One-Dimensional Fermions with neither Luttinger-Liquid nor Fermi-Liquid Behavior

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It is well known that, generically, one-dimensional interacting fermions cannot be described in terms of a Fermi liquid. Instead, they present a different phenomenology, that of a Tomonaga-Luttinger liquid: the Landau quasiparticles are ill defined, and the fermion occupation number is continuous at the Fermi energy. We demonstrate that suitable fine tuning of the interaction between fermions can stabilize a peculiar state of one-dimensional matter, which is dissimilar to both Tomonaga-Luttinger and Fermi liquids. We propose to call this state a quasi-Fermi liquid. Technically speaking, such a liquid exists only when the fermion interaction is irrelevant (in the renormalization group sense). The quasi-Fermi liquid exhibits the properties of both a Tomonaga-Luttinger liquid and a Fermi liquid. Similar to a Tomonaga-Luttinger liquid, no finite-momentum quasiparticles are supported by the quasi-Fermi liquid; on the other hand, its fermion occupation number demonstrates a finite discontinuity at the Fermi energy, which is a hallmark feature of a Fermi liquid. A possible realization of the quasi-Fermi liquid with the help of cold atoms in an optical trap is discussed.

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Introduction.—An important goal of modern many-body physics is the search for exotic states of matter. Appropriate examples are spin liquids [1–3], the Majorana fermion [4–8], topological insulators, semimetals [9–11], and others. A peculiar state of one-dimensional (1D) fermionic matter deviating from known types of interacting Fermi systems is the subject of this Letter.

Let us remind ourselves that the most basic model of interacting fermions is that of the Fermi liquid. It successfully describes a variety of interacting fermion systems (e.g., electrons in solids, atoms of helium-3) [12]. The approach is based on the Landau conjecture that both the ground state of a Fermi liquid and its low-lying excitations are adiabatically connected to states of a noninteracting Fermi gas. If the interaction is weak, this hypothesis implies that the perturbation theory in the interaction strength is valid. The latter supplies theorists with a tool to study specific examples.

A known system for which the Landau conjecture fails is a 1D liquid of interacting fermions. The interacting 1D fermions constitute a separate universality class, the so-called Tomonaga-Luttinger liquid [13,14]: unlike a Fermi liquid, the Tomonaga-Luttinger ground and excited states have zero overlap with the corresponding noninteracting states, and the Tomonaga-Luttinger liquid properties cannot be calculated perturbatively with interaction strength as a small parameter.

In 1D the Tomonaga-Luttinger liquid is a generic state of matter. However, recent progress in fabrication and control over the properties of many-particle systems allows us to look for more fragile types of 1D correlated liquids. Specifically, consider a gas of Fermi atoms in a 1D trap [15]. It is within modern experimental capabilities to vary the effective interaction constant of optically trapped atoms, and even tune the constant to zero [16,17]. Below we will demonstrate that such nullification of the effective coupling constant does not imply vanishment of all microscopic interactions. Some residual interactions remain, and in 1D they stabilize a peculiar state of matter, which we propose to call a quasi-Fermi liquid. The latter state appears to be a hybrid of both Fermi and Tomonaga-Luttinger liquids: its ground state is perturbatively connected to the ground state for free fermions, yet the perturbatively defined quasiparticles do not exist. That is, in the case of a quasi-Fermi liquid, the Landau conjecture is valid only for the ground state, but not for excitations. Of course, there is nothing special about cold atoms, and the quasi-Fermi liquid may be realized in other fermion systems, which allow adequate fine tuning of the coupling.

The presentation below has the following structure. First, we formally introduce our model. Second, the self-energy is evaluated perturbatively, which allows us to determine both the quasiparticle residue and the occupation number corrections. Third, analyzing these quantities we will be able to define the quasi-Fermi liquid as a distinct state of fermionic matter. Fourth, we discuss the possible implementation of such a quantum liquid using optically trapped cold atoms. Finally, we formulate our conclusions. In the Supplemental Material [18] we present the extension of our calculations beyond second-order perturbation theory, and discuss other subtleties.

