Sequestering the Standard Model Vacuum Energy

Nemanja Kaloper[1] and Antonio Padilla[2]
[1]Department of Physics, University of California, Davis, California 95616, USA
[2]School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom
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We propose a very simple reformulation of general relativity, which completely sequesters from gravity all of the vacuum energy from a matter sector, including all loop corrections and renders all contributions from phase transitions automatically small. The idea is to make the dimensional parameters in the matter sector functionals of the 4-volume element of the Universe. For them to be nonzero, the Universe should be finite in spacetime. If this matter is the standard model of particle physics, our mechanism prevents any of its vacuum energy, classical or quantum, from sourcing the curvature of the Universe. The mechanism is consistent with the large hierarchy between the Planck scale, electroweak scale, and curvature scale, and early Universe cosmology, including inflation. Consequences of our proposal are that the vacuum curvature of an old and large universe is not zero, but very small, that $w_{\rm DE} \simeq -1$ is a transient, and that the Universe will collapse in the future.

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The cosmological constant problem is the most severe naturalness problem in fundamental physics [1–3]. It follows from the equivalence principle of general relativity (GR) which asserts that *all* forms of energy curve spacetime. So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions. One can add a classical piece to the cosmological constant and tune it with tremendous precision to cancel the vacuum energy. However, this tuning is unstable: any change of the matter sector parameters or addition of loop corrections to vacuum energy dramatically shifts the value of vacuum energy, by $\mathcal{O}(1)$ in the units of the UV cutoff. To neutralize it one must retune the classical term by hand, order by order, in perturbation theory [4].

In this Letter we present a mechanism which provides a remedy, ensuring that *all* the vacuum energy from a matter sector is sequestered from gravity. This includes matter loop corrections (not involving virtual gravitons) which are invisible to gravity, and contributions from phase transitions, which are automatically small at late times. Our idea is to make all scales in this matter sector functionals of the 4-volume element of the Universe. For the scales to be nonzero, the Universe should be finite in spacetime, collapsing in the future. If the matter sector is the standard model of particle physics, our mechanism prevents it from generating large contributions to the net cosmological constant, and, therefore, to the curvature of the background universe. The mechanism is a very minimal modification of general relativity, without any new propagating degrees of freedom. We formulate it adding to the action auxiliary fields with an extra term, which is *not* integrated over and is completely covariant. This subtracts "historic averages" of the matter stress energy from the gravitational sources, and removes the vacuum energy contributions from the field equations. Nonetheless, there is still an effective net nonzero cosmological term, but now (i) it is purely classical, set by the complete evolution of the geometry, (ii) it is a "cosmic average" of the values of nonconstant sources, and so (iii) it is automatically small in universes which grow large and old [5]. In the limit of (semi) classical gravity there are absolutely no dynamical pathologies. All the propagating degrees of freedom obey standard second order field equations compatible with local Poincaré symmetry and diffeomophism invariance, and the spectrum of fluctuations is the same as in conventional GR with minimally coupled matter. The mechanism is consistent with phenomenological requirements, specifically with large hierarchies between the Planck scale, electroweak scale, and vacuum curvature scale, and with early Universe cosmology including inflation.

Our mechanism can be described by the action as

$$S = \int d^4x\sqrt{g}\left[\frac{M_{Pl}^2}{2}R - \Lambda - \lambda^4\mathcal{L}(\lambda^{-2}g^{\mu\nu},\Phi)\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right), \tag{1}$$

where matter couples minimally to the rescaled metric $\tilde{g}_{\mu\nu} = \lambda^2 g_{\mu\nu}$. The parameter λ sets the hierarchy between the matter scales and the Planck scale, since $m_{\rm phys}/M_{Pl} \propto \lambda m/M_{Pl}$, where $m_{\rm phys}$ is the physical mass scale and m is the bare mass in the Lagrangian. In conventional GR (or unimodular gravity) the variable Λ would be an arbitrary classical contribution to the total cosmological constant. We treat the parameters Λ and λ as dynamical variables without *any* local dynamics—i.e., just as auxiliary fields. We then vary (1) with respect to Λ and λ, in addition to other variables with local propagating modes, as in formulations of unimodular gravity [6–9]. Here, in

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contrast to old approaches, we add the function σ *outside* of the integral, to fix the matter scales as functionals of $\int d^4x\sqrt{g}$. The external function σ is an odd (to allow for solutions with vacuum energy of either sign for $\lambda > 0$) differentiable function, to be determined by phenomenology. Without gravity, it would completely drop out of the calculation of any observables, but with gravity turned on it affects the dynamics of the metric determinant $g = -\det(g_{\mu\nu})$ sector. The scale μ is also chosen phenomenologically.

