Effective Helium Burning Rates and the Production of the Neutrino Nuclei

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Effective values for the key helium burning reaction rates, triple-α and ^{12}C$(\alpha, \gamma)^{16}$O, are obtained by adjusting their strengths so as to obtain the best match with the solar abundance pattern of isotopes uniquely or predominately made in core-collapse supernovae. These effective rates are then used to determine the production of the neutrino isotopes. The use of effective rates considerably reduces the uncertainties in the production factors arising from uncertainties in the helium burning rates, and improves our ability to use the production of ^{11}B to constrain the neutrino emission from supernovae.

Uncertainties in the reaction rates for stellar helium burning have long limited the accuracy with which one can predict nucleosynthesis in massive stars [1–5]. In this Letter we outline a new approach, the use of effective reaction rates (ERRs), obtain a candidate ERR, and apply it to the production of the neutrino nuclei. It appears that this procedure considerably reduces the uncertainties in the predictions.

Early work along these lines by Weaver and Woosley [1] and by M. M. Boyes [6] (unpublished, but quoted in Ref. [7]) concentrated on the reaction rate, $r_{\alpha,\gamma}$, of the ^{12}C$(\alpha, \gamma)^{16}$O reaction. Boyes used the KEPLER code [8–11] to calculate the pre-supernova abundance of nine isotopes ranging from ^{16}O to ^{40}C, for various values of $r_{\alpha,\gamma}$ and found that the smallest spread in their production factors, as measured by their statistical variance, σ^2, occurred for a rate about 1.2 times that of Buchmann [12]. This rate was used in most subsequent calculations with KEPLER. For details see Ref. [7].

Later, Tur *et al.* [2] improved this procedure by using a larger set of stars and taking into account supernova explosive nucleosynthesis, which modified some of the reference abundances. The resulting best value was slightly changed, to 1.3 times that of Buchmann [12]. A problem with these approaches was that the value of the triple-α rate, $r_{3\alpha}$, was fixed at its experimental value, and since this value was itself uncertain, the overall validity of the process was difficult to assess.

In these attempts to determine a reaction rate, an implicit assumption was that uncertainties in the calculations themselves were substantially smaller than those resulting from uncertainties in the helium burning reaction rates. It is, however, not certain that this is the case, since the simulations do not include all phenomena that might influence nucleosynthesis. Although the KEPLER code can calculate the effects of rotation and magnetic fields,

such calculations are more cumbersome, and these effects were not included in the above calculations. In addition, many reaction rates are uncertain, as are opacities and mass loss rates. Perhaps the most important uncertainties are related to convection (KEPLER uses the Ledoux criterion), semiconvection, and overshooting. Helium burning reactions are strong sources of energy in the star and it is well known that a small change in these rates can have a major influence on nucleosynthesis processes affected by convection [4,10].

These issues are not particular to KEPLER but inherent to most stellar evolution codes. Imbriani *et al.* [13] studied the influence on stellar evolution of changes in the ^{12}C$(\alpha, \gamma)^{16}$O reaction, in combination with variations of the mixing processes; it appeared that these two uncertainties cannot be treated separately. In their work, however, they did not vary the triple-α rate, and did not follow nucleosynthesis beyond Zn. Sukhbold *et al.* [14] studied the sensitivity of stellar structure changes to mixing processes (semiconvection, overshooting) and compared different stellar evolution codes; they found that while there are significant differences in the outcomes using the default values for the codes, parameters for the mixing physics can be adjusted to give comparable results. It seems clear that uncertainties in the two reaction rates and in the mixing physics are to some extent intertwined and that all are important.

A possible approach in such a situation is to view the operators as "effective," with their parameters fixed by comparing the results of calculations to data. One example of this approach is the use of effective interactions in the description of nuclear structure using the nuclear shell model [15]. The effective interaction is determined by fitting low-lying energy levels of a set of nuclei. This procedure has been remarkably successful, and is the basis

of most modern large-basis shell-model calculations. In
many cases one cannot show in detail why the process
works well; its justification lies in the fact that the
procedure works for many observables.

