## Microscopic Theory of the Inverse Edelstein Effect

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We provide a precise microscopic definition of the recently observed inverse Edelstein effect in which a nonequilibrium spin accumulation in the plane of a two-dimensional (interfacial) electron gas drives an electric current perpendicular to its own direction. The drift-diffusion equations that govern the effect are presented and applied to the interpretation of the experiments.

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Introduction.-The spin Hall effect (SHE) and the inverse spin Hall effect (ISHE) are well established phenomena [1-11], which play an important role in experimental spintronic devices [12-17]. In the SHE, an electric current  $J_x$ , driven by an electric field  $E_x$  produces a z spin current in the y direction, denoted by  $J_{y}^{z}$ . In the ISHE, which is the Onsager reciprocal of the SHE, a spin current  $J_{\nu}^{z}$ , driven by a "spin electric field"  $E_{y}^{z}$  produces an electric current  $J_x$  in the x direction. Both effects are characterized by the spin-Hall conductivity, which can be as large as  $10^5(\Omega \cdot m)^{-1}$  in bulk metals such as Pt [18,19]. The SHE plays an important role in technology as a source of spin currents that can, for example, excite a spin wave in a ferromagnet or flip the magnetization of an element of a spin valve structure [18]. Likewise, the ISHE has been exploited for the detection of spin currents [19–21].

Another well-known effect, intimately related to the SHE, is the so-called Edelstein effect (EE) [22] (notice, however, the paper published at about the same time by Lyanda-Geller and Aronov [23]). In this effect, a steady current  $J_x$ , driven by an electric field  $E_x$  produces a steady non-equilibrium spin polarization  $S^y$ . The effect has been observed experimentally [24,25] and can be understood, on a basic level, as the result of the effective magnetic field (due to spin-orbit coupling) "seen" by the drifting electrons in their own reference frame [26].

After much theoretical work in the past decade, an intuitive and useful drift-diffusion theory of the SHE, ISHE, and EE has recently emerged (see Refs. [27–29]). This theory is firmly grounded in quantum kinetic equations and diagrammatic calculations for systems in which the spin-orbit interaction is linear in  $\mathbf{k}$  and can, therefore, be described by an SU(2) vector potential. In the meanwhile, relatively little attention has been paid to the inverse Edelstein effect (IEE), by which we mean the Onsager reciprocal of the normal Edelstein effect. Although the spin-galvanic effect observed almost a decade ago by Ganichev *et al.* [30] in GaAs may be interpreted as a

manifestation of the IEE, to the best of our knowledge, the latter was first introduced and quantitatively described in a very recent experimental paper by Rojas Sánchez *et al.* [31]. The possibility of spin-voltage conversion was also recognized theoretically, but only in very specific devices such as quantum dots connected to leads [32] and spin-polarized p-n junctions in semiconductors [33–35]. In this Letter, we provide a precise theoretical characterization of this effect, independent of any specifics of the spin injection mechanism, and introduce the drift-diffusion equations that describe it.

At first sight, the IEE is puzzling: a static magnetic field  $B^{y}$ , which couples linearly to the spin density  $S^{y}$ , will not create an electric current in the x direction; rather, it will change the value of the equilibrium spin polarization  $S^{y}$ . This reasoning fails to recognize the essential difference that exists between the spin polarization created by a static magnetic field in equilibrium and the nonequilibrium spin polarization that arises from a steady spin injection. In both cases, the spin polarization is constant in time, but similar to the spin-galvanic effect in p-n junctions [33], it is only the nonequilibrium part of the polarization that drives an electric current. The theoretical problem is to identify the mechanical field that is reciprocal, in the sense of Onsager's reciprocity relations, to the electric field  $E_x$ . This is the field that describes the physical process of spin injection-an effect that is more commonly treated as a source term in the Boltzmann collision integral.