From (1) one can see that all vacuum energy contributions coming from the Lagrangian $\sqrt{g}\lambda^4\mathcal{L}(\lambda^{-2}g^{\mu\nu},\Phi)$, must scale with λ as λ^4, even after the logarithmic corrections are included, provided that a regulator of the QFT is defined to also couple minimally to $\tilde{g}_{\mu\nu}$ [10]. This follows from diffeomorphism invariance of the theory, which guarantees that the full effective Lagrangian computed from $\sqrt{g}\lambda^4\mathcal{L}(\lambda^{-2}g^{\mu\nu},\Phi) = \sqrt{\tilde{g}}\mathcal{L}(\tilde{g}^{\mu\nu},\Phi)$, including all quantum corrections, still couples to the exact same $\tilde{g}_{\mu\nu}$ [11]. In this Letter we consider gravity as a purely (semi) classical theory, and focus on the quantum effects from matter alone [12]. After canonically normalizing the matter fields in $\mathcal{L}$, the matter mass scales that enter in physical observables scale as $m_{\rm phys} \propto \lambda m$, where m are "bare" parameters in $\mathcal{L}$. So the vacuum energy, including all loop contributions to $\mathcal{L}_{\rm eff}$, scales as $V_{\rm vac} = \lambda^4\langle 0|\mathcal{L}_{\rm eff}|0\rangle$.

The field equations that follow from varying the action (1) with respect to (the "constants") Λ, λ are

$$\frac{\sigma'}{\lambda^4\mu^4} = \int d^4x\sqrt{g}, \qquad 4\Lambda\frac{\sigma'}{\lambda^4\mu^4} = \int d^4x\sqrt{g}\lambda^4\tilde{T}^{\mu}{}_{\mu}, \quad (2)$$

where $\tilde{T}_{\mu\nu} = -(2/\sqrt{\tilde{g}})(\delta S_m/\delta\tilde{g}^{\mu\nu})$ is the energy-momentum tensor defined in the "Jordan frame." To rewrite it in the "physical" frame, in which the matter sector is canonically normalized, we note that $T^{\mu}{}_{\nu} = \lambda^4\tilde{T}^{\mu}{}_{\nu}$. Here $\sigma' = (d\sigma(z)/dz)$, and as long as it is nonzero [13], we can eliminate it from the two Eqs. (2) to find $\Lambda = \frac{1}{4}\langle T^{\mu}{}_{\mu}\rangle$, where we defined the 4-volume average of Q by $\langle Q\rangle = \int d^4x\sqrt{g}Q/\int d^4x\sqrt{g}$.

The variation of (1) with respect to $g_{\mu\nu}$ yields $M_{Pl}^2 G^{\mu}{}_{\nu} = -\Lambda\delta^{\mu}{}_{\nu} + \lambda^4\tilde{T}^{\mu}{}_{\nu}$, which, by eliminating Λ and canonically normalizing the matter sector, becomes

$$M_{Pl}^2 G^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle T^{\alpha}{}_{\alpha}\rangle, \quad (3)$$

where $G^{\mu}{}_{\nu}$ is the standard Einstein tensor. Eq. (3) is the key: it is the full system of ten field equations, with the trace equation included, and with the trace of the 4-volume historic average of the stress energy tensor of matter subtracted from the right-hand side. This is unlike unimodular gravity [6–9], where, although the restricted variation removes the trace equation that involves the vacuum energy, it comes back along with an arbitrary integration constant, after using the Bianchi identity. Here there are *no* hidden equations nor integration constants, all the sources are automatically accounted for in (3).