In this Letter we describe a first attempt to obtain ERRs
for the helium burning reactions and to apply them to the
production of the neutrino isotopes ^{7}Li, ^{11}B, ^{19}F, ^{138}La, and
^{180}Ta. We obtain the ERRs by fitting the production of
intermediate-mass and s-only isotopes, taking advantage of
the extensive supernova calculations of West *et al.* [5].
In that work, KEPLER was used to model the evolution of
a group of 12 stars (initial masses 12, 13, 14, 15, 16, 17, 18,
20, 22, 25, 27, and 30 $M_\odot$) from central hydrogen burning
to core collapse; a piston placed at the base of the oxygen
shell was then used to simulate the explosion yielding a
total kinetic energy of the ejecta of 1.2×10^{51} erg. The
calculations were carried out for a matrix of rates, covering
$\pm 2\sigma$ for both $r_{\alpha,\gamma}$ and $r_{3\alpha}$ (176 rate pairs). This involved a
total of $12 \times 176 = 2112$ simulations. In the following
discussion the rates are characterized by a multiple of the
standard values as is described in Tur *et al.* [2]. The results
were then averaged over a Salpeter initial mass function
(IMF). For each reaction pair, the *standard deviations* of
the IMF averaged production factors of the intermediate-
mass isotopes (^{16}O, ^{18}O, ^{20}Ne, ^{23}Na, ^{24}Mg, ^{27}Al, ^{28}Si, ^{32}S,
^{36}Ar, ^{40}Ca) and the s-only isotopes (^{70}Ge, ^{76}Se, ^{80}Kr, ^{82}Kr,
^{86}Sr, ^{87}Sr) were obtained, and the standard deviations from
each of the two isotope lists were averaged, thereby giving
equal weight to the intermediate-mass and s-only isotopes.

Before we performed a comparison to observed abun-
dances, two corrections were made. First, models that
were likely to collapse to a black hole were filtered out
by including only models with a compactness factor
[14,16,17] satisfying $\xi_{2.5} < 0.25$. In addition, the observed
s-only abundances were corrected for the contributions
of other processes. For details of these calculations see
West *et al.* [5]. The results are shown graphically in Fig. 1.

These results are somewhat surprising. We expected that
both the rates would be individually constrained, but
instead we find that the best-fit points lie within a band.
A large range of $r_{3\alpha}$ is allowed, but the relationship between
$r_{\alpha,\gamma}$ and $r_{3\alpha}$ is constrained. A best-fit curve is shown.
It passes through the overall minimum of points, each of
which is the minimum local standard deviation of the fitted
abundances of intermediate-mass and weak-s isotopes,
determined as described above. There is no strong reason
for choosing one point on the ERR rate line over another;
this line is taken as the best available description of the
ERR. Clearly, the best value of one of the rates depends on
what the other rate is chosen to be. If a new measurement
showed reliably that the actual value of $r_{3\alpha}$ was 1.2 (0.8)
instead of 1.0, one would choose a significantly larger
(smaller) value of $r_{\alpha,\gamma}$ to best predict the nucleosynthesis of
the intermediate-mass and weak-s isotopes; the range of
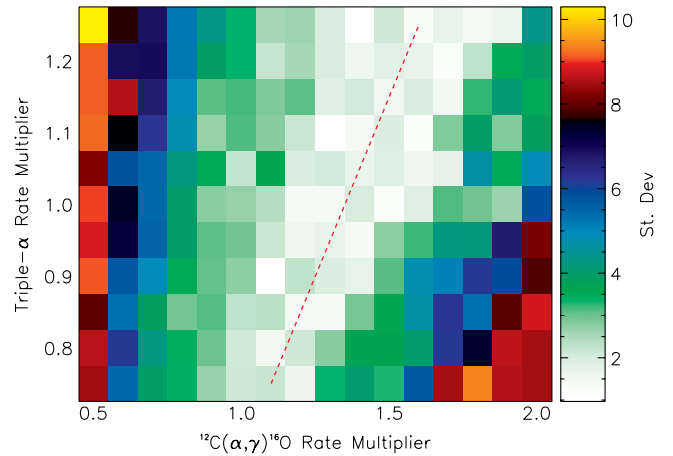values is 35%.

FIG. 1 (color online). The standard deviations for the IMF
average of the production factors of the intermediate mass and
weak s-only isotopes, including explosive yields and yields
from stellar winds. Only models that satisfied the compactness
condition $\xi_{2.5} < 0.25$ were included. The dashed line
($r_{\alpha,\gamma} = r_{3\alpha} + 0.35$) was drawn through the overall minima of
the calculated standard deviations. Adapted from Fig. 13 of [5]

We now apply this ERR to study the production of ^{7}Li,
^{11}B, ^{19}F, ^{138}La, and ^{180}Ta in the neutrino process. The
fundamental picture is simple: neutrinos emitted by the
protoneutron star resulting from core collapse interact with
relatively abundant nuclei in the stellar envelope to form
the precursors of the neutrino isotopes. After decay and
processing in the ensuing shock wave, these become
the observed isotopes. Austin *et al.* [18] concluded that
production of ^{11}B in the neutrino process might serve to
constrain the average neutrino production in supernovae.
The uncertainties arising from the uncertainties in the
helium burning rates, however, were relatively large, and
it seemed unrewarding to pursue the issue until one had a
better handle on the helium burning rates.

