Relativistic Transfer Ionization and the Breit Interaction

O. Yu. Andreev,[1] E. A. Mistonova,[1] and A. B. Voitkiv[2]
[1]*Physics Department, St. Petersburg State University, 198504 St. Petersburg, Russia*
[2]*Max-Planck-Institut für Kernphysik, D-69117 Heidelberg, Germany*
(Received 6 January 2014; published 12 March 2014)

We consider correlated transfer ionization in relativistic collisions between a highly charged ion and a light atom. In this process two quasifree electrons of the atom interact with each other during the short collision time that results in the capture of one of them by the ion and emission of the other. We show that this process is strongly influenced by the generalized Breit interaction already at modest relativistic impact energies.

DOI: 10.1103/PhysRevLett.112.103202 PACS numbers: 34.10.+x, 34.50.Fa

Charged particles moving with velocities much smaller than the speed of light c ($c \approx 137$ a.u.) interact with each other mainly via the Coulomb force. When the velocities increase and approach the speed of light the electromagnetic fields generated by the particles start to noticeably deviate from the Coulomb law, which in turn influences the form of their interaction. Nevertheless, in many cases the effect of the relativistic corrections to the Coulomb force remains quite modest unless the impact energies reach extremely high values.

For instance, the total cross section for ionization of a light atom by collision with an ion is proportional to $\ln av + \ln\gamma - v^2/2c^2$ (see, e.g., [1]), where v is the collision velocity (in a.u.), $\gamma = 1/\sqrt{1 - v^2/c^2}$, and $a \sim$ 1-10 is a quantity whose value depends on the properties of the atom. The last two terms arise due to the relativistic corrections to the Coulomb interaction between the ion and the atomic electron. In order that these terms change the cross section by 2 or 5 times one should have $\gamma \sim 10^2$ or 10^8, respectively, which corresponds to impact energies of the order of 100 GeV/u or 10^5 TeV/u.

When the interacting particles are electrons the (leading) relativistic correction to the Coulomb force is given by the generalized Breit interaction (GBI) [2]. The latter is usually derived within the scope of quantum electrodynamics appearing as a result of the exchange of a transverse photon between the electrons. However, as a rule, the contribution of the GBI remains a small correction to that due to the Coulomb force.

For ionization of a light atom by high-energy electrons, the situation with the importance of the relativistic correction is quite similar to that in ionization by the ion impact. In particular, in order that the GBI increases the total cross section for ionization by 2 or 5 times, the electron impact energy should reach enormous values ($\sim$100 MeV or 10^2 TeV, respectively).

Compared to ionization of light atoms the GBI is much more important for excitation and "ionization" of very highly charged ions [like, e.g., $U^{91+}(1\,s)$] by electron impact [3], [4]. However, even in such a case, in order that the GBI could strongly dominate these processes the impact energy also has to be extremely high.

The electron-electron interaction plays an important role in the projectile-electron loss [5]. In this process a partially stripped projectile-ion looses an electron due to the collision with a neutral atom. The interaction acting between the electrons of the ion and atom may result in the simultaneous electron loss from the ion and ionization of the atom. The effect of the GBI in such a process, however, remains modest no matter how high the impact energy [6].

The GBI can influence dielectronic recombination. However, it can dominate this process only provided some special transitions are considered [7] and remains on overall more or less minor correction to the Coulomb force.

Here we consider correlated transfer ionization in relativistic collisions between a highly charged ion (HCI) and a light atom. In this process two atomic electrons interact with each other during the very short collision time that results in the capture of one of them by the ion and emission of the other. It turns out that, in contrast to the processes mentioned above, the correlated transfer ionization is profoundly influenced by the GBI already at very modest relativistic impact energies.

Note that while transfer ionization in the relativistic domain was not yet explored, the different aspects (correlated and uncorrelated) of this process in nonrelativistic collisions ($v \ll c$) with low charged ions were extensively studied (see [8–14] and references therein).

Atomic units ($\hbar = m_e = e = 1$) are used throughout except where otherwise stated.

