we find “singular points” of the decoherence, which can be used to faithfully broadcast a specific classical message through the noisy environment.

DOI: 10.1103/PhysRevLett.112.120402

PACS numbers: 03.65.Yz, 03.67.Hk

The uninterrupted series of successes of quantum mechanics supports a belief that quantum formalism applies to all of physical reality. Thus, in particular, the objective classical world of everyday experience should emerge naturally from the formalism. This has been a long-standing problem, already present from the very dawn of quantum mechanics [1,2]. Recently, a crucial step was made in a series of works (see, e.g., Refs. [3–5]) introducing quantum Darwinism—a refined model of decoherence [6], based on a multiple environment paradigm: A quantum system of interest  $S$  interacts with multiple environments  $E_1, \dots, E_N$  instead of just one. The authors assumed [3] that each of these independent fractions effectively measures the system and argued that after the decoherence (with some time scale  $\tau_D$ ), it carries nearly complete classical information about the system, meaning that the information propagates in the environment with a huge redundancy. A further step was made in Ref. [7] by dropping any explicit assumptions on the dynamics and applying a operational definition of objectivity [4] directly to the postdecoherence quantum state. This, together with the Bohr’s criterion of nondisturbance [8], allowed us to derive a universal state structure—a spectrum broadcast form (cf. Ref. [9]), responsible for the appearance of classical objectivity in a model- and dynamics-independent way [7].

There appears an objectively existing state of the system  $S$  if the time-asymptotic joint quantum state of  $S$  and the observed fraction of the environment  $fE$  is of a spectrum broadcast form:

$$\begin{aligned} \varrho_{S:fE}(\infty) &= \sum_i p_i |\vec{x}_i\rangle \langle \vec{x}_i| \otimes \varrho_i^{E_1} \otimes \dots \otimes \varrho_i^{E_{fN}}, \\ \varrho_i^{E_k} \varrho_{i' \neq i}^{E_k} &= 0, \end{aligned} \quad (1)$$

with  $\{|\vec{x}_i\rangle\}$  a pointer basis [10],  $p_i$ ’s initial pointer probabilities, and  $\varrho_i^{E_1}, \dots, \varrho_i^{E_{fN}}$  some states of the environments  $E_1, \dots, E_{fN}$  with mutually orthogonal supports.

The states [Eq. (1)] “work” by faithfully encoding the same classical information about the system (index  $i$ ) in each portion of the environment—they describe redundant proliferation (broadcasting) of information, necessary for objectivity [4,7]. A process of formation of a state [Eq. (1)] is what we call state information broadcasting [7]. It is a weaker form of quantum state broadcasting [9,11].

In this Letter, we apply the above novel results to the celebrated model of a dielectric sphere illuminated by photons [12–16] to show how an objectively existing state of a system [4,7] is actually formed in the course of the quantum evolution with a general (not only thermal) noisy environment. In contrast to the earlier studies [15,16], we show it directly on the fundamental level of quantum states, proving the emergence of the broadcast structure [Eq. (1)], rather than using information-theoretical conditions, which so far are only known to be necessary, while their sufficiency is still not known [7]. We thus prove robustness and a generic character of the emergence of objectivity—a well known property of the everyday life. In other words, the state information broadcasting process still works even if the environment is noisy, which, in principle, might cover or mismatch the proliferation of emerging classical information about the system. Moreover, with the help of the classical Perron-Frobenius theorem [17], we show a surprising effect of how the decoherence mechanism can be used to faithfully broadcast a specific message into the environment.

*The model* [12].—A dielectric sphere  $S$  of radius  $a$  and relative permittivity  $\epsilon$  is bombarded by a constant flux of

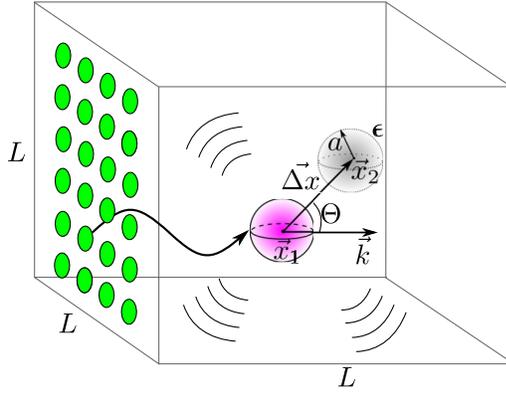


FIG. 1 (color online). The illuminated sphere model [12]. Green dots represent the photons, which constitute the environments  $E_1, \dots, E_N$  of the sphere. The sphere and the photons are enclosed in a large cubic box of edge  $L$ ; photon momentum eigenstates  $|\vec{k}\rangle$  obey the periodic boundary conditions.