The studied model.—One-dimensional interacting fermions are commonly described by the Tomonaga-Luttinger Hamiltonian:

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$$H_{\rm TL} = H_{\rm kin} + H_{\rm int}, \quad (1)$$

$$H_{\rm kin} = iv_F \int dx(:\psi_{\rm L}^\dagger \nabla\psi_{\rm L}: - :\psi_{\rm R}^\dagger \nabla\psi_{\rm R}:), \quad (2)$$

$$H_{\rm int} = g\int dx \rho_{\rm L}\rho_{\rm R}, \quad (3)$$

where ψ_p is the field operator for the right-moving ($p = R$) and left-moving ($p = L$) fermions, operators $\rho_p = :\psi_p^\dagger \psi_p:$ are the densities of the left and right movers, v_F is the Fermi velocity, and g is the coupling constant. Colons denote the normal ordering.

The Tomonaga-Luttinger liquid differs from the Fermi liquid: the perturbatively defined quasiparticles are absent, the Fermi occupation number $n_k^p = \langle c_{pk}^\dagger c_{pk}\rangle$ has no discontinuity at the Fermi point, and the Tomonaga-Luttinger ground state has zero overlap with the free fermion ground state.

The culprit responsible for these abnormalities is the fermion-fermion interaction $H_{\rm int}$, which is marginal in the renormalization group sense. Perturbation theory in orders of g has additional divergences absent in higher-dimensional systems. For example, the Matsubara single-particle self-energy is equal to [19–21]

$$\Sigma_{\rm TL}^p = \frac{g^2}{16\pi^2 v_F^2}(i\nu - pv_F k)\ln\left(\frac{v_F^2 k^2 + \nu^2}{4v_F^2\Lambda^2}\right) + \cdots, \quad (4)$$

where the ellipsis stands for the less singular terms, $p = +1$ ($p = -1$) for the right-moving (left-moving) fermions, and Λ is the ultraviolet cutoff. This self-energy corresponds to the following expression

$$\delta Z_{\rm TL}^p = \frac{g^2}{16\pi^2 v_F^2}\ln\left(\frac{4v_F^2\Lambda^2}{v_F^2 k^2 + \nu^2}\right) + \cdots, \quad (5)$$

for correction to the quasiparticle residue $Z_{\rm TL}^p = 1 - \delta Z_{\rm TL}^p$. The correction diverges for small ν and k. As a result, the conventional Fermi quasiparticles are ill defined, and the occupation number function has a power-law singularity instead of the discontinuity. The properties of $H_{\rm TL}$, Eq. (1), are now well understood [13,14].

However, it is sometimes required to include irrelevant operators into consideration. There are two least irrelevant operators:

$$H_{\rm nl} = v_F' \int dx[:(\nabla\psi_{\rm L}^\dagger)(\nabla\psi_{\rm L}): + :(\nabla\psi_{\rm R}^\dagger)(\nabla\psi_{\rm R}):], \quad (6)$$

$$H_{\rm int}' = ig' \int dx\{\rho_{\rm R}[:\psi_{\rm L}^\dagger(\nabla\psi_{\rm L}): - :(\nabla\psi_{\rm L}^\dagger)\psi_{\rm L}:] - \rho_{\rm L}[:\psi_{\rm R}^\dagger(\nabla\psi_{\rm R}): - :(\nabla\psi_{\rm R}^\dagger)\psi_{\rm R}:]\}. \quad (7)$$

Here $H_{\rm nl}$ is the quadratic correction to the linear dispersion of the fermions, and $H_{\rm int}'$ is the irrelevant interaction. Both $H_{\rm nl}$ and $H_{\rm int}'$ have a scaling dimension of 3 (the dimension of the gradient operator is 1, each field operator has a dimension of 1/2). Other irrelevant operators have higher scaling dimensions; therefore, their effects are less pronounced.