Let us consider the collision between a highly charged bare ion with a charge Z_i and a light atom with atomic number Z_a ($Z_a \ll Z_i$) which has two K-shell electrons. Our consideration employs the semiclassical approximation in which the electrons are treated as quantum particles whereas the heavy nuclei are described classically. It is convenient to perform the basic consideration using the rest frame of the HCI and take its position as the origin. In this frame the nucleus of the atom moves along a straight-line classical trajectory, $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$, where $\mathbf{b}$, $\mathbf{v}$, and t are the impact parameter, the velocity of the atom, and the time, respectively. Note that in relativistic collisions v is much larger than the typical orbiting velocities $\sim Z_a$ of the electrons in the atom.

If two free electrons are incident on an ion one of them can be captured into a bound state of the ion due to the interaction with the other electron which carries away the energy excess in such a three-body recombination process. In the ion-atom collision the electrons are initially bound in the incident atom. However, if the change in their momenta in the collision is much larger than their orbiting momenta in the atom they can be regarded as (quasi) free. Such an approximation is very well established in the calculations of radiative and nonradiative capture of a single electron (see, e.g., [15]) and is also valid for other collision processes characterized by large changes in the momentum of the atomic electrons (see, e.g., [16–18]). In our case the change in the momenta of the electrons is $\sim \gamma v$ ($\gamma = 1/\sqrt{1 - v^2/c^2}$); it greatly exceeds their orbiting momenta $\sim Z_a$ in the atom and the approximation of quasifree electrons is well justified.

The correlated transfer ionization proceeds via the (single) interaction between the quasifree atomic electrons during the short collision time when the ion and atom are very close to each other. The transition amplitude for this process, which is of the first order in the electron-electron interaction, consists of the “direct” and “exchange” contributions. The direct one is given by [18]

$$a^d_{fi}(\mathbf{b}) = -\frac{i}{c^2}\sum_{\mu=0}^{3}\int d^4x\, j_{1\mu}(x,\mathbf{b})A^{\mu}(x,\mathbf{b}). \tag{1}$$

Here, $j_{1\mu}(x)$ ($\mu = 0, 1, 2, 3$) is electromagnetic transition four-current generated by one of the electrons in the collision at a space-time point $x = (x_0, \mathbf{x})$ and $A^{\mu}(x, \mathbf{b})$ is the four-potential of the electromagnetic field created by the other electron at the same point. It satisfies the Maxwell equations

$$\left(\frac{\partial^2}{\partial x_0^2} - \Delta\right)A^{\mu}(x,\mathbf{b}) = \frac{4\pi}{c}\, j_2^{\mu}(x,\mathbf{b}), \tag{2}$$

where $j_2^{\mu}(x, \mathbf{b})$ is the transition four-current generated by the other electron in the collision.

The field of the HCI, because of its strength, has crucial impact on the motion of the electrons not only in the final but also in the initial channel [15], [18], [19]. Taking into account the effect of the HCI’s field on the initial state in the impulse approximation and using Eqs. (1)–(2) one can show that the Fourier transform of (1) into the momentum space is given by

$$S^d_{fi}(\mathbf{q}_\perp) = -\frac{i}{2\pi\gamma v}\int d^3\boldsymbol{\kappa}\,\xi_a(\boldsymbol{\kappa}, \mathbf{q} - \kappa) \times \int d^3\mathbf{r}_1 \int d^3\mathbf{r}_2 (L_{\text{Coul}} + L_{\text{GBI}}), \tag{3}$$

where ξ_a is the momentum distributions of the electrons in the initial atomic state as viewed in the rest frame of the atom (the atomic Compton profile) and the terms

$$L_{\text{Coul}} = \chi_b^{\dagger}(\mathbf{r}_2)\chi^{(+)}_{\mathbf{p}_2}(\mathbf{r}_2)\frac{1}{r_{12}}\chi^{(-)\dagger}_{\mathbf{p}}(\mathbf{r}_1)\chi^{(+)}_{\mathbf{p}_1}(\mathbf{r}_1)$$