photons, constituting the sphere's environment see Fig. 1. The sphere can be at two possible locations  $\vec{x}_1$  and  $\vec{x}_2$ , and the photons are assumed not energetic enough to individually resolve the displacement  $\Delta x \equiv |\vec{x}_2 - \vec{x}_1|$ :

$$k\Delta x \ll 1, \quad (2)$$

where  $\hbar k$  is some characteristic momentum, but they are able to do so collectively: If the sphere is initially in a superposition of the localized states  $|\vec{x}_1\rangle$  and  $|\vec{x}_2\rangle$ , the scattering photons will localize it via collisional decoherence [12]. Here, we show that during this process, a broadcast state [Eq. (1)] is formed for the radiation that is initially a general mixture of plane waves, concentrated around [Eq. (2)]

$$q_0^{\text{ph}} = \sum_{\vec{k}} p(\vec{k}) |\vec{k}\rangle \langle \vec{k}|, \quad (3)$$

with  $p(\vec{k})$  significantly different from zero only for  $|\vec{k}|\Delta x \ll 1$ . (In the previous studies, only thermal states were considered [12,15,16].) Following Refs. [12,13,15,16], we use box normalization (cf. Fig. 1): the sphere and the photons are enclosed in a box of a volume  $V \equiv L^3$ . We remove it through the thermodynamic limit (signified by  $\cong$ ) [15,16]:  $V \rightarrow \infty$ ,  $N \rightarrow \infty$ , and  $N/V = \text{const}$ , where  $N$  is the total number of photons and  $N/V$  is the radiation density. The interaction time  $t$  enters through the number of scattered photons up to time  $t$  (a “macroscopic time”); see Fig. 1:

$$N_t \equiv L^2 \frac{N}{V} ct, \quad (4)$$

where  $c$  is the speed of light. We will work with a fixed  $t$  and pass to the decoherence limit  $t/\tau_D \rightarrow \infty$  (denoted by  $\approx$  or  $\infty$ ) at the very end. The sphere-photon interaction is of a controlled-unitary type (symmetric environments):

$$U_{S:E}(t) \equiv \sum_{i=1,2} |\vec{x}_i\rangle \langle \vec{x}_i| \otimes \underbrace{\mathbf{S}_i \otimes \dots \otimes \mathbf{S}_i}_{N_t}, \quad (5)$$

where (assuming translational invariance)  $\mathbf{S}_i \equiv \mathbf{S}_{\vec{x}_i} = e^{-i\vec{x}_i \cdot \vec{k}} \mathbf{S}_0 e^{i\vec{x}_i \cdot \vec{k}}$  is the scattering matrix (see, e.g., Ref. [18]).

*Macrofractions.*—We introduce a crucial environment coarse graining: we divide the full photonic environment into a number of macroscopic fractions, each containing  $mN_t$  photons, with  $0 \leq m \leq 1$ . By macroscopic, we will understand “scaling with the total number of photons  $N_t$ .” By definition, these are the environment fractions accessible to independent observers, searching for an objective state of the sphere [7]. In typical situations, detectors used to monitor the environment, e.g., eyes, have some minimum detection thresholds, and the fractions  $mN_t$  are meant to reflect it. The concrete fraction size is irrelevant here—it is enough that it scales with  $N_t$  [19]. The detailed initial product state of the environment  $(q_0^{\text{ph}})^{\otimes N_t}$  can thus be trivially rewritten as

$$\begin{aligned} \underbrace{q_0^{\text{ph}} \otimes \dots \otimes q_0^{\text{ph}}}_{N_t} &= \underbrace{q_0^{\text{ph}} \otimes \dots \otimes q_0^{\text{ph}}}_{mN_t} \otimes \dots \otimes \underbrace{q_0^{\text{ph}} \otimes \dots \otimes q_0^{\text{ph}}}_{mN_t} \\ &\equiv \underbrace{q_0^{\text{mac}} \otimes \dots \otimes q_0^{\text{mac}}}_M, \end{aligned} \quad (6)$$

where  $M \equiv 1/m$  is the number of macrofractions and  $q_0^{\text{mac}} \equiv (q_0^{\text{ph}})^{\otimes mN_t}$  is the initial state of each of them.