Hence, the hard cosmological constant, be it a classical contribution to $\mathcal{L}$ in (1), or quantum vacuum correction calculated to any order in the loop expansion, never contributes to the field equations (3). Indeed, if we write $\mathcal{L} = \Lambda_0 + V_{\rm vac} + \mathcal{L}_{\rm local}$, by our definition of the historic average, $\langle\Lambda_0 + V_{\rm vac}\rangle \equiv \Lambda_0 + V_{\rm vac}$. Next, defining

$$\tau_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{g}\lambda^4\mathcal{L}_{\rm local}(\lambda^{-2}g^{\mu\nu},\Phi),$$

we can write $T^{\mu}{}_{\nu} = \lambda^4(\Lambda_0 + V_{\rm vac})\delta^{\mu}{}_{\nu} + \tau^{\mu}{}_{\nu}$, and so

$$T^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle T^{\alpha}{}_{\alpha}\rangle = \tau^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle\tau^{\alpha}{}_{\alpha}\rangle$$

$\Lambda_0 + V_{\rm vac}$ completely dropped out from the source in (3). There remains a 'leftover' cosmological constant: the historic average $\langle\tau^{\mu}{}_{\mu}\rangle/4$ contributes to the curvature of the Universe, but without the classical and vacuum loop contributions. Therefore, we can write

$$M_{Pl}^2 G^{\mu}{}_{\nu} = \tau^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle\tau^{\alpha}{}_{\alpha}\rangle, \quad (4)$$

setting the sum of the classical Lagrangian and its quantum corrections to zero, and forgetting them in what follows, at least in the limit of (semi) classical gravity.

This is consistent since our action (1) has *two* approximate symmetries which ensure the cancellations of the vacuum energy and protect the curvature from both large classical and quantum corrections [2,3]. The first is the scaling $\lambda \to \Omega\lambda$, $g_{\mu\nu} \to \Omega^{-2}g_{\mu\nu}$ and $\Lambda \to \Omega^4\Lambda$, broken only by the gravitational sector. The second involves the shift of Λ and $\mathcal{L}$ in (1) by $\alpha\lambda^4$ and $-\alpha$, so the action only changes by $\delta S = \sigma((\Lambda/\lambda^4\mu^4) + (\alpha/\mu^4)) - \sigma(\Lambda/\lambda^4\mu^4) \simeq \sigma'(\alpha/\mu^4)$. The scaling ensures that the vacuum energy at arbitrary order in the loop expansion couples to the gravitational sector exactly the same way as the classical piece. The "shift symmetry" of the bulk action then cancels the matter vacuum energy and its quantum corrections [14]. The scaling is broken by the gravitational action, but the breaking is mediated to the matter by the cosmological evolution, through the scale dependence on $\int d^4x\sqrt{g}$, and so is weak. The residual cosmological constant is small: substituting the first of Eqs. (2) and using $\lambda \propto m_{\rm phys}/M_{Pl}$, we see that $\delta S \simeq \alpha\lambda^4\int d^4x\sqrt{g} \propto \alpha(m_{\rm phys}/M_{Pl})^4$, and is small when $m_{\rm phys}/M_{Pl} \ll 1$, vanishing in the conformal limit [15] $\lambda \propto m_{\rm phys} \to 0$. So, the bulk shift symmetry and the approximate scaling symmetry render a small residual curvature technically natural.

Quantum corrections from the matter sector to the Planck scale can be estimated by canonically normalizing $\mathcal{L}$ in (1), and performing one loop renormalization of the Einstein-Hilbert Lagrangian. The corrections to M_{Pl} from each species in the loop are given by [16] $\Delta M_{Pl}^2 \simeq \mathcal{O}(1)\times(M_{\rm UV}^{\rm phys})^2+\mathcal{O}(1)\times m_{\rm phys}^2\ln(M_{\rm UV}^{\rm phys}/m_{\rm phys})+\mathcal{O}(1)\times m_{\rm phys}^2+\cdots$, where $M_{\rm UV}^{\rm phys}=\lambda M_{\rm UV}$ is the matter UV regulator mass and $m_{\rm phys}$ the mass of the virtual particle in the loop. Thus, the Planck scale is radiatively stable [17] as long as $M_{\rm UV}^{\rm phys}\le M_{Pl}$, which is easily achieved in a sufficiently large and old universe. This is in contrast to the model discussed in [18], which does share some similarities with our mechanism. Indeed, imagine that instead of action (1), we started with

$$S=\int d^4x\sqrt{g}\left[\frac{\lambda^4 M_{Pl}^2}{2}R-\Lambda-\lambda^4\mathcal{L}(g^{\mu\nu},\Phi)\right]+\frac{\Lambda}{\lambda^4\mu^4},$$

where we have chosen a linear function $\sigma(z)=z$, and added a scaling with λ in the Einstein-Hilbert term, but removed it from the matter Lagrangian. We can readily integrate out Λ, λ, using $\lambda^4=(\mu^4\int d^4x\sqrt{g})^{-1}$ and $(\Lambda/\lambda^4\mu^4)=\int d^4x\sqrt{g}[(\lambda^4M_{Pl}^2/2)R-\lambda^4\mathcal{L}(g^{\mu\nu},\Phi)]$, so that

$$S_{\rm eff}=\frac{\int d^4x\sqrt{g}[\frac{M_{Pl}^2}{2}R-\mathcal{L}(g^{\mu\nu},\Phi)]}{\mu^4\int d^4x\sqrt{g}}. \quad (5)$$