We followed the general procedures outlined in Heger
et al. and Austin *et al.* [18,19], but calculated the produc-
tion of the neutrino isotopes for ten ERR points along the
best-fit curve of Fig. 1. We used Fermi-Dirac neutrino
spectra, with temperatures for ν_e, $\bar{\nu}_e$, and ν_x of 4 MeV,
5 MeV and 6 MeV; x stands for μ and τ. The results are
shown in Fig. 2. For the entire range of the ERR line of
Fig. 1, the deviations from a constant value are typically
$\pm 10\%$ or less. The remaining variations arise (at least
mainly) from binning and aliasing effects, because the ERR
line does not pass precisely through the values of $r_{\alpha,\gamma}$ and
$r_{3\alpha}$ used in the models. Note that in Fig. 2, as well as in
Figs. 3 and 4, the values of $r_{3\alpha}$ shown on the abscissa lie
along the ERR line, and hence describe implicitly the
values of $r_{\alpha,\gamma}$.

This is to be compared to the much larger ranges found
in Austin *et al.* [18] when uncorrelated uncertainties of $r_{\alpha,\gamma}$
and $r_{3\alpha}$ were considered. These uncertainties are also
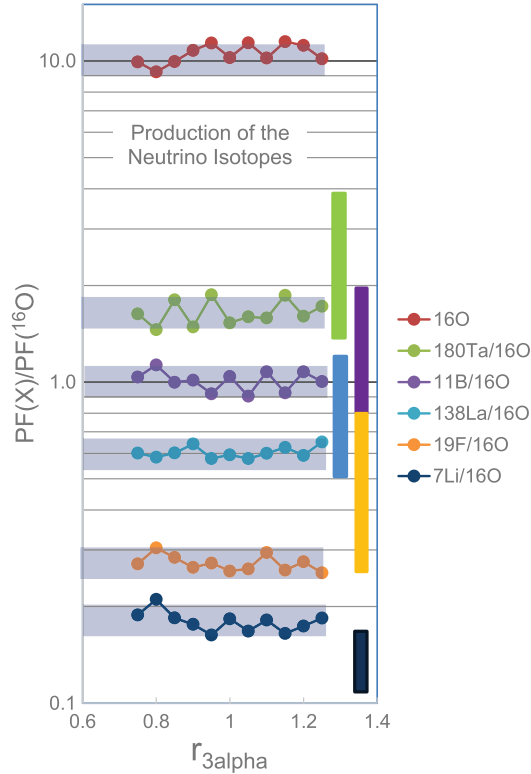shown as bars near the right-hand ordinate of Fig. 2.

FIG. 2 (color online). Production factors for the neutrino isotopes normalized to those for ^{16}O. The abscissa is the position along the ERR line of Fig. 1, parameterized by the value of $r_{3\alpha}$. The narrow bands covering $\pm 10\%$ give an indication of the precision of the present results. The bars on the right of the graph show the spread of uncorrelated errors obtained in Austin, *et al.* [18]. As noted in that paper, the values of the production ratio for ^{11}B that would agree with observation is about 0.4.

It is not straightforward to assess the accuracy of the ERRs. We obtained a rough estimate of possible effects of moving the line to the left by changing $r_{\alpha,\gamma}$ by -0.1, corresponding to the ERR that is obtained by fitting the intermediate isotopes only. This changed the production of the neutrino isotopes by between 5% and 12%.

One must ask whether these encouraging results are reliable. As a minimum, the use of ERRs allows one to deal with the effects of uncertainties in the reaction rates and the weaknesses of the model calculations in a unified way. It is striking that the production factors for the neutrino nuclei, which owe their origins to different shells in the star [19,20], vary so little with position along the line.