Recently, the Hamiltonian

$$H = H_{\rm TL} + H_{\rm nl} + H_{\rm int}' \quad (8)$$

and its modifications have been investigated actively [22–40]. These studies have demonstrated that the combined effect of the marginal and the irrelevant operators has important and measurable consequences for a system's properties.

In this Letter we will discuss the model of the 1D fermions without the marginal interaction at all:

$$H_{\rm ii} = H_{\rm kin} + H_{\rm nl} + H_{\rm int}', \quad (9)$$

where "ii" stands for "irrelevant interaction." We may name two examples where $H_{\rm ii}$ is applicable. First, consider cold Fermi atoms in a 1D trap. Under rather general conditions the suitable Hamiltonian is given by Eq. (1), see Refs. [17,41]. However, the interaction between the atoms is highly adjustable [16,17], which may be used to our advantage: below we will offer an argument suggesting that the system parameters can be tuned in such a manner that g [or, more precisely, renormalized coupling $g^{\rm eff} = g + \mathcal{O}((g')^2)$] vanishes, but $g' \neq 0$.

Our second case requires no fine tuning. Using the unitary transformation of Ref. [42], it has been demonstrated that the Tomonaga-Luttinger Hamiltonian with nonlinear dispersion, Eq. (8), may be mapped [24,26,34] on Hamiltonian $H_{\rm ii}$ (see also Ref. [43]). Therefore, the properties of $H_{\rm ii}$ are important for the theoretical description of the generic model H.

Superficially, one expects that, since $H_{\rm ii}$ has only the irrelevant interaction, it describes a kind of 1D Fermi liquid. Indeed, using perturbation theory, we will demonstrate that the correction to the fermion occupation number n_k^p is finite and small. However, in a drastic departure from the Fermi liquid picture, the quasiparticle residue correction diverges on the mass surface. Thus, Hamiltonian $H_{\rm ii}$ describes a state of 1D matter that lies halfway between the Fermi liquid and the Tomonaga-Luttinger liquid: n_k^p has finite discontinuity at the Fermi energy, but no perturbatively defined quasiparticles exist. This is our quasi-Fermi liquid.

Self-energy correction.—To implement the outlined plan, we must calculate the self-energy. For definiteness, consider the self-energy for right movers. The corresponding diagram is shown in Fig. 1.

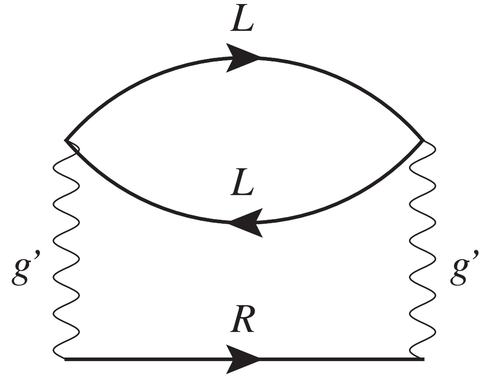

FIG. 1. The leading self-energy correction diagram. The solid lines with arrows and L, R chirality labels correspond to the fermion propagators. The wiggly lines are irrelevant interactions.

The expression that must be evaluated is

$$\Sigma^R_{k,i\nu} = -(g')^2 T^2 \sum_{i\Omega, i\nu'} \int_{Q,q} (2q - 2k)^2 G^{R,0}_{k-Q,i\nu - i\Omega} \times G^{L,0}_{q-Q,i\nu'} G^{L,0}_{q,i\Omega + i\nu'}. \quad (10)$$

In this equation $\int_k \cdots = \int (dk/2\pi) \cdots$; the free Matsubara propagator is $G^{p,0}_{k,i\omega} = (i\omega - \varepsilon^p_k)^{-1}$, where the fermion dispersion is $\varepsilon^p_k = p v_F k + v'_F k^2$. The factor $(2k - 2q)^2$ appears because each interaction line contributes a factor of $g'(2k - 2q)$ to the diagram. The overall minus sign accounts for the presence of a single fermion loop. Calculating the momentum integrals we assume that

$$|q|, \quad |Q| < \Lambda < k_F = \frac{v_F}{2v'_F}, \quad (11)$$

where k_F is the Fermi momentum. This way we may avoid complications arising from spurious zeros of ε^p_k, which are located at $k = -2pk_F$.