It will be shown below that the sought "spin injection field" is simply a magnetic field that varies linearly in time. Such a field produces a spin density that, at each instant of time, lags slightly behind the instantaneous equilibrium spin density by an amount proportional to the spin relaxation rate. The difference between the true spin polarization and the instantaneous equilibrium polarization is the proper nonequilibrium spin density injected by the field. The efficacy of the injection is, thus, limited by the spin relaxation time  $\tau_s$ .

Figure 1 shows the qualitative picture of the IEE for the paradigmatic case of the Rashba spin-orbit coupling, i.e., a spin-orbit coupling of the form  $\alpha(\mathbf{z} \times \mathbf{p}) \cdot \boldsymbol{\sigma}$ , where  $\mathbf{z}$  is the unit vector perpendicular to the plane of the electrons,  $\alpha$  is the Rashba velocity, and **p** is the momentum of the electron. If spin injection could be selectively done at fixed momentum, it would be, then, obvious in order to produce a charge current along the x direction to inject and extract electrons at  $(-k_F, 0)$  and  $(k_F, 0)$ , respectively, where the Rashba field  $\alpha(\mathbf{z} \times \mathbf{p})$  aligns the spin in the y direction (see Fig. 1). This may happen at the surface of a topological insulator, where momentum and spin are locked [36]. However, in the Rashba model, y-polarized spin injection may occur at any momentum, irrespective of the direction of the internal field  $\alpha(\mathbf{z} \times \mathbf{p})$ . Take, for instance, the apparently least favorable case at  $(0, k_F)$  or  $(0, -k_F)$ . Under the action of the Rashba field  $\alpha(\mathbf{z} \times \mathbf{p})$ , the spin of an injected electron, initially pointing along y, acquires a z component that is positive or negative according to whether  $p_{y}$  is positive or negative. The resulting correlation between  $p_{y}$  and  $S^{z}$  is the signature of a spin current  $J_{y}^{z}$ . At this point, the regular inverse spin Hall effect takes hold [37], converting part of the spin current to a perpendicular charge current  $J_{x}$ . This shows, as we will derive later in a quantitative way, that spin density in the Rashba model is intimately related to both spin and charge currents in such a



FIG. 1 (color online). Schematic description of the Rashba model and of the generation of a spin current from injected spin in the *y* direction. The thick arrows denote the Rashba field around which the spin (thin arrow) precesses. The two insets (a) and (b) zoom in on the dynamics of the spin near points  $(0, k_F)$  and  $(0, -k_F)$  in momentum space. In the first case the spin, initially pointing in the *y* direction, acquires a positive *z* component and travels in the +*y* direction. In the second case, the spin acquires a negative *z* component and travels in the -*y* direction.

way that the final result is a direct proportionality between  $J_x$  and the incoming current of  $S^y$  spin

$$J_x = \lambda_{\rm IEE} J_s^y,\tag{1}$$

where  $\lambda_{\text{IEE}}$  connects an areal current density to a volume spin current density and, therefore, has the dimensions of a length if, as we do in this Letter, we express both charge and spin in the same units (this is achieved by multiplying the spin by  $-\hbar/e$ ). In the simplest case of the pure Rashba model we find  $\lambda_{\text{IEE}} = \alpha \tau$ , where  $\tau$  is the momentum relaxation time.

In this Letter, we provide a precise formal definition of the IEE in terms of the Kubo formula for the response of the current  $J_x$  to the  $\dot{B}_y$  field, which is the Onsager reciprocal of the electric field  $E_x$ . We also develop the drift-diffusion theory of the IEE along the lines of Refs. [27–29] in the presence of both intrinsic and extrinsic (but linear in **k**) spin-orbit coupling. In the pure Rashba limit, these equations yield Eq. (1). Finally, we make contact with the recent experimental work on the generation of a charge current by spin injection into a Ag-Bi interface [31] and identify the proper relaxation time to be used in the expression for  $\lambda_{\text{IEE}}$ as the momentum relaxation time.