Another striking qualitative feature, shown in Fig. 3, is that the values of the central mass fraction at the end of helium burning are nearly constant along the ERR line. A similar statement (see Fig. 3) can be made for the baryonic mass of the progenitor of the remnant of that star. The larger variability for the 15 M$_\odot$ star apparently reflects a sensitivity to small changes in the reaction rates [14] that cannot be described by an ERR. There is, however, only a very weak overall trend with the value of $r_{3\alpha}$. It has been pointed
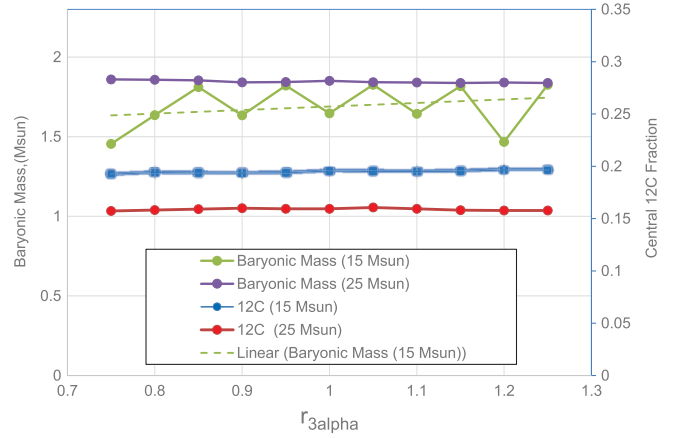

FIG. 3 (color online). Values of the central carbon fraction at the end of helium burning (right-hand ordinate) and of the baryonic mass of the progenitor of the stellar remnant (left-hand ordinate)[5].

out [13] that advanced stages of evolution are strongly influenced by the central ^{12}C abundance at the end of helium burning. The constant value of the carbon fraction along the ERR line may then provide an understanding of why the ERR apparently works well. See also the detailed study of Sukhbold *et al.* [14].

It may be, however, that fitting additional information could provide a better ERR, or illuminate other processes. With this in mind we examined the production factors of the intermediate and *s*-only isotopes. The intermediate isotopes provide no obvious additional information, but the *s*-only isotopes do. We find (Fig. 4) that for larger $r_{3\alpha}$, their average production factor, normalized to that of ^{16}O, is smaller, decreasing significantly from 0.85 and reaching a plateau of 0.55 for $r_{3\alpha} \gtrsim 1.0$. The standard deviations of the production factors, fitted in deriving the ERR, do not change significantly. This behavior arises from the temperature sensitivity of the ^{22}Ne$(\alpha, n)^{25}$Mg reaction. At lower
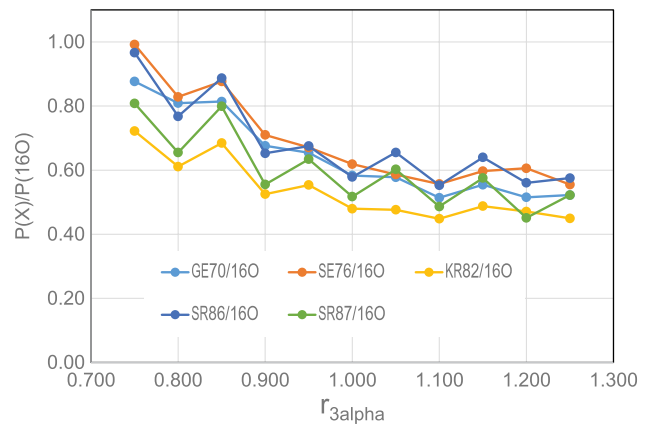

FIG. 4 (color online). The production factors of the *s*-only nuclei. ^{80}Kr is omitted because KEPLER does not treat the T dependence of ^{79}Se decay which affects ^{80}Kr production.
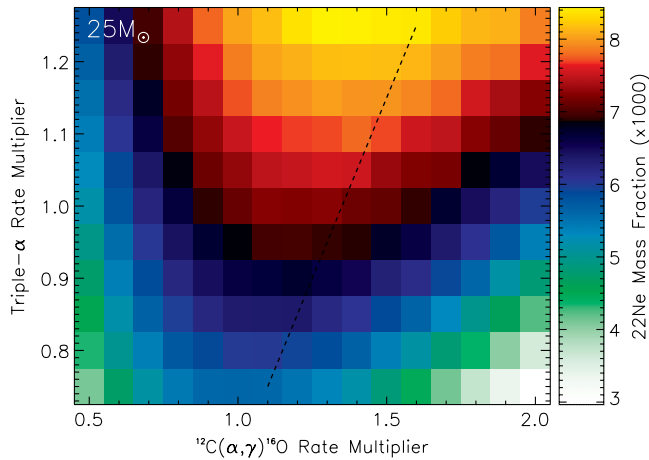
FIG. 5 (color online).   Amount of ^{22}Ne left in the core after end of central He burning for 25 $M_\odot$ stars. The dashed line indicates the ERR valley from Figure 1.