Performing the standard summation over $i\Omega$ and $i\nu$ and taking the limit $T \to 0$ we find

$$\Sigma^R = (g')^2 \int_{Q,q} (2q - 2k)^2 [\theta(-\varepsilon^L_q) - \theta(-\varepsilon^L_{q-Q})] \times \frac{\theta(\varepsilon^L_{q-Q} - \varepsilon^L_q) - \theta(\varepsilon^R_{k-Q})}{i\nu - \varepsilon^R_{k-Q} - \varepsilon^L_q + \varepsilon^L_{q-Q}}. \quad (12)$$

For our purposes it is convenient to evaluate the imaginary part of the retarded self-energy:

$$\mathrm{Im}\Sigma^R_{\mathrm{ret}} = -\pi(g')^2 \int_{Q,q} (2q - 2k)^2 [\theta(-\varepsilon^L_q) - \theta(-\varepsilon^L_{q-Q})] \times [\theta(\varepsilon^L_{q-Q} - \varepsilon^L_q) - \theta(\varepsilon^R_{k-Q})] \delta(\nu - \varepsilon^R_{k-Q} - \varepsilon^L_q + \varepsilon^L_{q-Q}). \quad (13)$$

Now we integrate over Q:

$$\mathrm{Im}\Sigma^R_{\mathrm{ret}} = -(g')^2 \int_q \frac{(k-q)^2}{v_F + v'_F(k-q)} [\theta(q) - \theta(q - Q^*)] \times [\theta(\varepsilon^L_{q-Q^*} - \varepsilon^L_q) - \theta(\varepsilon^R_{k-Q^*})], \quad (14)$$

where $Q^*(q)$ delivers zero to the argument of the delta function in Eq. (13):

$$Q^* = -\frac{\Delta\nu}{2v_F + 2v'_F(k-q)}, \qquad \Delta\nu = \nu - \varepsilon^R_k. \quad (15)$$

Thus, we need to evaluate the integral

$$I = \int_0^{q^*} dq \frac{(k-q)^2 [\theta(\varepsilon^L_{q-Q^*} - \varepsilon^L_q) - \theta(k - Q^*)]}{v_F + v'_F(k-q)}, \quad (16)$$

where the upper limit of the integration $q^* \approx -\Delta\nu/2v_F$ satisfies the equation $q^* = Q^*(q^*)$. It is easy to check that

$$\theta(\varepsilon^L_{q-Q^*} - \varepsilon^L_q) = \theta(Q^*[v_F - v'_F(2q - Q^*)]) = \theta(Q^*). \quad (17)$$

Further, analyzing Eq. (15), we determine that the sign of Q^* coincides with the sign of $(\varepsilon^R_k - \nu)$. Consequently,

$$\theta(\varepsilon^L_{q-Q^*} - \varepsilon^L_q) = \theta(\varepsilon^R_k - \nu), \quad (18)$$

where the function on the right-hand side is independent of the integration variable q.

The second step function $\theta(k - Q^*)$ in Eq. (16) can be evaluated easily near the mass surface $\nu = \varepsilon^R_k$. When the mass surface is approached, $Q^* \to 0$; consequently, $\theta(k - Q^*) = \theta(k)$.

