Universal Fluctuation-Driven Eccentricities in Proton-Proton, Proton-Nucleus, and Nucleus-Nucleus Collisions

Li Yan* and Jean-Yves Ollitrault†

CNRS, URA2306, IPhT, Institut de Physique Théorique de Saclay, F-91191 Gif-sur-Yvette, France

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We show that the statistics of fluctuation-driven initial-state anisotropies in proton-proton, proton nucleus and nucleus-nucleus collisions is to a large extent universal. We propose a simple parametrization for the probability distribution of the Fourier coefficient ε_n in harmonic n, which is in good agreement with Monte Carlo simulations. Our results provide a simple explanation for the 4-particle cumulant of triangular flow measured in Pb-Pb collisions and for the 4-particle cumulant of elliptic flow recently measured in p-Pb collisions. Both arise as natural consequences of the condition that initial anisotropies are bounded by unity. We argue that the initial rms anisotropy in harmonic n can be directly extracted from the measured ratio $v_n\{4\}/v_n\{2\}$: this gives direct access to a property of the initial density profile from experimental data. We also make quantitative predictions for the small lifting of degeneracy between $v_n\{4\}$, $v_n\{6\}$, and $v_n\{8\}$. If confirmed by future experiments, they will support the picture that long-range correlations observed in p-Pb collisions at the LHC originate from collective flow proportional to the initial anisotropy.

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Introduction.—A breakthrough in our understanding of high-energy nuclear collisions is the recognition [1,2] that quantum fluctuations in the wave functions of projectile and target, followed by hydrodynamic expansion, result in unique long-range azimuthal correlations between outgoing particles. The importance of these fluctuations was pointed out in the context of detailed analyses of elliptic flow in nucleus-nucleus collisions [1,3]. It was later realized that fluctuations produce triangular flow [2], which has subsequently been measured in nucleus-nucleus collisions at RHIC [4,5] and LHC [6–8]. Recently, fluctuations were predicted to generate significant anisotropic flow in proton-nucleus collisions [9], which quantitatively explains [10] the long-range correlations observed by LHC experiments [11–13].

Recently, the ATLAS and CMS experiments reported the observation of a nonzero 4-particle cumulant of azimuthal correlations, dubbed $v_2\{4\}$, in proton-nucleus collisions [14,15]. The occurrence of a large $v_2\{4\}$ in proton-nucleus collisions is not fully understood, even though it is borne out by hydrodynamic calculations with fluctuating initial conditions [16]. Such higher-order cumulants were originally introduced [17,18] to measure elliptic flow in the reaction plane of noncentral nucleus-nucleus collisions and isolate it from other, "nonflow" correlations. It turns out that the simplest fluctuations one can think of, namely, Gaussian fluctuations, do not contribute to $v_2\{4\}$ [19]. Since flow in proton-nucleus collisions is thought to originate from fluctuations in the initial geometry, one naively expects $v_2\{4\} \sim 0$, *even if* there is collective flow in the system.

In this Letter, we argue that the values observed for $v_2\{4\}$ in p-Pb collisions are naturally explained by non-Gaussian fluctuations, which are expected for small systems. Our explanation differs from that recently put forward by Bzdak *et al.* [20] that it is due to symmetry breaking [see Eq. (3) and discussion below]. As do Bzdak *et al.*, we assume that anisotropic flow v_n scales like the corresponding initial-state anisotropy ε_n on an event-by-event basis. This is known to be a very good approximation in ideal [21] and viscous [22] hydrodynamics. Thus, flow fluctuations directly reflect ε_n fluctuations. Now, ε_n is bounded by unity by definition. On the other hand, Gaussian fluctuations are not bounded, which is the reason why they fail to model small systems. We propose a simple alternative to the Gaussian parametrization that naturally satisfies the constraint $\varepsilon_n < 1$. We show that it provides an excellent fit to all Monte Carlo calculations.