Although the variation removes the tree-level part of the cosmological constant [18], the radiative corrections survive. After conformally rescaling the metric in (5) so that M_{Pl} is independent of λ, we see that the Λ term scales as $\sim 1/\lambda^4$, and the physical masses as $m_{\rm phys}\simeq m/\lambda^2$. This implies that the radiative corrections to vacuum energy scale as $\sim 1/\lambda^8$, which differs from the scaling of the tree-level part, $\Lambda\sim 1/\lambda^4$. It was also noted that the theory (5) has Planck scale radiative instabilities. This stems from $\lambda^4=(\mu^4\int d^4x\sqrt{g})^{-1}$ being small in big and old universes, which makes the matter UV regulator mass and the matter physical masses large (they scale like $\sim 1/\lambda^2$) relative to M_{Pl}, so that M_{Pl} is susceptible to the renormalization effects from them. None of this is a problem for our mechanism in (1).

Let us consider now our historic average, $\langle\tau^\alpha{}_\alpha\rangle$. In our case, the individual factors in the ratio must be finite too. First, $\int d^4x\sqrt{g}$ must be finite: (i) we require $\sigma(z)$ to be differentiable, to get field equations (4); (ii) hence, divergent $\int d^4x\sqrt{g}$ would generically force λ to vanish; (iii) but $\lambda\neq 0$ since $m_{\rm phys}\propto\lambda$ in the matter sector. Fortunately, there is a diffeomorhism invariant regulator for these integrals: spacetime singularities. A spatially compact universe of finite lifetime, starting in a bang and ending with a crunch, has finite integral $\int d^4x\sqrt{g}=\mathcal{O}(1){\rm Vol}_3/H_{\rm age}^4$, where ${\rm Vol}_3$ is the comoving spatial volume, and $H_{\rm age}^{-1}$ is the scale of the lifetime of the Universe. Furthermore, for sources which obey the standard energy conditions ($|p/\rho|\le 1$), we can estimate [19] $\int d^4x\sqrt{g}\tau^\mu{}_\mu\sim -{\rm Vol}_3\int_{t_{\rm bang}}^{t_{\rm crunch}}dta^3\rho$, in comoving coordinates. The only potentially divergent contributions come from the end points, where ρ scales as $\rho\sim 1/(t-t_{\rm end})^2$, by virtue of the Friedman equation, where $t_{\rm end}$ is either of the instants of bang or crunch. In this limit, $a^3\sim(t-t_{\rm end})^{2/(1+w)}$, and so the integrand is $a^3\rho\sim(t-t_{\rm end})^{-2w/(1+w)}$. The integral will not diverge provided $|w|\le 1$ [20]. So, for realistic matter sources, our historic averages will always be finite in a bang or crunch universe. Next, it is straightforward to show [19] that the largest contribution to $\langle\tau^\mu{}_\mu\rangle$ will come from the turnaround region, when the Universe is close to its maximal size. We then find that $\langle\tau^\mu{}_\mu\rangle\simeq\mathcal{O}(1)M_{Pl}^2H_{\rm age}^2$, where we recall that the scale of the lifetime of the Universe, $H_{\rm age}^{-1}>H_0^{-1}$, where H_0^{-1} is its current age. This would yield a naturally small cosmological constant in our Universe (with the sign controlled by the pressure of the dominant contribution) if it begins to collapse in, say, 100 billion years or so. This might happen if the current acceleration were a transient, with the net potential turning negative some time in the future, and/or our Universe were spatially closed, with a small but nonzero positive spatial curvature. For example, the current LHC data suggest that the Higgs potential may indeed have an unstable phase, with the Higgs vacuum expectation value close to the precipice [21]. Curiously, a warning about this has been raised in the prescient paper by Wilczek quite a while ago [2].