*Formation of the broadcast state.*—After all the  $N_t$  photons have scattered and  $(1-f)M$ ,  $0 \leq f \leq 1$ , macrofractions went unobserved (the necessary loss of information), the postscattering “out state”  $q_{S:FE}(t) \equiv \text{Tr}_{(1-f)E} \times U_{S:E}(t) q_{S:E}(0) U_{S:E}(t)^\dagger$  is given from Eqs. (5) and (6) for a product initial state  $q_{S:E}(0) \equiv q_0^S \otimes (q_0^{\text{ph}})^{\otimes N_t}$  by

$$\begin{aligned} q_{S:FE}(t) &= \sum_{i=1,2} \langle \vec{x}_i | q_0^S | \vec{x}_i \rangle |\vec{x}_i\rangle \langle \vec{x}_i| \otimes [q_i^{\text{mac}}(t)]^{\otimes fM} \\ &+ \sum_{i \neq j} \langle \vec{x}_i | q_0^S | \vec{x}_j \rangle (\text{Tr} \mathbf{S}_i q_0^{\text{ph}} \mathbf{S}_j^\dagger)^{(1-f)N_t} |\vec{x}_i\rangle \langle \vec{x}_j| \end{aligned} \quad (7)$$

$$\otimes (\mathbf{S}_i q_0^{\text{ph}} \mathbf{S}_j^\dagger)^{\otimes fN_t}, \quad (8)$$

where  $q_i^{\text{mac}}(t) \equiv (\mathbf{S}_i q_0^{\text{ph}} \mathbf{S}_i^\dagger)^{\otimes mN_t}$ ,  $i = 1, 2$ . We demonstrate that in the soft scattering sector [Eq. (2)], the above state approaches asymptotically the broadcast form [Eq. (1)] by showing that for  $t \gg \tau_D$ , (1) the postscattering coherent part  $q_{S:FE}^{i \neq j}(t)$ , defined by Eq. (8), vanishes in the trace norm (decoherence):

$$\|q_{S:FE}^{i \neq j}(t)\|_{\text{tr}} \equiv \text{Tr} \sqrt{[q_{S:FE}^{i \neq j}(t)]^\dagger q_{S:FE}^{i \neq j}(t)} \approx 0, \quad (9)$$

and (2) the postscattering macrostates  $q_i^{\text{mac}}(t)$  become perfectly distinguishable:  $q_1^{\text{mac}}(t) q_2^{\text{mac}}(t) \approx 0$ , or equivalently, using mixed state fidelity [20],

$$B[q_1^{\text{mac}}(t), q_2^{\text{mac}}(t)] \equiv \text{Tr} \sqrt{\sqrt{q_1^{\text{mac}}(t) q_2^{\text{mac}}(t)} \sqrt{q_1^{\text{mac}}(t)}} \approx 0, \quad (10)$$

despite the individual photon states (microstates) becoming equal in the thermodynamic limit.

The first mechanism above is the usual decoherence of  $S$  by  $fE$ . Some form of quantum correlations may still survive it, since the resulting state [Eq. (7)] is generally of a classical-quantum form [21], but they are damped by the second mechanism, and  $q_{S:fE}(\infty)$  becomes a spectrum broadcast state [Eq. (1)] for  $p_i = \langle \vec{x}_i | \rho_0^S \vec{x}_i \rangle$ .

The decoherence mechanism alone [Eq. (9)] has been extensively studied in the model for thermal initial states  $q_0^{\text{ph}}$  (see., e.g., Refs. [12–16]). We recall that the decay of the off-diagonal part  $q_{S:fE}^{i \neq j}(t)$ , defined by Eq. (8), is governed by the decoherence factor  $|\text{Tr} \mathbf{S}_1 \rho_0^{\text{ph}} \mathbf{S}_2^\dagger|$ , since  $\|q_{S:fE}^{i \neq j}(t)\|_{\text{tr}} = 2|\langle \vec{x}_1 | \rho_0^S \vec{x}_2 \rangle| |\text{Tr} \mathbf{S}_1 \rho_0^{\text{ph}} \mathbf{S}_2^\dagger|^{(1-f)N_i}$ . For pure  $q_0^{\text{ph}}$ , it reads in the regime [Eq. (2)] [12–16]