$$L_{\text{GBI}} = -\chi_b^{\dagger}(\mathbf{r}_2)\alpha_2\chi^{(+)}_{\mathbf{p}_2}(\mathbf{r}_2)\frac{e^{iK_0 r_{12}}}{r_{12}}\chi^{(-)\dagger}_{\mathbf{p}}(\mathbf{r}_1)\alpha_1\chi^{(+)}_{\mathbf{p}_1}(\mathbf{r}_1) + \chi_b^{\dagger}(\mathbf{r}_2)\chi^{(+)}_{\mathbf{p}_2}(\mathbf{r}_2)\frac{e^{iK_0 r_{12}} - 1}{r_{12}}\chi^{(-)\dagger}_{\mathbf{p}}(\mathbf{r}_1)\chi^{(+)}_{\mathbf{p}_1}(\mathbf{r}_1) \tag{4}$$

arise due to the Coulomb and GBI parts, respectively, of the electron-electron interaction. In (4) α_j are the Dirac matrices for the jth electron ($j = 1, 2$), $\chi^{(+)}_{\mathbf{p}_1}$ and $\chi^{(+)}_{\mathbf{p}_2}$ with $\mathbf{p}_1 = \boldsymbol{\kappa}_\perp + \gamma(\kappa_z + v\varepsilon_a^{(1)}/c^2)\mathbf{v}/v$ and $\mathbf{p}_2 = \mathbf{q}_\perp - \boldsymbol{\kappa}_\perp + \gamma(q_z - \kappa_z + v\varepsilon_a^{(2)}/c^2)\mathbf{v}/v$ describe the motion of the incident electrons in the field of the HCI ($\varepsilon_a^{(1)}$ and $\varepsilon_a^{(2)}$ are the electron energies in the initial atomic state), and χ_b and $\chi^{(-)}_{\mathbf{p}}$ are the final states of the captured and emitted electrons, respectively. $\mathbf{q} = (\mathbf{q}_\perp, q_z)$ is the momentum transfer in the collision, where $\mathbf{q}_\perp$ and $q_z = (\varepsilon_p + \varepsilon_b - \gamma\varepsilon_a)/\gamma v$ are its transverse and longitudinal parts, respectively, ε_b is the energy of the captured electron, and ε_p is the energy of the emitted electron, $\varepsilon_a = \varepsilon_a^{(1)} + \varepsilon_a^{(2)}$ and $K_0 = |\epsilon_b - \gamma\epsilon_a^{(1)} - \gamma v\kappa_z|/c$.

Equations (3)–(4) were derived using the Feynman gauge. Note that in a gauge-invariant treatment the same result is obtained no matter which gauge (e.g., Feynman or Coulomb) is employed [20].

Because of the term $\xi_a(\boldsymbol{\kappa}, \mathbf{q} - \boldsymbol{\kappa})$ the integrand in Eq. (3) becomes very small if $|\boldsymbol{\kappa}|$ and/or $|\mathbf{q} - \boldsymbol{\kappa}|$ substantially exceed the typical orbiting momenta ($\sim Z_a$) of the electrons in the atom. Therefore, taking into account that $Z_a \ll v$ and neglecting the binding energy of the electrons in the atom compared to their rest energy, $\varepsilon_a^{(1,2)} \approx mc^2$, one can set $\mathbf{p}_1 = \mathbf{p}_2 = m\gamma\mathbf{v}$ and $K_0 = |\varepsilon_b - \gamma mc^2|/c$.

The exchange contribution S^e_{fi} to the transition amplitude can be obtained from (3)–(4) by interchanging the electrons in the initial (or final) states. The amplitude S_{fi} for the correlated transfer ionization is $S_{\text{fi}} = S^d_{\text{fi}} - S^e_{\text{fi}}$.

The cross section for the transfer ionization, differential in energy and angle of the emitted electron, reads

$$\frac{d^2\sigma}{d\varepsilon \sin\vartheta_p d\vartheta} = \int_0^{2\pi} d\varphi_p \int d^2\mathbf{q}_\perp |S_{fi}(\mathbf{q}_\perp)|^2, \tag{5}$$

where ϑ_p is the polar emission angle. In (5) the integration runs over the transverse part of the momentum transfer and the azimuthal angle φ_p of the emission.