*Formal definition of IEE.*—The direct Edelstein effect is defined by the proportionality

$$S^{y}(\omega) = \sigma_{\text{DEE}}(\omega)E_{x}(\omega), \qquad (2)$$

where we have allowed a periodic variation of the field and the induced density at a frequency  $\omega$ . To formalize the calculation of the Edelstein conductivity  $\sigma_{\text{DEE}}(\omega)$ , we introduce the Kubo response function of the homogeneous spin density  $S^y$  to a vector potential  $A_x$ , such that  $E_x = -\dot{A}_x$ . Since  $A_x$  couples linearly to the current density, we denote this response by  $\langle\langle \hat{S}^y; \hat{J}_x \rangle\rangle_{\omega}$ , where  $\hat{J}_x$  is the operator of the physical current (obtained by differentiating the Hamiltonian with respect to  $A_x$ ) and  $\hat{S}^y$  is the operator of the spin density. The double bracket denotes the Kubo product  $\langle\langle \hat{A}; \hat{B} \rangle\rangle_{\omega} \equiv -(i/\hbar) \int_0^t \langle[\hat{A}(t), \hat{B}(0)]e^{i\omega t} dt$ . Since the electric field is related to the vector potential by  $\mathbf{E}(\omega) = i\omega \mathbf{A}(\omega)$ , we immediately see that

$$\sigma_{\text{DEE}}(\omega) = \frac{\langle\langle \hat{S}^{y}; \hat{J}_{x} \rangle\rangle_{\omega}}{i\omega}.$$
(3)

In the dc limit, the numerator vanishes by gauge invariance because a static and uniform vector potential does not change  $S^{y}$ . Then we obtain the dc Edelstein conductivity

$$\sigma_{\text{DEE}}(0) = \lim_{\omega \to 0} \frac{\Im m \langle \langle \hat{S}^{y}; \hat{J}_{x} \rangle \rangle_{\omega}}{\omega}, \qquad (4)$$

which is a real quantity.

The inverse Edelstein effect is similarly defined by the proportionality

$$J_{x}(\omega) = \sigma_{\text{IEE}}(\omega)[g\mu_{B}\dot{B}_{y}(\omega)], \qquad (5)$$

where, as described in the introduction,  $\dot{B}^{y}$  is the field that injects a nonequilibrium spin density  $S^{y}$ . We then introduce the Kubo response function of the homogeneous current density to a magnetic field that couples linearly to the spin density, namely,  $\langle \langle \hat{J}^{x}; \hat{S}^{y} \rangle \rangle_{\omega}$ . Noting that,  $-\dot{B}^{y}(\omega) = i\omega B^{y}(\omega)$  yields

$$\sigma_{\rm IEE}(\omega) = \frac{\langle \langle \hat{J}_x; \hat{S}^y \rangle \rangle_{\omega}}{i\omega}.$$
 (6)

In the dc limit,

$$\sigma_{\text{IEE}}(0) = \lim_{\omega \to 0} \frac{\Im m \langle \langle \hat{J}_x; \hat{S}^y \rangle \rangle_{\omega}}{\omega}.$$
 (7)

The Onsager relation, namely, the equality of the conductivities  $\sigma_{\text{DEE}}$  and  $\sigma_{\text{IEE}}$ , follows immediately from wellknown properties of the Kubo product for a system governed by a time-reversal invariant Hamiltonian. This establishes the IEE as described above as the proper Onsager reciprocal of the standard Edelstein effect.