$$\begin{aligned} \langle \vec{k} | \mathbf{S}_2^\dagger \mathbf{S}_1 \vec{k} \rangle &= 1 + i \frac{8\pi \Delta x k^5 \tilde{a}^6}{3L^2} \cos \Theta - \frac{2\pi \Delta x^2 k^6 \tilde{a}^6}{15L^2} \\ &\times (3 + 11 \cos^2 \Theta) + O\left[\frac{(k\Delta x)^3}{L^2}\right], \end{aligned} \quad (11)$$

where  $\Theta$  is the angle between  $\vec{k}$  and  $\vec{\Delta x} \equiv \vec{x}_2 - \vec{x}_1$ ,  $\tilde{a} \equiv a[(\epsilon - 1)/(\epsilon + 2)]^{1/3}$ , while for a general distribution [Eq. (3)], it is given in the leading order in  $1/L$  by [12,15,16,22]

$$\begin{aligned} |\text{Tr} \mathbf{S}_1 \rho_0^{\text{ph}} \mathbf{S}_2^\dagger|^{(1-f)N_i} &\cong \left[ 1 - \frac{2\pi \Delta x^2 \tilde{a}^6}{15L^2} \right. \\ &\left. \times \sum_{\vec{k}} p(\vec{k}) k^6 (3 + 11 \cos^2 \Theta_{\vec{k}}) \right]^{(1-f)N_i} \end{aligned} \quad (12)$$

$$\xrightarrow{\text{therm}} \exp\left[-\frac{(1-f)}{\tau_D} t\right], \quad (13)$$

where  $\tau_D^{-1} \equiv (2\pi/15)(N/V)\Delta x^2 c \tilde{a}^6 \int d\vec{k} p(\vec{k}) k^6 (3 + 11 \cos^2 \Theta_{\vec{k}})$  is the decoherence time.

Completing the step [Eq. (10)] is more involved. For the microstates  $q_i^{\text{mic}} \equiv \mathbf{S}_i \rho_0^{\text{ph}} \mathbf{S}_i^\dagger$ , we obtain under Eq. (2) [22]

$$B(q_1^{\text{mic}}, q_2^{\text{mic}}) = 1 - \frac{\bar{\eta} - \bar{\eta}'}{L^2} \xrightarrow{\text{therm}} 1, \quad (14)$$

where

$$\bar{\eta} \equiv \frac{L^2}{2} \left( 1 - \sum_{\vec{k}} p(\vec{k}) |\langle \vec{k} | \mathbf{S}_1^\dagger \mathbf{S}_2 \vec{k} \rangle|^2 \right) \cong \left( \frac{N}{V} c \right)^{-1}, \quad (15)$$

$$\bar{\eta}' \equiv \frac{L^2}{2} \sum_{\vec{k}} \sum_{\vec{k}' \neq \vec{k}} p(\vec{k}) |\langle \vec{k} | \mathbf{S}_1^\dagger \mathbf{S}_2 \vec{k}' \rangle|^2, \quad (16)$$

so that  $q_{1,2}^{\text{mic}}$  become equal and encode no information about  $S$ . The same holds if the observed portion  $\mu$  of the environment  $E$  is microscopic, i.e., not scaling with  $N_i$ :

$$\begin{aligned} q_{S:\mu E}(0) &= q_0^S \otimes (q_0^{\text{mac}})^{\otimes \mu} \xrightarrow[\text{therm}]{t \gg \tau_D} q_{S:\mu E}(\infty) \\ &= \left( \sum_{i=1,2} p_i |\vec{x}_i\rangle \langle \vec{x}_i| \right) \otimes [q^{\text{mic}}]^{\otimes \mu}. \end{aligned} \quad (17)$$

This is a “product phase” [7], in which the mutual information  $I[q_{S:\mu E}(\infty)] = 0$ .