Our results for the cross section (5) (which determines the energy-angular spectrum of the emitted electrons) are shown in Figs. 1 and 2 for collisions of carbon atoms with different HCIs at two impact energies. The K-shell atomic electrons are mostly tightly packed and, as a result, have the strongest interaction with each other. Therefore, only these electrons were taken into account in our calculations [21]. Note also that the results shown in the figures were computed by describing the states χ_b, $\chi_{\mathbf{p}_1}$, $\chi_{\mathbf{p}_2}$, and $\chi_{\mathbf{p}}$ fully relativistically (as a Dirac bound state and Dirac-Coulomb continuum states, respectively).

The results, displayed in the left column of these figures, were obtained by neglecting the GBI. In such an approximation the calculated spectra contain two maxima centered at $\vartheta_p = 0°$ and $\vartheta_p \approx 25°$. The shape of these spectra is quite similar to that obtained for the transfer ionization in nonrelativistic ($v \ll c, Z_i \ll c$) collisions with HCIs [19,22].

In the nonrelativistic case the maximum at $\vartheta_p = 0°$ is caused by the so-called electron-electron Auger (EEA) mechanism of transfer ionization [22] (which was proposed in [12] and confirmed experimentally in [13]). The other maximum appears due to the distortion of the motion of both electrons in the continuum by the Coulomb field of the ion [22] [the corresponding mechanism might be termed the electron-electron Coulomb distortion (EECD) mechanism]. In contrast to the EEA maximum it vanishes if the charge of the ion becomes small ($Z_i \ll v$) [23].

The right column in Figs. 1 and 2 presents the results obtained when the full relativistic form of the electron-electron interaction is taken into account. Comparing the right and left columns one can conclude that the GBI strongly influences the shape of the electron emission.

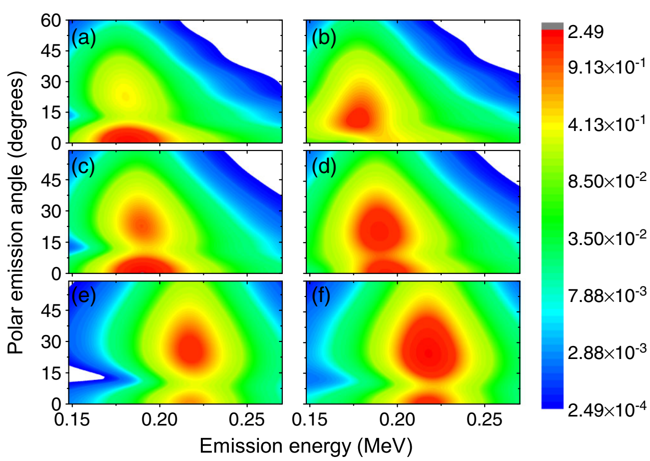

FIG. 1 (color online). Energy-angular spectrum [in mb/(MeV rad)] of the emitted electrons in 152 MeV/u $X^{Z_i+} + \mathrm{C} \longrightarrow X^{(Z_i-1)+}(1\ s) + \mathrm{C}^2 + e^-$ collisions ($v = 70$ a.u.). $Z_i = 30$ (a), (b), $Z_i = 40$ (c),(d), $Z_i = 60$ (e),(f). The spectrum is given in the rest frame of the HCI, the emission angle is counted from the velocity of the atom. The results shown in the right (left) column are obtained using the full relativistic (Coulomb) electron-electron interaction.