Since both step functions are independent of the integration variable q, the integral I can be trivially evaluated to the lowest order in $\nu - \varepsilon^R_k$. Keeping the most singular term, we derive

$$\mathrm{Im}\Sigma^R_{\mathrm{ret}} = -\frac{(g'k)^2}{4\pi v_F^2} (\varepsilon^R_k - \nu)[\theta(\varepsilon^R_k - \nu) - \theta(k)] + \delta\Sigma, \quad (19)$$

where $\delta\Sigma$ stands for less singular terms. To obtain $\mathrm{Re}\Sigma^R_{\mathrm{ret}}$ we use the Kramers-Kronig relations. For the first term in Eq. (19) the Kramers-Kronig integral can be easily calculated analytically (with Λ playing the role of the high-energy cutoff):

$$\Sigma^R_{\mathrm{ret}} = \frac{(g'k)^2}{4\pi^2 v_F^2} (\nu - \varepsilon^R_k) \ln\left(\frac{\nu - \varepsilon^R_k + i0}{v_F \Lambda}\right) + \cdots. \quad (20)$$

The less-singular contribution due to $\delta\Sigma$ is replaced by the ellipsis.

Equation (20) resembles Eq. (4): both have singularities at the mass surface. Yet, there is an important difference:

the expression in Eq. (20) has an extra k^2 factor, which acts to weaken the singular contribution at small k. We will see that the peculiar properties of our system may be traced back to this feature of the self-energy.

The quasiparticle residue [44]

$$Z^R(k) = \frac{1}{1 - \partial\Sigma^R_{\rm ret}/\partial\nu}\Bigg|_{\nu=\varepsilon^R_k}, \tag{21}$$

$$\frac{\partial\Sigma^R_{\rm ret}}{\partial\nu} = \frac{(g'k)^2}{4\pi^2 v_F^2}\ln\left(\frac{\nu - \varepsilon^R_k + i0}{v_F\Lambda}\right) + \cdots \tag{22}$$

vanishes for any finite k due to the divergence of $\partial\Sigma^R_{\rm ret}/\partial\nu$ on the mass surface. Thus, like the Tomonaga-Luttinger model, our system does not support the perturbatively defined quasiparticles. However, since the interaction is irrelevant, in the Matsubara domain $\partial\Sigma^R/\partial\nu$ remains finite, while the expression in Eq. (5) diverges. The divergence is sensitive to temperature: if one replaces the step functions in Eq. (13) by appropriate Fermi functions, the resultant $\mathrm{Im}\Sigma^R_{\rm ret}$ becomes a continuous function of its arguments. Consequently, $\mathrm{Re}\Sigma^R_{\rm ret}$ becomes finite. Note, also, that Eq. (22) does not contain v_F' explicitly. Thus, the destruction of the quasiparticles occurs even for systems with linear dispersion, provided that the interaction is nonzero $g' \neq 0$.

Despite the absence of the quasiparticles, the fermionic occupation numbers $n^p_k = \langle c^\dagger_{pk} c_{pk}\rangle$ remain well defined. This is not surprising: any finite-order correction to a ground-state matrix element due to irrelevant interaction is finite (since Z_k is a property of an excited state, it is exempt from this rule). To calculate δn^p_k explicitly we start with the formula $n^R_k = -\int_{-\infty}^0 (d\nu/\pi)\mathrm{Im}G^R_{{\rm ret},k,\nu}$. Therefore, the second-order correction is equal to

$$\delta n^R_k = -\int_{-v_F\Lambda}^0 \frac{d\nu}{\pi}\mathrm{Im}[(G^{R,0}_{\rm ret})^2\Sigma^R_{\rm ret}]. \tag{23}$$