Passing to macrostates, the situation changes as now:

$$\begin{aligned} B[q_1^{\text{mac}}(t), q_2^{\text{mac}}(t)] &= \left( \text{Tr} \sqrt{\sqrt{q_1^{\text{mic}} q_2^{\text{mic}}} \sqrt{q_1^{\text{mic}}}} \right)^{mN_i} \\ &\cong \left( 1 - \frac{\alpha \bar{\eta}}{L^2} \right)^{mN_i} \xrightarrow{\text{therm}} \exp\left[-\frac{\alpha m}{\tau_D} t\right], \end{aligned} \quad (18)$$

where  $\alpha \equiv (\bar{\eta} - \bar{\eta}')/\bar{\eta}$  [16] and Eq. (15) was used. Thus, whenever  $\alpha \neq 0$  ( $\alpha = 0$ , e.g., for an isotropic illumination [22]),  $B[q_1^{\text{mac}}(t), q_2^{\text{mac}}(t)] \approx 0$  for  $t \gg \tau_D/\alpha$ , despite Eq. (14); i.e., the macrostates become perfectly distinguishable via orthogonal projectors on their supports. The latter are contained in  $\text{span}\{\vec{k} : \vec{k} \in \text{supp } p\}^{\otimes mN_i}$  [cf. Eq. (3)], rotated by  $\mathbf{S}_1^{\otimes mN_i}$  and  $\mathbf{S}_2^{\otimes mN_i}$ , respectively. Equations (13) and (18) together imply an asymptotic formation of the spectrum broadcast state [Eq. (1)]:

$$\begin{aligned} q_{S:fE}(0) &= q_0^S \otimes [q_0^{\text{mac}}]^{\otimes fM} \xrightarrow[\text{therm}]{t \gg \tau_D} q_{S:fE}(\infty) \\ &= \sum_{i=1,2} \langle \vec{x}_i | \rho_0^S \vec{x}_i \rangle |\vec{x}_i\rangle \langle \vec{x}_i| \otimes [q_i^{\text{mac}}(\infty)]^{\otimes fM}, \end{aligned} \quad (19)$$

with  $q_1^{\text{mac}}(\infty) q_2^{\text{mac}}(\infty) = 0$  [25]. The scattering [Eq. (19)] is thus a combination of the localization measurement in the pointer basis  $|\vec{x}_i\rangle$  and spectrum broadcasting of the result, described by a classical-classical-type channel [9]:

$$\Lambda_\infty^{S \rightarrow fE}(q_0^S) \equiv \sum_i \langle \vec{x}_i | \rho_0^S \vec{x}_i \rangle [q_i^{\text{mac}}(\infty)]^{\otimes fM}. \quad (20)$$

As a consequence of Eq. (19), it follows that [22]

$$I[q_{S:fE}(t)] \xrightarrow[\text{therm}]{t \gg \tau_D} H_S \quad \text{for any } 0 < f < 1; \quad (21)$$

i.e., the mutual information becomes asymptotically independent of the fraction size  $f$  (as long as it is macroscopic). This is the entropic objectivity condition of quantum Darwinism, leading to the characteristic classical plateau [4]. We stress that here, Eq. (21) is derived as a consequence of the state information broadcasting [Eq. (19)], and we call this regime a “broadcasting phase” [7]. When the whole  $E$  is observed, modulo a microfraction, there appears from Eq. (8) a “full information phase,” when quantum correlations are retained and  $I[\rho_{S:fE}(t)] \approx I_{\max}$ .

Comparing Eqs. (13) and (18), we observe that, unlike in the pure case [7], the time scales of decoherence [Eq. (9)] and distinguishability [Eq. (10)] are *a priori* different (cf. Ref. [16]):  $\bar{\tau}_D$  and  $\bar{\tau}_D/\alpha$ , respectively. Since  $0 \leq \alpha \leq 1$ , the broadcast state is fully formed for  $t \gg \bar{\tau}_D/\alpha$ . Environment noise thus slows down the formation of the broadcast state [26].

“Singular points” of decoherence.—Let the initial state of the sphere have a diagonal representation  $\rho_0^S = \sum_i \lambda_{0i} |\phi_i\rangle\langle\phi_i|$ . Then, in Eq. (19), there appears a stochastic matrix  $P_{ij}(\phi) \equiv |\langle\phi_i|\vec{x}_j\rangle|^2$ , which by the Perron-Frobenius theorem [17] possesses at least one stable probability distribution  $\lambda_{*i}(\phi)$ :  $\sum_j P_{ij}(\phi)\lambda_{*j}(\phi) = \lambda_{*i}(\phi)$ . It exists for any initial eigenbasis  $|\phi_i\rangle$ . Let us choose it as the initial spectrum  $\lambda_{0i} \equiv \lambda_{*i}(\phi)$ . Then, the scattering [Eq. (19)] not only leaves this distribution unchanged, but broadcasts it into the environment:

$$\begin{aligned} & \left[ \sum_i \lambda_{*i}(\phi) |\phi_i\rangle\langle\phi_i| \right] \otimes (\rho_0^{\text{mac}})^{\otimes fM} \xrightarrow[\text{therm}]{t \gg \tau_D} \rho_{S:fE}(\infty) \\ &= \sum_i \left( \sum_j P_{ij}(\phi) \lambda_{*j}(\phi) \right) |\vec{x}_i\rangle\langle\vec{x}_i| \otimes (\rho_i^{\text{mac}})^{\otimes fM} \\ &= \sum_i \lambda_{*i}(\phi) |\vec{x}_i\rangle\langle\vec{x}_i| \otimes (\rho_i^{\text{mac}})^{\otimes fM}. \end{aligned} \quad (22)$$

The initial spectrum does not “decohere.” This surprising Perron-Frobenius broadcasting [9] can thus be used to faithfully (in the asymptotic limit above) broadcast the classical message  $\{\lambda_{*i}(\phi)\}$  through the environment macrofractions, however noisy they are.

Final remarks.—There is one straightforward generalization to many parties. Consider several spheres, each with its own photonic environment, separated by distances  $D$ ,  $kD \gg 1$  [cf. Eq. (2)]. The interaction is then a product of Eq. (5), e.g., for two spheres,  $U_{S_1 S_2 : E_1 E_2}(t) \equiv \sum_{i,j=1}^2 2|\vec{x}_i\rangle\langle\vec{x}_i| \otimes |\vec{y}_j\rangle\langle\vec{y}_j| \otimes \mathbf{S}_i^{\otimes N_i} \otimes \tilde{\mathbf{S}}_j^{\otimes N_j}$ , where  $\vec{x}_i$  and  $\vec{y}_j$  are the spheres’ positions and  $\mathbf{S}_i$  and  $\tilde{\mathbf{S}}_j$  are the corresponding scattering matrices. The asymptotic state [Eq. (19)] provides objectivization of classical correlations [9], e.g.,  $p_{ij} \equiv \langle\vec{x}_i, \vec{y}_j| \rho_0^S |\vec{x}_i, \vec{y}_j\rangle$ , measurable by observers who have access to photons originating from all the spheres.

In the studied model, as in the majority of decoherence models, the system-environment interaction is of a form

$$H_{\text{int}} = g A_S \sum_{k=1}^N X_{E_k}, \quad (23)$$

where  $g$  is a coupling constant, and  $A_S$  and  $X_{E_1}, \dots, X_{E_N}$  are some observables on the system and the environments, respectively. The eigenbasis of  $A = \sum_i a_i |i\rangle\langle i|$  becomes the pointer basis—it is arguably put by hand by the choice of the form [Eq. (23)]. It is then an interesting question if there are more general interactions, without an *a priori* privileged basis (see, e.g., Ref. [27]), which lead to an asymptotic formation of spectrum broadcast states [Eq. (1)].

An investigation of application of the state information broadcasting process developed here to the theory of continuous observation of quantum systems (see, e.g., Ref. [28]) seems also an interesting direction.

Finally, it would be extremely interesting to test our findings experimentally. In fact, our central object, the broadcast state [Eq. (1)], is, in principle, directly observable through, e.g., quantum state tomography—a well developed, successful, and widely used technique [29].

This work is supported by the ERC Advanced Grant QOLAPS and National Science Centre Project Maestro DEC-2011/02/A/ST2/00305. We thank W. H. Zurek and C. J. Riedel for discussions. P. H. and R. H. acknowledge discussions with K. Horodecki, M. Horodecki, and K. Życzkowski.