Drift-diffusion theory.—An elegant description of the direct spin Hall effect and Edelstein effect has recently been derived based on the methods of the quasiclassical Keldysh Green function technique [27–29]. We recall here the main aspects of this description for the case of the two-dimensional electron gas with Hamiltonian

$$H = \frac{p^2}{2m} + V(\mathbf{r}) + \alpha (p_y \sigma^x - p_x \sigma^y) + \lambda [\mathbf{p} \times \nabla V(\mathbf{r})] \cdot \boldsymbol{\sigma}$$
(8)

where  $V(\mathbf{r})$  is the impurity potential. The orbital motion takes place in the (x, y) plane, whereas the spin is three dimensional. The last two terms represent the intrinsic and the extrinsic spin-orbit coupling, respectively;  $\lambda$  is the extrinsic coupling constant, directly related to the square of the effective "Compton wavelength" for the conduction band;  $\alpha$  is the Rashba velocity, proportional to the electric field  $E_z$  perpendicular to the plane, and approximately given by  $\lambda E_z$ . Since the complete formal derivation of the drift-diffusion theory presented below has been provided in Ref. [27], we limit ourselves here to providing an heuristic justification. We exploit the fact that the spin-orbit coupling is linear in electron momentum **p** and can, therefore, be represented by a constant SU(2) vector potential  $p^2/2m +$  $\alpha(p_y\sigma^x - p_x\sigma^y) = (\mathbf{p} + \sum_{a=x,y,z} e\mathbf{A}^a\sigma^a/2)^2/2m$ , where the only nonzero components are  $-eA_y^x = eA_x^y = -2m\alpha$ . The non-Abelian character of the SU(2) group entails the appearance of covariant derivatives  $\nabla_i$  defined as follows:

$$(\nabla_i O)^a = \partial_i O^a - e \sum_{b,c} \epsilon_{abc} A^b_i O^c, \qquad (9)$$

where  $O^a$  is a generic vector function (in spin space) on which the derivative acts. Notice that the second term in Eq. (9) differs from zero even for a homogeneous function. An immediate consequence is the appearance of a SU(2)magnetic field different from zero even for uniform and constant vector potential. It is given by the covariant curl of the SU(2) vector potential:  $-eB_i^a = (i/2)\epsilon_{ijk}[A_j, A_k]^a$  with only nonzero component  $-eB_z^2 = -(2m\alpha)^2$ . Once one accepts the above SU(2) language, it is not too surprising that the coupled equations for charge and spin currents take the form first introduced in Ref. [27], namely,

$$\dot{S}^a = -\nabla_i J^a_i - \frac{\delta S^a}{\tau_{\rm EY}},\tag{10}$$

$$J_i^a = -D(\nabla_i \delta S)^a + \sigma_s E_i^a + \frac{e\tau}{2m} \epsilon_{ijk} J_j \mathcal{B}_k^a, \qquad (11)$$

$$J_i = -D\partial_i n + \sigma_D E_i + \frac{e\tau}{2m} \epsilon_{ijk} J^a_j \mathcal{B}^a_k, \qquad (12)$$

where  $D = v_F^2 \tau/2$  is the diffusion constant,  $v_F$  is the Fermi velocity of the 2DEG, and  $\sigma_D = \sigma_s = ne\tau/m$  is the Drude conductivity. These equations must be solved, in general, in conjunction with appropriate boundary conditions [38,39].

The first equation of the set is the continuity equation for the spin density. The nonconservation of the spin, due to the action of the Rashba field, is taken into account via the replacement of the ordinary derivative by the SU(2) covariant derivative, whereas the additional spin relaxation due to impurity scattering is taken into account via the phenomenological Elliot-Yafet relaxation time  $\tau_{\rm EY}$  [40]. The quantity  $\delta S^a = S^a - S^a_e$  is the deviation of the spin density from the instantaneous equilibrium spin density  $S^a_e = \chi_s B^a(t)$ , where  $\chi_s = -g\mu_B(m/\pi)$  is the static spin susceptibility.

The last two equations express the spin current and the charge current densities as sums of *diffusive*, *drift*, and *Hall-like* terms. Whereas in uniform circumstances the charge current does not have a diffusion contribution, the spin current, due to the SU(2) covariant derivative, does have an anomalous diffusion contribution—the first term on the right hand side of Eq. (11). This anomalous diffusion describes the spin current that arises from the precessional motion of the spins in the Rashba field. Its origin is described qualitatively in Fig. 1.