Distribution of the initial anisotropy.—In each event, the anisotropy in harmonic n is defined (for $n = 2$, 3) by [23]

$$\varepsilon_{n,x} \equiv -\frac{\int r^n \cos(n\phi)\rho(r,\phi) r dr d\phi}{\int r^n \rho(r,\phi) r dr d\phi},$$
$$\varepsilon_{n,y} \equiv -\frac{\int r^n \sin(n\phi)\rho(r,\phi) r dr d\phi}{\int r^n \rho(r,\phi) r dr d\phi}, \quad (1)$$

where $\rho(r,\varphi)$ is the initial transverse density profile near midrapidity in a centered polar coordinate system.

Figure 1 displays the histogram of the distribution of ε_2 in a p-Pb collision at 5.02 TeV obtained in a Monte Carlo Glauber calculation [24]. We use the PHOBOS implementation [25] with a Gaussian wounding profile [26,27]. We assume that the initial density $\rho(r,\varphi)$ is a sum of Gaussians of width $\sigma_0 = 0.4$ fm, centered around each participant nucleon with a normalization that fluctuates [28]. These

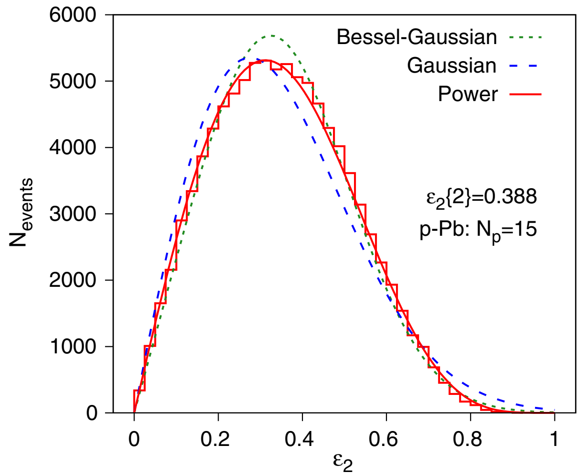

FIG. 1 (color online). Histogram of the distribution of ε_2 obtained in a Monte Carlo Glauber simulation of a p-Pb collision at LHC and fits using Eqs. (2)–(4).

fluctuations, which increase anisotropies [29], are modeled as in Ref. [20]. We have selected events with number of participants $14 \leq N \leq 16$, corresponding to typical values in a central p-Pb collision.

We now compare different parametrizations of this distribution, which we use to fit our numerical results. The first is an isotropic two-dimensional Gaussian (we drop the subscript n for simplicity)

$$P(\varepsilon) = \frac{2\varepsilon}{\sigma^2}\exp\left(-\frac{\varepsilon^2}{\sigma^2}\right), \tag{2}$$

where $\varepsilon \equiv \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$ and the distribution is normalized: $\int_0^\infty P(\varepsilon)d\varepsilon = 1$. This form is motivated by the central limit theorem, assuming that the eccentricity solely originates from event-by-event fluctuations and neglecting fluctuations in the denominator. Note that this distribution does not strictly satisfy the constraint $\varepsilon < 1$, which follows from the definition (1). When fitting our Monte Carlo results, we have therefore multiplied Eq. (2) by a constant to ensure normalization between 0 and 1. The rms ε has been fitted to that of the Monte Carlo simulation. Figure 1 shows that Eq. (2) gives a reasonable approximation to our Monte Carlo results, but not a good fit.