\*jaroslaw.korbicz@ug.edu.pl

- [1] N. Bohr, in *Albert Einstein: Philosopher-Scientist*, Library of Living Philosophers Vol. 7, edited by P. A. Schilpp (Library of Living Philosophers, Evanston, IL, 1949).
- [2] W. Heisenberg, *Philosophic Problems in Nuclear Science* (Faber and Faber, London, 1952).
- [3] R. Blume-Kohout and W. H. Zurek, *Phys. Rev. A* **73**, 062310 (2006).
- [4] W. H. Zurek, *Nat. Phys.* **5**, 181 (2009).
- [5] M. Zwolak and W. H. Zurek, *Sci. Rep.* **3**, 1729 (2013).
- [6] E. Joos *et al.*, *Decoherence and the Appearances of a Classical World in Quantum Theory* (Springer, Berlin, 2003); M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition* (Springer, Berlin, 2007).
- [7] R. Horodecki, J. K. Korbicz, and P. Horodecki, arXiv:1312.6588.
- [8] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935); N. Bohr, *Phys. Rev.* **48**, 696 (1935); H. M. Wiseman, *Ann. Phys. (Amsterdam)* **338**, 361 (2013).
- [9] J. K. Korbicz, P. Horodecki, and R. Horodecki, *Phys. Rev. A* **86**, 042319 (2012).
- [10] W. H. Zurek, *Phys. Rev. D* **24**, 1516 (1981).
- [11] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, *Phys. Rev. Lett.* **76**, 2818 (1996).
- [12] E. Joos and H. D. Zeh, *Z. Phys. B* **59**, 223 (1985).
- [13] M. R. Gallis and G. N. Fleming, *Phys. Rev. A* **42**, 38 (1990).
- [14] K. Hornberger and J. E. Sipe, *Phys. Rev. A* **68**, 012105 (2003).

- [15] C. J. Riedel and W. H. Zurek, *Phys. Rev. Lett.* **105**, 020404 (2010).
- [16] C. J. Riedel and W. H. Zurek, *New J. Phys.* **13**, 073038 (2011).
- [17] R. A. Horn and C. R. Johnson, *Matrix Analysis* (Cambridge University Press, Cambridge, England, 1985).
- [18] A. Messiah, *Quantum Mechanics* (Dover, Mineola, 1999).
- [19] This is analogous to, e.g., a description of liquids: each point of a liquid (a macrofraction here) is in reality composed of a suitably large number of microparticles (individual photons). The same technique is also used in a mathematical approach to von Neumann measurements using so-called macroscopic observables; see, e.g., G. Sewell, *Rep. Math. Phys.* **56**, 271 (2005) and the references therein.
- [20] C. A. Fuchs and J. van de Graaf, *IEEE Trans. Inf. Theory* **45**, 1216 (1999).
- [21] M. Piani, P. Horodecki, and R. Horodecki, *Phys. Rev. Lett.* **100**, 090502 (2008); M. Piani, S. Gharibian, G. Adesso, J. Calsamiglia, P. Horodecki, and A. Winter, *Phys. Rev. Lett.* **106**, 220403 (2011).
- [22] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.112.120402> for the details, which include Refs. [23] and [24].
- [23] M. Fannes, *Commun. Math. Phys.* **31**, 291 (1973); K. M. R. Audenaert, *J. Phys. A* **40**, 8127 (2007).
- [24] R. Alicki and M. Fannes, *J. Phys. A* **37**, L55 (2004).
- [25] By Eqs. (13) and (18), the asymptotic formation of the spectrum broadcast state [Eq. (19)] relies on the full product form of the initial state  $\rho_{S:E}(0)$  and the interaction [Eq. (5)] in each block  $i$ . However, one can allow for correlated or entangled fractions of photons, as long as they stay microscopic, i.e., do not scale with  $N_i$ . The corresponding terms then factor out in front of the exponentials in Eqs. (13) and (18), and the formation of the spectrum broadcast states is not affected.
- [26] If the difference  $\overline{\tau_D}/\alpha - \overline{\tau_D}$  is sufficiently large, then for  $\overline{\tau_D} \ll t < \overline{\tau_D}/\alpha$ , the state  $\rho_{S:fE}(t)$  is approximately a classical-quantum state, whose mutual information is given by the Holevo quantity [A. S. Holevo, *Probl. Inf. Transm.* **9**, 177 (1973)]:  $I[\rho_{S:fE}(t)] = S_{\text{vN}}[\sum_i p_i \rho_i^{\text{mac}}(t)^{\otimes fM}] - (fM) \sum_i p_i S_{\text{vN}}[\rho_i^{\text{mac}}(t)]$ .
- [27] M. J. Everitt, W. J. Munro, and T. P. Spiller, *Phys. Lett. A* **374**, 2809 (2010).
- [28] T. Bhattacharya, S. Habib, and K. Jacobs, *Phys. Rev. Lett.* **85**, 4852 (2000).
- [29] *Quantum State Estimation*, Lecture Notes in Physics Vol. 649, edited by M. Paris and J. Řeháček (Springer, Berlin, 2004).