What about the contributions to the cosmological constant from phase transitions in the early Universe [22–24]? In our setup they do *not* drop out from (3,4), but they become *automatically* small at times after the transition in a large and old universe. To see it, we model them with a step function potential

$$V=V_{\rm before}[1-\Theta(t-t_*)]+V_{\rm after}\Theta(t-t_*),$$

where $\Theta(t-t_*)$ is the step function, and t_* the transition time. Substituting into (4), *after* the transition we find

$$\begin{aligned}\tau^\mu{}_\nu-\frac{1}{4}\delta^\mu{}_\nu\langle\tau^\alpha{}_\alpha\rangle &\simeq \delta^\mu{}_\nu\frac{\int d^4x\sqrt{g}(V-V_{\rm after})}{\int d^4x\sqrt{g}}\\ &\simeq \delta^\mu{}_\nu\left(\frac{\Delta V}{M_{Pl}^2H_*^2}\right)M_{Pl}^2H_0^2\left(\frac{H_{\rm age}}{H_0}\right)^2\\ &\quad\times\left(\frac{H_{\rm age}}{H_*}\right)^{(1-w/1+w)},\end{aligned} \quad (6)$$

where $\Delta V=V_{\rm before}-V_{\rm after}$, and H_* is the curvature scale during the transition, of the order of $\sqrt{V_{\rm before}}/M_{Pl}\gtrsim\sqrt{\Delta V}/M_{Pl}$. For simplicity, we took the matter from the transition to turnaround to be a single component fluid with a fixed w; a more precise estimate would merely give corrections of order one, provided we restrict attention to physically reasonable matter sources with $|p/\rho|\le 1$ [19]. In any event, as long as $H_*\gg H_{\rm age}$, which is true for the standard model, the vacuum energy contributions from early phase transitions are far smaller than the current critical density $M_{Pl}^2 H_0^2$.

How could a universe become so big in our framework? The simplest mechanism to explain it is inflation. To incorporate it in the theory, we can add an extra sector to (1) which contains an inflaton, outside of the protected sector $\mathcal{L}$. A slightly nontrivial issue is that once inflation ends, the universe needs to reheat by particle production in the protected sector, so the inflaton must couple to the fields given in $\mathcal{L}$. A model which realizes this without spoiling the sequestration of vacuum energy from $\mathcal{L}$ is the original inflation of Starobinsky [25], which is actually the model preferred by the current data anyway [26]. So we just add a term $\int d^4x\sqrt{g}\beta R^2$ to the action (1) where $\beta \sim \mathcal{O}(10^6)$ is a dimensionless parameter. This is radiatively stable under the protected sector loops due to β being so large [27]. In line with our philosophy here, we will treat this term as a semiclassical term in the theory, still ignoring any loops with virtual gravitons. In the axial gauge, extracting the Starobinsky scalaron χ by the field redefinition [28],

$$\bar{g}_{\mu\nu} = \left(1 + \frac{4\beta}{M_{Pl}^2} R\right) g_{\mu\nu}, \qquad \chi = \sqrt{\frac{3}{2}} M_{Pl} \ln\left(1 + \frac{4\beta}{M_{Pl}^2} R\right),$$

we treat χ as a (semi) classical field too, omitting any processes where it appears in loops. The scalaron has the potential $V_\chi = (M_{Pl}^4/16\beta)[1 - \exp(-\sqrt{\frac{2}{3}}\chi/M_{Pl})]^2$ and the matter couples to both $\bar{g}_{\mu\nu}$ and to χ, via

$$S = \int d^4x\sqrt{\bar{g}}\left[\frac{M_{Pl}^2}{2}\bar{R} - \frac{1}{2}(\bar{\partial}\chi)^2 - V_\chi - \Lambda e^{-2\sqrt{(2/3)}(\chi/M_{Pl})} - \lambda^4 e^{-2\sqrt{(2/3)}(\chi/M_{Pl})}\mathcal{L}\left(\lambda^{-2} e^{\sqrt{(2/3)}(\chi/M_{Pl})}\bar{g}^{\mu\nu}, \Phi\right)\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right). \tag{7}$$

The dynamics of inflation and reheating is almost the same as in the Starobinsky model. The only difference is that now $(\sigma'/\lambda^4\mu^4) = \int d^4x\sqrt{g}e^{-2\sqrt{(2/3)}(\chi/M_{Pl})}$, which involves χ, only shifts the numerical value of λ by very little. This is because $\chi \neq 0$ only during inflation, while the dominant contribution comes from the full history of the Universe.