Whereas the second term on the right hand side of Eq. (11) is the well-known drift term due to the spin-electric field arising from the spin accumulation potential of classical spintronics, the first and third terms are features of the Rashba model and are responsible for the EE and the SHE, respectively. To appreciate this, we recall that under very general conditions, the coupling between charge and spin currents can be described [37] in terms of a single parameter  $\tilde{\gamma}$ , the spin-Hall angle, which must be evaluated from a microscopic model. If we now consider a definite geometry where the external electric field (which is

present in both effects) is applied along the x direction, Eqs. (10)–(12) read

$$\dot{\delta S^{y}} = -2m\alpha J_{y}^{z} - \frac{\delta S^{y}}{\tau_{\rm EY}} + \chi_{s} \dot{B}^{y}, \qquad (13)$$

$$J_{y}^{z} = 2m\alpha D\delta S^{y} + \sigma_{s} E_{y}^{z} + \tilde{\gamma} J_{x}, \qquad (14)$$

$$J_x = \sigma_D E_x - \tilde{\gamma} J_y^z. \tag{15}$$

In the first equation,  $\chi_s B^{\gamma}$  is the spin injection rate. For the pure Rashba model the spin-Hall angle is  $\tilde{\gamma} = -2m\alpha^2 \tau$  [41], which corresponds to the  $\omega_c \tau$  of the classical magnetotransport theory, where the cyclotron frequency  $\omega_c$  is replaced by  $2m\alpha^2$ . When  $\lambda \neq 0$ , the parameter  $\tilde{\gamma}$  gets additional contributions proportional to  $\lambda$ , due to the so-called side-jump and skew-scattering mechanisms. We refer to Ref. [29] for details. Clearly, the spin Hall effect is a consequence of the Hall-like term with  $\tilde{\sigma}_{\text{SHE}} = \tilde{\gamma} \sigma_D$  [42].

Solving the coupled Eqs. (13)–(15) yields expression for  $\delta S^y$ ,  $J_y^z$ , and  $J_x$ , which capture the phenomenology of the direct and inverse Edelstein effects and spin Hall effects, including the effects of extrinsic impurity scattering, which are quite nonintuitive in the case of the DEE (see Refs. [28,29]). In particular, setting  $E_x = E_y^z = 0$  yields

$$\delta S^{y} = \frac{\tau_{s} \chi_{s} B_{y}}{1 - i\omega \tau_{s}},\tag{16}$$

$$J_{y}^{z} = 2m\alpha D \frac{\tau_{s} \chi_{s} B_{y}}{1 - i\omega \tau_{s}},$$
(17)

$$J_x = -\frac{2\pi}{e} \alpha \tilde{\sigma}_{\text{SHE}} \frac{\tau_s \chi_s \tilde{B}_y}{1 - i\omega \tau_s},\tag{18}$$

where the total relaxation time is given by  $\tau_s = 1/(\tau_{\rm EY}^{-1} + \tau_{\rm DP}^{-1})$  with  $\tau_{\rm DP}^{-1} = (2m\alpha)^2 D$  the standard D'yakonov-Perel' spin relaxation time. In the low-frequency limit, the coefficient of the IEE reads

$$\sigma_{\rm IEE} = \frac{2}{e} \alpha m \tau_s \tilde{\sigma}_{\rm SHE}.$$
 (19)

In the most interesting regime in which the Rashba spin precession dominates extrinsic processes, the EY spin relaxation process is negligible and the spin-Hall conductivity is given by  $\tilde{\sigma}_{\text{SHE}} \simeq -(e/8\pi)(4\tau/\tau_{\text{DP}})$ . In this regime, which is directly relevant to the experiments of Rojas Sánchez *et al.* [31], we obtain

$$\sigma_{\rm IEE} = -\alpha m \tau / \pi. \tag{20}$$

This is the result that would have been obtained by computing the anomalous part of the current  $J_x$  in the presence of Rashba coupling, using for the expectation

value of  $\delta S^{y}$  the nonequilibrium spin polarization injected by the source  $\chi_{s} \dot{B}^{y}$ . Although derived for the diffusive regime, this result remains valid in the ballistic regime, due to the cancellation of the spin relaxation rate contained in  $\delta S_{y}$  against the one contained in the denominator of  $\tilde{\sigma}_{SHE}$ .