If the charge of the HCI is noticeably smaller than the collision velocity v the full relativistic calculation predicts only one maximum in the spectrum of the emitted electrons [see (b) in Figs. 1 and 2]. Its position differs from those of the maxima obtained neglecting the GBI. When, at a fixed velocity v, the charge of the HCI increases, this maximum stretches expanding to smaller and larger emission angles ϑ_p (not shown in the figures) and eventually splits into two well-separated maxima centered at $\vartheta_p = 0°$ and $\vartheta_p \approx 25°$ [see (d) and (f) in Figs. 1 and 2). The maximum at $\vartheta_p = 0°$ is caused by the EEA mechanism, whereas the other one appears due to the EECD. Similarly to the nonrelativistic case [19,22], the relative importance of the EEA (EECD) decreases (increases) when the HCI's charge grows.

In order to get insight into the role of the distortion of the electron motion by the Coulomb field of the HCI we have performed two more sets of calculations. In one of them the functions $\chi_{\mathbf{p}_1}$, $\chi_{\mathbf{p}_2}$, and $\chi_{\mathbf{p}}$ were approximated by the relativistic (Dirac) plane waves. In the second, the function $\chi_{\mathbf{p}_1}$ for the electron to be captured was taken as the Dirac-Coulomb continuum state, whereas the functions $\chi_{\mathbf{p}_2}$ and $\chi_{\mathbf{p}}$ for the other electron were again approximated by the plane waves.

According to such calculations the effect of the relativistic part of the electron-electron interaction on the shape of the emission spectra is weak. In particular, in all these calculations the spectrum has only one maximum centered at $\vartheta_p = 0°$ no matter whether the electron-electron interaction is taken in its full relativistic form or approximated by the Coulomb interaction. It means that the effect of the GBI in the correlated transfer ionization is intimately connected with the distortion of the motion of *both* electrons in the Coulomb field of the highly charged ion.

The GBI not only qualitatively changes the shape of the electron emission pattern but also has a strong impact on the absolute values of the cross section. In Table I we

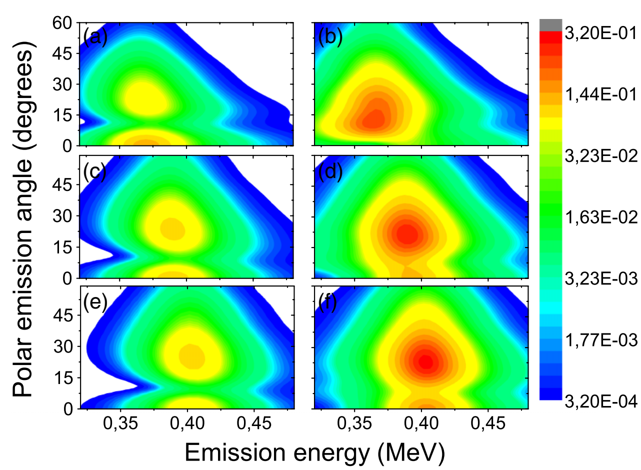

FIG. 2 (color online). Same as in Fig. 1 but for an impact energy of 304 MeV/u ($v = 90$ a.u.) and $Z_i = 50$ (a),(b), $Z_i = 63$ (c),(d) and $Z_i = 70$ (e),(f).

TABLE I. The total cross section for the correlated transfer ionization. The first and second columns show the collision velocity and the colliding system, respectively, the third and fourth columns display the cross section calculated by ignoring ($\sigma_{\rm Coul}$) and including ($\sigma_{\rm full}$) the GBI, the last column gives the ratio $R = \sigma_{\rm full}/\sigma_{\rm Coul}$.

v (a.u.)	Colliding particles	$\sigma_{\rm Coul}$ (mb)	$\sigma_{\rm full}$ (mb)	R
70	$Zn^{30+} + C$	2.74	4.93	1.8
	$Zr^{40+} + C$	5.61	9.51	1.7
	$Nd^{60+} + C$	8.91	17.74	2.0
90	$Sn^{50+} + C$	0.47	1.55	3.3
	$Eu^{63+} + C$	0.74	2.32	3.1
	$Yb^{70+} + C$	0.81	2.67	3.3
110	$Yb^{70+} + C$	0.04	0.39	8.3
	$At^{85+} + C$	0.06	0.49	7.7
	$U^{92+} + C$	0.07	0.92	13.1

present the total cross section for the correlated transfer ionization. This cross section was calculated by neglecting ($\sigma_{\rm Coul}$) and including ($\sigma_{\rm full}$) the GBI.