Substituting the expressions for $G^{R,0}_{\rm ret}$ and $\Sigma^R_{\rm ret}$ it is easy to show that

$$(G^{R,0}_{\rm ret})^2\Sigma^R_{\rm ret} = \frac{(g'k)^2}{8\pi^2 v_F^2}\frac{\partial}{\partial\nu}\left[\ln\left(\frac{\nu - \varepsilon^R_k + i0}{v_F\Lambda}\right)\right]^2 + \cdots, \tag{24}$$

where, as above, the ellipsis stands for the less-singular contributions to $\Sigma^R_{\rm ret}$. With the help of this formula the integral in Eq. (23) can be trivially evaluated

$$\delta n^R_k \approx \frac{(g'k)^2}{4\pi^2 v_F^2}\theta(\varepsilon^R_k)\ln\left(\frac{\varepsilon^R_k}{v_F\Lambda}\right) + \cdots, \tag{25}$$

which is finite and small for any $|k| < \Lambda$, provided that g' is small.

Quasi-Fermi liquid.—The calculations presented above prove that the quasi-Fermi liquid of 1D spinless fermions constitutes a distinct state of matter. Indeed, it is not a Tomonaga-Luttinger liquid: since δn^R_k, Eq. (25), is small, the quasi-Fermi liquid occupation number is discontinuous at the Fermi energy, while the Tomonaga-Luttinger's n^p_k is continuous. [This dissimilarity is a consequence of the fact that the marginal interaction in the Tomonaga-Luttinger Hamiltonian induces a stronger singularity of the self-energy diagram than the singularity of Eq. (20). As a result, for the Tomonaga-Luttinger liquid the occupation number correction diverges for small k.]

On the other hand, the state of matter we are dealing with is not a Fermi liquid because it has no perturbatively defined fermionic quasiparticles. (Heuristic nonperturbative construction of excitations for $H_{\rm ii}$ is discussed in the Supplemental Material [18].) However, the system retains certain features of the Fermi liquid: as we have mentioned in the previous paragraph, the occupation number exhibits a finite discontinuity at $k = 0$. This discontinuity exists even though the quasiparticles do not.

Let us now discuss the experimental identification of the quasi-Fermi liquid. Because of its peculiar nature, the quasi-Fermi liquid may present itself in experiment as an ordinary Fermi liquid, unless the measurements are done at sufficiently high energy. Indeed, formally, the correction to the quasiparticle residue diverges for any finite k; however, the divergence becomes progressively weaker as k approaches the Fermi point.

To appreciate the latter point imagine that the single-fermion spectral function is measured, and the quasiparticle residue is extracted. For an experimental apparatus with finite resolution width Ω the measured value of $\delta Z^{R,\Omega}_k$ is never divergent

$$|\delta Z^{R,\Omega}| = \frac{(g'k)^2}{4\pi^2 v_F^2}\ln\left(\frac{v_F\Lambda}{\Omega}\right) < \infty. \tag{26}$$

In this expression the divergence of δZ^R_k, Eq. (22), is cut at the energy scale $\sim\Omega$. Nonetheless, it is possible that $|\delta Z^{R,\Omega}| > 1$, provided that k is not too small: $k > k^\times$, where $k^\times$ is equal to

$$k^\times = \frac{2\pi v_F}{g'\sqrt{\ln(\frac{v_F\Lambda}{\Omega})}}. \tag{27}$$

The quantity $k^\times$ defines the crossover scale: for momenta smaller than $k^\times$ the experimental behavior of the system is indistinguishable from the usual Fermi liquid. Indeed, $|k| < k^\times \Leftrightarrow |\delta Z^{R,\Omega}| < 1$. Thus, the characteristic divergence of the quasiparticle residue may be measured only for momenta k in the interval $k^\times < |k| < \Lambda$. If the resolution is so poor that $k^\times > \Lambda$, the experimentally measured behavior of the system is indistinguishable from the Fermi liquid for any k. This imposes a restriction on Ω: it has to be smaller than $\Omega_{\rm max} = v_F\Lambda\exp[-(2\pi v_F/g'\Lambda)^2]$. Therefore, unless we have an apparatus with exponentially sharp resolution, the phenomenology of the quasi-Fermi liquid may be observed only if g' is not too small. However, at larger g' our perturbation theory becomes less accurate. Can the quasi-Fermi liquid survive in the nonperturbative regime? We hypothesize that the quasi-Fermi liquid, much like Fermi or Tomonaga-Luttinger liquids, constitutes its own separate universality class, and the quasi-Fermi liquid phenomenology extends beyond the small-g' region.