Discussion of experiments.-In a recent experiment, Rojas Sánchez et al. [31] observed the inverse Edelstein effect at the Ag/Bi interface. The Ag/Bi interface hosts a 2DEG of surface density  $n \simeq 6 \times 10^{13} \text{ cm}^{-2}$ , corresponding to a Fermi wave vector  $k_F \simeq 0.2$  Å<sup>-1</sup> [43,44]. These electrons reside in states bound to the interface and propagate only in the plane of the interface with an effective mass  $m^* \simeq 0.35m$  [43]. They are subjected to an unusually large Rashba spin-orbit field,  $\hbar \alpha \simeq 1$  eV Å, and they are well described by the Rashba 2DEG Hamiltonian of Eq. (8). In practice, rather than using a time-dependent magnetic field as we proposed above, Rojas Sánchez et al. inject the nonequilibrium spin polarization by a spin current generated by ferromagnetic resonance of a remote NiFe laver. It is believed [31] that the injected spin current perpendicular to the interface does not propagate but is almost entirely absorbed, leading to a nonequilibrium spatially uniform spin accumulation at the interface. Thus, the observed inplane charge current cannot be explained by the ISHE of the interfacial electron gas (the signal is demonstrated to not be due to the ISHE in the bulk Ag or Bi [31]). Because the spin accumulation is uniform, the spatial derivatives vanish and no boundary condition is needed.

Obviously, in the absence of external magnetic field, the equilibrium distribution is unpolarized, i.e.,  $\delta S^y \equiv S^y$ . Moreover, the spin pumping term in Eq. (13),  $\chi_s B_y$ , should be replaced by the injected spin current density  $J_s^y$ (polarized along the y direction). Notice that this latter spin current density is three dimensional (i.e., related to number of electrons per unit volume) in contrast to the charge current density, which is a surface density. Hence, the ratio  $J_x/J_s^y$  must have the dimensions of a length. Therefore, the induced charge current is expressed by

$$J_x = -\frac{2\pi}{e} \alpha \tau_s \tilde{\sigma}_{\text{SHE}} J_s^y \xrightarrow[\text{dominant}]{\text{Intrinsic}} \alpha \tau J_s^y.$$
(21)

The result in the intrinsic limit is similar to that suggested by the simple two-band model in the experimental paper,  $J_x = \alpha \tau_s J_s^{y}$  [31]. Whereas in Ref. [31] it is suggested that the relaxation time in this formula effectively takes into account the coupled spin-momentum dynamics, our theory provides a full microscopic derivation of it. In particular, our theory shows that the relaxation time present in the ratio between induced charge current and injected spin current should be the momentum relaxation time, even though the magnitude of the spin polarization itself is proportional to the spin relaxation time [see Eq. (16)]. This property is also demonstrated by our calculation from Kubo's formula (not shown). The underlying physics is that the generation of charge current from a spin polarization is mediated by an in-plane spin current (generated by precession in the Rashba field; see Fig. 1, which is, therefore, proportional to the SHE coefficient, introducing a factor  $\tau/\tau_{\rm DP}$  as shown in Eq. (21). With the measured value of  $\lambda_{\rm IEE} = \alpha \tau = 0.3$  nm, we estimate  $\tau = 2 \times 10^{-15}$  s and  $\tau_s = 3 \times 10^{-15}$  s, which puts us at the borderline of the spin-diffusive regime [45].

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