It follows from Table I that the GBI substantially influences the total cross section already at a modest relativistic impact energy of 152 MeV/u ($v = 70$ a.u.), where the corresponding collisional Lorentz factor is still not far from unity ($\gamma = 1.16$). Moreover, the GBI may increase the cross section by more than an order of magnitude at an impact energy of 630 MeV/u ($v = 110$ a.u.) for which the Lorentz factor remains well below 2 ($\gamma = 1.68$). This could be compared with the electron impact ionization of $U^{91+}(1s)$ where an order of magnitude increase in the total cross section due to the GBI would be reached at impact energies $\sim 10^{15}$ eV, corresponding to the Lorentz factor of $\sim 10^9$.

In Fig. 3 we present the emission spectrum for the correlated transfer ionization (152 MeV/u $Zn^{30+} + C \longrightarrow Zn^{29+} + C^2 + e^-$, $v = 70$ a.u.) in the laboratory frame in which the atom is at rest and the ion moves with a velocity $\mathbf{v}_p = (0, 0, -v)$. It follows from the figure that in this frame the electron is emitted with a large energy in the direction opposite to the motion of the ion. The effect of the GBI is clearly seen also in the laboratory frame that makes it possible to verify it in experiment.

In collisions with HCIs the so-called independent transfer ionization yields the main contribution to the total transfer ionization. However, this channel is not of interest for the present study because the electron-electron interaction plays in it essentially no role since capture of one electron and emission of another proceed independently of each other. In the rest frame of the atom this channel leads to the emission of low energy electrons which preferably move in the direction of the motion of the ion. Therefore, in relativistic collisions the emission patterns due to the correlated and independent transfer ionization do not overlap.

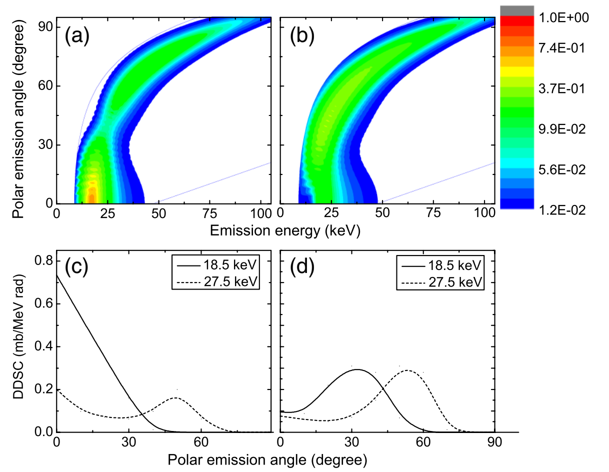

FIG. 3 (color online). Panels (a) and (b) correspond to Figs. 1(a) and 1(b) but the spectrum is given in the atomic (laboratory) frame in which the HCI moves under 180°. Panels (c) and (d) show the "cuts" of the spectrum for emission energies of 18.5 and 27 keV as a function of the emission angle.

In conclusion, we have considered the correlated transfer ionization in relativistic collisions between a highly charged ion and a light atom. We have shown that this process is profoundly influenced by the generalized Breit interaction already at modest relativistic impact energies. This interaction qualitatively changes the shape of the emission pattern and strongly increases the emission. These effects can be verified experimentally by detecting high-energy electrons emitted in the laboratory frame under large angles (~120°-180°) with respect to the motion of the highly charged ion.

O. Y. A. and E. A. M. are grateful to the MPIK Heidelberg for the hospitality during their visits and to the DFG for financial support, project SU 658/2-1. The work of O. Y. A. and E. A. M. was supported by the RFBR Grants No. 12-02-91340-NIIO$_a$, No. 14-02-00188 A and by the Ministry of Education and Science of the Russian Federation under the project 8420. The work of E. A. M. was also supported by German-Russian Interdisciplinary Science Center (G-RISC) Grant No. P-2013a-1. A. B. V. acknowledges support from the Extreme Matter Institute EMMI and the DFG under the Project No. VO 1278/2-1.