As we noted above, the parameter λ controls the physical scales in $\mathcal{L}$, setting $m_{\rm phys} = \lambda m$. It cannot protect the hierarchy between $m_{\rm phys}$ and M_{Pl}, and the hierarchies between different physical masses in $\mathcal{L}$. But it can help set it, coexisting with models which address particle hierarchies and help them solve the vacuum energy problem. As an example, the regulator of the protected sector in $\mathcal{L}$ can be as high as $M_{\rm UV}^{\rm phys} \sim M_{Pl}$. This requires $\lambda \sim \mathcal{O}(1)$, and may imply a vacuum energy as high as $\Lambda \sim M_{Pl}^4$, which is nevertheless sequestered from gravity by our mechanism. If we take the Universe to have a lifetime $\mathcal{O}(10)H_0^{-1}$, the first of Eqs. (2) implies $\sigma' \sim (10\mu/H_0)^4$. Taking $\mu \sim 0.1 M_{Pl}$ and $\sigma \simeq e^{(\Lambda/\lambda^4\mu^4)}$ we can account for such a large vacuum energy [29]. The standard model may be embedded in $\mathcal{L}$ via some of its BSM extensions, such as, e.g., some variant of supersymmetry, in which case we could get a much lower value of Λ, given by the fourth power of the SUSY breaking scale. If this is as low as TeV, then we require $\mu \sim$ TeV. Either way, μ can be chosen to fit whatever mechanism protects the hierarchy within $\mathcal{L}$ so that our proposal can then be utilized to sequester the vacuum energy contributions to $\mathcal{L}$ which the BSM extension cannot remove.

Why can our mechanism sequester the vacuum energy of the protected sector, both classical and quantum? The problem, as explained in the context of the Weinberg's no-go theorem [3], is with the trace equation, which involves Λ as an *a priori* arbitrary source. Because g is a pure gauge mode, its variational equation does not provide any intrinsic boundary conditions—all are equally good, by symmetry. The integral $\int d^4x\sqrt{g}$ is gauge (i.e., diffeomorphism) invariant, but it is not independent: its variation is a linear combination of the variation of $g_{\mu\nu}$ at all spacetime points. So since $\int d^4x\sqrt{g}$ multiplies the vacuum energy, its variation yields a source $\propto \Lambda$ as opposed to constraining it to vanish or to be small. Our mechanism dramatically changes the role of $\int d^4x\sqrt{g}$. Since it is a true scalar, we make all the physical scales in the protected matter sector depend on it, which automatically forces the vacuum energy to drop out. As an example, if $\sigma(z) = z$ in (1) and $\mathcal{L}$ is literally the standard model, we can integrate Λ and λ out by using (2) and rewrite (1) as just Einstein-Hilbert action coupled to the Standard Model, with the only modification being that the Higgs vacuum expectation value v is replaced by $v/(\mu^4 \int d^4x\sqrt{g})^{1/4}$. Further, in (asymptotically) flat space, the integral $\int d^4x\sqrt{g}$ is infinite, which would send the physical matter scales to zero, yielding the same outcome as in GR, as sanctioned by Weinberg's no-go theorem. In a collapsing spacetime, however, $\int d^4x\sqrt{g}$ is finite, gapping the particle spectrum from zero, mediating cosmologically the scaling symmetry breaking in the gravitational sector (giving a residual cosmological constant $\langle\tau^\mu{}_\mu\rangle/4$, which is, however, completely independent of the cutoff and naturally small in a large old universe by virtue of the two approximate symmetries). This scale dependence on $\int d^4x\sqrt{g}$ is completely invisible to any nongravitational local experiment, by diffeomorphism invariance. Since no new propagating modes appear, locally the theory looks just like standard GR, in the (semi) classical limit but without a large cosmological constant.

Cosmic eschatology changes, however, since consistency requires that a universe should have a compact spacetime, whose signatures could be sought for in cosmology, both in the frozen sky and in its evolution.

The mechanism also predicts that there should be a residual cosmological constant, which is automatically small in an old and big universe, and it would be interesting to search for the right ingredients that could make it fit the current data. Since the universe eventually collapses, the residual cosmological constant cannot dominate forever, and so $w_{\mathrm{DE}} \simeq -1$ as determined from the data is a (possibly long lived) transient state.

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