$r_{3\alpha}$, the burning temperature at the end of helium burning is higher [3], and so is the ^{22}Ne$(\alpha, n)^{25}$Mg reaction rate, resulting in more destruction of ^{22}Ne (as shown in Fig. 5), more neutron production, and a stronger weak s-process. We note, however, that the ^{22}Ne burning rates are quite uncertain, so the effect may be stronger or weaker than that shown.

Assuming that we have a sufficient understanding of the observed s-only abundances, and of the weak s-process, these observations favor helium burning rates at the lower end of the present ERR. Caution is warranted, however, because of the need to correct for the significant contribution of other processes to the s-only isotopes, the small number of these isotopes [21], and the uncertainty in the ^{22}Ne$(\alpha, n)^{25}$Mg rate.

To summarize, we have made a first attempt at developing an ERR for the two helium burning reactions, based on minimizing the standard deviation in the production factors of two groups of isotopes: the intermediate-mass isotopes and the s-only isotopes. This results in a correlation between the best values of the ^{12}C$(\alpha, \gamma)^{16}$O and triple-α rates. We have taken this representation of the ERR, as shown in Fig. 1, and evaluated the production of the neutrino nuclei at various points along the ERR line. They are essentially the same at all ERR points, lending credence to the procedure used to determine the effective rates. The success of the ERR may be related to the fact that the central ^{12}C densities and remnant masses along the ERR line (for 15 $M_\odot$ and 25 $M_\odot$ stars) are very closely the same.

These results apparently remove what was a major hurtle to comparing neutrino isotope abundances to the predictions; namely, that they depended so strongly on poorly known helium burning rates. It now becomes meaningful to address other uncertainties: the explosion energy, the neutrino interaction cross section, the cross sections for

reactions that process the mass-11 products, and the nature of the neutrino spectrum, as outlined in Austin *et al.* [18] to see whether, as described there, one can use the abundance of ^{11}B to determine the average emission of neutrinos in supernova explosions.

We note that the derivation of the ERRs depends on the model and is only valid for KEPLER and the specific values used for input physics, including mixing processes, reaction rates, initial abundances, and metallicity.

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[1] T. A. Weaver and S. E. Woosley, Phys. Rep. **227**, 65 (1993).

[2] C. Tur, A. Heger, and S. M. Austin, Astrophys. J. **671**, 821 (2007).

[3] C. Tur, A. Heger, and S. M. Austin, Astrophys. J. **702**, 1068 (2009).

[4] C. Tur, A. Heger, and S. M. Austin, Astrophys. J. **718**, 357 (2010).

[5] C. West, A. Heger and S. M. Austin, Astrophys. J. **769**, 2 (2013).

[6] M. Boyes, B. S. Thesis, University of California Santa Cruz, 2001.

[7] S. E. Woosley and A. Heger, Phys. Rep. **442**, 269 (2007).

[8] T. A. Weaver, G. B. Zimmerman, and S. E. Woosley, Astrophys. J. **225**, 1021 (1978).

[9] S. E. Woosley and T. A. Weaver, Astrophys. J. Suppl. Ser. **101**, 181 (1995) .

[10] T. Rauscher, A. Heger, R. D. Hoffman, and S. E. Woosley, Astrophys. J. **576**, 323 (2002).

[11] S. E. Woosley, A. Heger, and T. A. Weaver, Rev. Mod. Phys. **74**, 1015 (2002).

[12] L. Buchmann, Astrophys. J., **468**, L127 (1996); **479**, L153 (E) (1997).

[13] G. Imbriani, M. Limongi, L. Gialanella, F. Terrasi, O. Straniero, and A. Chieffi, Astrophys. J. **558**, 903 (2001).

[14] T. Sukhbold and S. E. Woosley, Astrophys. J. **783**, 10 (2014).

[15] B. A. Brown and W. A. Richter, Phys. Rev. C **74**, 034315 (2006).

[16] E. Oconnor and C. Ott, Astrophys. J. **730**, 70 (2011).

[17] S. E. Woosley (private communication).

[18] S. M. Austin, A. Heger, and C. Tur, Phys. Rev. Lett. **106**, 15 (2011).

[19] A. Heger, E. Kolbe, W. C. Haxton, K. Langanke, G. Martinez-Pinedo, and S. E. Woosley, Phys. Lett. B **606**, 258 (2005).

[20] S. E. Woosley, D. H. Hartmann, R. D. Hoffman, and W. C. Haxton, Astrophys. J. **356**, 272 (1990).

[21] C. West and A. Heger, Astrophys. J. **774**, 75 (2013).