[1] U. Fano, Phys. Rev. **102**, 385 (1956).
[2] J. B. Mann and W. R. Johnson, Phys. Rev. A **4**, 41 (1971).
[3] R. E. Marrs, S. R. Elliott, and D. A. Knapp, Phys. Rev. Lett. **72**, 4082 (1994);C. J. Fontes, D. H. Sampson, and H. L. Zhang, Phys. Rev. A **51**, R12 (1995).
[4] A. Gumberidze *et al.*, Phys. Rev. Lett. **110**, 213201 (2013).
[5] A. B. Voitkiv and J. Ullrich, *Relativistic Collisions of Structured Atomic Particles* (Springer-Verlag, Berlin, 2008).

[6] A. B. Voitkiv, B. Najjari, and J. Ullrich, Phys. Rev. Lett. **92**, 213202 (2004).
[7] N. Nakamura, A. Kavanagh, H. Watanabe, H. Sakaue, Y. Li, D. Kato, F. Currell, and S. Ohtani, Phys. Rev. Lett. **100**, 073203 (2008);S. Fritzsche, A. Surzhykov, and Th. Stöhlker, Phys. Rev. Lett. **103**, 113001 (2009).
[8] V. Mergel, R. Dörner, Kh. Khayyat, M. Achler, T. Weber, O. Jagutzki, H. Lüdde, C. Cocke, and H. Schmidt-Böcking, Phys. Rev. Lett. **86**, 2257 (2001).
[9] H. T. Schmidt *et al.*, Phys. Rev. Lett. **89**, 163201 (2002).
[10] A. Godunov, C. T. Whelan, and H. R. J. Walters, J. Phys. B **37**, L201 (2004).
[11] H. T. Schmidt *et al.*, Phys. Rev. A **72**, 012713 (2005).
[12] A. B. Voitkiv, B. Najjari, and J. Ullrich, Phys. Rev. Lett. **101**, 223201 (2008); A. B. Voitkiv, J. Phys. B **41**, 195201 (2008).
[13] M. Schulz *et al.*, Phys. Rev. Lett. **108**, 043202 (2012).
[14] L. Gulyas, A. Igarashi, and T. Kirchner, Phys. Rev. A **86**, 024701 (2012).
[15] D. H. Jakubassa-Amundsen, Eur. Phys. J. D **41**, 267 (2007).
[16] V. L. Yakhontov and M. Ya. Amusia, Phys. Rev. A **55**, 1952 (1997).
[17] A. I. Mikhailov, I. A. Mikhailov, A. V. Nefiodov, G. Plunien, and G. Soff, Phys. Lett. A **328**, 350 (2004).
[18] B. Najjari and A. B. Voitkiv, Phys. Rev. A **85**, 052712 (2012).
[19] A. B. Voitkiv, Phys. Rev. Lett. **111** 043201 (2013).
[20] In particular, Eqs. (4) can be obtained from the well-known form of the GBI operator given by, e.g., Eq. (79) of I. Lindgren, S. Salomonson, and B. Åsén, Phys. Rep. **389** 161 (2004).
[21] A rough estimate for the contribution of the other pairs of electrons in a carbon atom can be obtained by considering effective localization volumes for these pairs (see [17]) which suggests that the other pairs yield about 25% contribution to the total cross section.
[22] A. B. Voitkiv, arXiv:1305.0417.
[23] The EECD should not be confused with the well-known electron-electron Thomas mechanism of transfer ionization [L. H. Thomas, Proc. R. Soc. A **114**, 561 (1927)] which is effective in collisions with fast low charged ions ($Z_i \ll v \ll c$) and for which the interaction between the ion and only one of the atomic electrons is necessary. In nonrelativistic collisions with HCIs, when $Z_i \sim v$, the Thomas mechanism is already not possible kinematically. In the relativistic domain this mechanism is not supported by the kinematics both for $Z_i \ll v$ and $Z_i \sim v$.