Bzdak *et al.* [20] have proposed to replace Eq. (2) by a "Bessel-Gaussian" (BG)

$$P(\varepsilon) = \frac{2\varepsilon}{\sigma^2} I_0\left(\frac{2\varepsilon\bar{\varepsilon}}{\sigma^2}\right)\exp\left(-\frac{\varepsilon^2+\bar{\varepsilon}^2}{\sigma^2}\right). \tag{3}$$

This parametrization introduces an additional free parameter $\bar{\varepsilon}$, corresponding to the mean eccentricity in the reaction plane in nucleus-nucleus collisions [19]. It reduces to Eq. (2) if $\bar{\varepsilon} = 0$. A nonzero value of $\bar{\varepsilon}$ is, however, difficult to justify for a symmetric system in which anisotropies are solely created by fluctuations. In Fig. 1, $\bar{\varepsilon}$ and σ have been chosen so that the first even moments $\langle\varepsilon^2\rangle$ and $\langle\varepsilon^4\rangle$ match exactly the Monte Carlo results, as suggested in Ref. [20]. The quality of the fit is not much improved compared to the Gaussian distribution, even though there is an additional free parameter. Note that the Bessel-Gaussian, like the Gaussian, does not take into account the constraint $\varepsilon < 1$.

We now introduce the one-parameter power-law distribution

$$P(\varepsilon) = 2\alpha\varepsilon(1-\varepsilon^2)^{\alpha-1}, \tag{4}$$

where $\alpha > 0$. Equation (4) reduces to Eq. (2) for $\alpha \gg 1$, with $\sigma^2 \equiv 1/\alpha$. The main advantage of Eq. (4) over previous parametrizations is that the support of $P(\varepsilon)$ is the unit disc: it satisfies for all $\alpha > 0$ the normalization $\int_0^1 P(\varepsilon)d\varepsilon = 1$. In the limit $\alpha \to 0^+$, $P(\varepsilon) \simeq \delta(\varepsilon - 1)$.

Equation (4) is the *exact* [30] distribution of ε_2 for N identical pointlike sources with a 2-dimensional isotropic Gaussian distribution, with $\alpha = (N-1)/2$, if one ignores the recentering correction [see Eq. (3.10) of Ref. [30]; what is derived there is the distribution of anisotropy in momentum space, but the algebra is identical for the distribution of eccentricity]. In a more realistic situation, Eq. (4) is no longer exact. We adjust α to match the rms ε from the Monte Carlo calculation. Figure 1 shows that Eq. (4) (with $\alpha \simeq 5.64$) agrees much better with Monte Carlo results than Gaussian and Bessel-Gaussian distributions.

Cumulants.—Cumulants of the distribution of ε are derived from a generating function, which is the logarithm of the two-dimensional Fourier transform of the distribution of $(\varepsilon_x, \varepsilon_y)$

$$G(k_x, k_y) \equiv \ln\langle\exp(ik_x\varepsilon_x + ik_y\varepsilon_y)\rangle, \tag{5}$$

where angular brackets denote an expectation value over the ensemble of events. If the system has azimuthal symmetry, by integrating over the relative azimuthal angle of $\mathbf{k}$ and $\boldsymbol{\varepsilon}$, one obtains

$$G(k) = \ln\langle J_0(k\varepsilon)\rangle, \tag{6}$$

where $k \equiv \sqrt{k_x^2 + k_y^2}$ and $\varepsilon \equiv \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$. The cumulant to a given order n, $\varepsilon\{n\}$, is obtained by expanding Eq. (6) to order k^n and identifying with the expansion of $\ln J_0(k\varepsilon\{n\})$ to the same order. This uniquely defines $\varepsilon\{n\}$ for all *even* n. One, thus, obtains [3] $\varepsilon\{2\}^2 = \langle\varepsilon^2\rangle$, $\varepsilon\{4\}^4 = 2\langle\varepsilon^2\rangle^2 - \langle\varepsilon^4\rangle$. Expressions of $\varepsilon\{6\}$ and $\varepsilon\{8\}$ are given in Ref. [20].

Expressions of the first four cumulants are listed in Table I. For the power-law distribution (4), these results are obtained by expanding the generating function (6) whose expression is

TABLE I. Values of the first eccentricity cumulants for the Gaussian (2), Bessel-Gaussian (3), and power-law (4) distributions.