Cold atoms.—Finally, let us discuss the possible implementation of the quasi-Fermi liquid with the help of cold fermion atoms in a trap [15]. To characterize the gas, instead of using a full interatomic potential $V(x)$, the interactions in such systems are modeled by an effective delta-function-like potential with the corresponding coupling g. Such formalism is equivalent to our $H_{\rm int}$ [see Eq. (3)]. Experimentally, it is possible to control the magnitude and sign of the coupling g. Moreover, g can be nullified. When this nullification occurs, however, the atoms will not behave as a noninteracting gas. Indeed, the vanishing of $H_{\rm int}$ does not imply the vanishing of the irrelevant $H'_{\rm int}$, which drives the system toward the quasi-Fermi liquid.

To be more specific, consider the following toy model: a 1D fermions gas with weak interaction $\int dx dx' V(x-x')\rho(x)\rho(x')$. For such a situation the effective low-energy Hamiltonian of the form H, Eq. (8), may be derived. The (bare) coupling constants are

$$g = 2\int V(x)[1-\cos(2k_F x)]dx, \tag{28}$$

$$g' = \int xV(x)\sin(2k_F x)dx. \tag{29}$$

Usually, it is enough to retain g, and g' is discarded due to its irrelevance.

Imagine now that we adjust V to cancel g. [Strictly speaking, we must eradicate the renormalized coupling $g^{\rm eff} = g + \mathcal{O}((g')^2)$; however, when V is small, the corrections to the bare coupling are insignificant.] In a generic situation g' remains finite even when $g=0$. Of course, in this case g' cannot be neglected, and $H_{\rm ii}$ [see Eq. (9)] is realized. The aim of this discussion is to demonstrate that upon destruction of the marginal interaction one does not arrive at the free fermion theory. Rather, the new effective theory has the irrelevant interaction term, and our system becomes the quasi-Fermi liquid.

Conclusions.—To conclude, we have shown that the system of 1D spinless fermions with irrelevant interaction is neither a Fermi liquid, nor is it a Tomonaga-Luttinger liquid. Instead, our system constitutes a distinct state of matter, which we propose to call the quasi-Fermi liquid. The generic Tomonaga-Luttinger Hamiltonian with nonlinear dispersion is known to be unitary equivalent to the Hamiltonian of such a quasi-Fermi liquid. In addition, we have speculated that the quasi-Fermi liquid may be realized using cold atoms in a 1D trap.

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[43] Note that, despite the fact that Hamiltonians H and H_{ii} are unitary equivalent, they belong to different universality classes. As we know, the universality class is defined by the asymptotic behavior of different propagators. If we were to calculate a Tomanaga-Luttinger propagator $G = \langle O(\tau)\exp(-\tau H)O^{\dagger}\rangle_{\mathrm{TL}}$, we could use the unitary equivalence of the Hamiltonians $H = UH_{\mathrm{ii}}U^{-1}$ to express G as follows: $G = \langle [U^{-1}O(\tau)U]\exp(-\tau H_{\mathrm{ii}})[U^{-1}OU]^{\dagger}\rangle_{\mathrm{ii}}$. This propagator, however, is not equal to the propagator $G_{\mathrm{ii}} = \langle O(\tau)\exp(-\tau H_{\mathrm{ii}})O^{\dagger}\rangle_{\mathrm{ii}}$, and may have different asymptotic properties.
[44] G. Mahan, *Many-Particle Physics* (Kluwer Academic/ Plenum Publishers, New York, 2000).