	Gaussian	Bessel-Gaussian	Power law
$\varepsilon\{2\}$	σ	$\sqrt{\sigma^2+\bar{\varepsilon}^2}$	$1/\sqrt{1+\alpha}$
$\varepsilon\{4\}$	0	$\bar{\varepsilon}$	$[2/(1+\alpha)^2(2+\alpha)]^{1/4}$
$\varepsilon\{6\}$	0	$\bar{\varepsilon}$	$[6/(1+\alpha)^3(2+\alpha)(3+\alpha)]^{1/6}$
$\varepsilon\{8\}$	0	$\bar{\varepsilon}$	$[48(1+(5\alpha/11))/(1+\alpha)^4(2+\alpha)^2(3+\alpha)(4+\alpha)]^{1/8}$

$$G(k) = \ln\left[\int_0^1 J_0(k\varepsilon)P(\varepsilon)d\varepsilon\right] = \ln\left[\frac{2^\alpha \alpha!}{k^\alpha} J_\alpha(k)\right]. \quad (7)$$

General results have been obtained previously in the case of N pointlike sources and in the large N limit for $\varepsilon_2\{2\}$ [31] and $\varepsilon_2\{4\}$ [32]. Our results derived from Eq. (4) are exact for a Gaussian distribution of sources and, therefore, agree with these general results for $N \gg 1$. Similar results have also been derived for $\varepsilon_3\{2\}$ and $\varepsilon_3\{4\}$ [33] but not for cumulants of order 6 or higher.

Figure 2 displays the cumulants $\varepsilon\{2\}$ to $\varepsilon\{8\}$ as a function of N, as predicted by Eq. (4) for pointlike sources (here we assume that the recentering correction effectively reduces by one unit the number of independent sources; we thus replace N by $N-1$ in the exact result of Ref. [30]). These results are similar to those obtained in full Monte Carlo Glauber calculations [20]. In the limit $N \gg 1$, the power-law distribution yields $\varepsilon\{k\} \propto N^{(1-k)/k}$. It, thus, predicts a strong ordering $\varepsilon\{8\} \ll \varepsilon\{6\} \ll \varepsilon\{4\} \ll \varepsilon\{2\} \ll 1$, unlike the Bessel-Gaussian that predicts $\varepsilon\{4\} = \varepsilon\{6\} = \varepsilon\{8\}$. For fixed N, however, the cumulant expansion quickly converges, as illustrated in Fig. 2. In practice, for typical values of N in p-Pb collisions, one observes $\varepsilon\{4\} \simeq \varepsilon\{6\} \simeq \varepsilon\{8\}$, in agreement with numerical findings of Bzdak *et al.* [20]. This rapid convergence can be traced back to the fact that the generating function $G(k)$ in Eq. (7) has a singularity at the first zero of $J_\alpha(k)$, denoted by $j_{\alpha 1}$. This causes the cumulant expansion to quickly converge to the value [34]

$$\varepsilon\{\infty\} = \frac{j_{01}}{j_{\alpha 1}}. \quad (8)$$

This asymptotic limit is also plotted in Fig. 2. It is hardly distinguishable from $\varepsilon\{6\}$ and $\varepsilon\{8\}$ for these values of N.

Testing universality.—The power-law distribution (4) predicts the following parameter-free relation between the first two cumulants:

$$\varepsilon\{4\} = \varepsilon\{2\}^{3/2}\left(\frac{2}{1+\varepsilon\{2\}^2}\right)^{1/4}. \quad (9)$$

This relation can be used to test the universality of the distribution (4). For p-Pb collisions at 5.02 TeV, we run two different types of Monte Carlo Glauber calculations: a full Monte Carlo calculation identical to that of Fig. 1 and a second one where fluctuations and smearing are switched off (identical pointlike sources). We calculate ε_2 and ε_3 for each event. Events are then binned according to the number of participants N, mimicking a centrality selection. For p-p collisions at 7 TeV, we use published results [35] obtained with the event generator DIPSY [36], which are binned according to multiplicity. Results are shown in Fig. 3 (left).

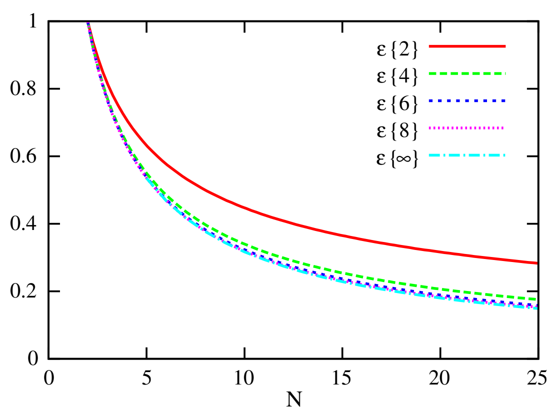

FIG. 2 (color online). Cumulants of the eccentricity distribution as a function of the number of participants N for the power-law distribution of Eq. (4), where we have set $\alpha = (N-2)/2$.

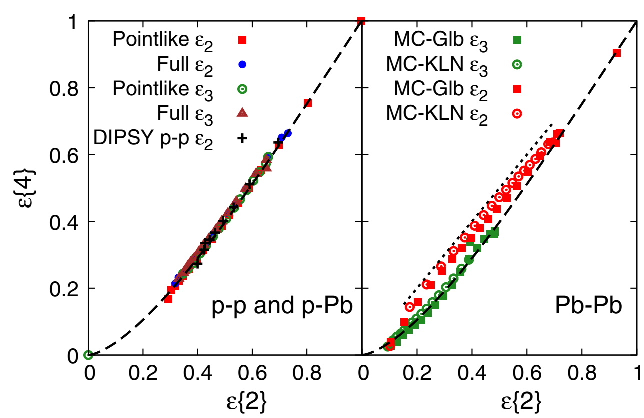

FIG. 3 (color online). $\varepsilon\{4\}$ versus $\varepsilon\{2\}$. The dashed line in both panels is Eq. (9). Left: p-Pb collisions. "Full" refers to Gaussian sources associated with each participant and fluctuations in the weights of each source. "Pointlike" refers to pointlike identical sources. DIPSY results for p-p collisions are replotted from Ref. [35]. Right: Pb-Pb collisions. The dotted line is $\varepsilon\{4\} = \varepsilon\{2\}$, corresponding to a nonzero mean eccentricity and negligible fluctuations.

Each symbol of a given type corresponds to a different bin. All Monte Carlo results are in very good agreement with those of Eq. (9). A closer look at the results shows that the full Monte Carlo Glauber calculations are above the line by ∼0.015 (for both ε_2 and ε_3), the pointlike results for ε_3 by ∼0.005, and the pointlike results for ε_2 (where our result is exact, up to the recentering correction) by ∼0.002. DIPSY results are above the line by ∼0.01.

For Pb-Pb collisions at 2.76 TeV (Fig. 3 right), we use the results obtained in Ref. [37] using the Monte Carlo Glauber [25] and Monte Carlo KLN [38] models. These results are in 5% centrality bins. For ε_3, both models are in very good agreement with Eq. (9) (within 0.01 or so). Note that Pb-Pb collisions probe this relation closer to the origin, in the large N limit where more general results are available [33]. These general results predict $\varepsilon\{4\} \propto \varepsilon\{2\}^{3/2}$ for $N \to \infty$, but with a proportionality constant that depends on the density profile. Our results show that it is in practice very close to the value predicted by Eq. (9) , namely, $2^{1/4}$.

Monte Carlo results for ε_2 in Pb-Pb differ from Eq. (9). This is expected, since ε_2 in midcentral Pb-Pb collisions is mostly driven by the almond shape of the overlap area between colliding nuclei [30], not by fluctuations. In the limiting case where fluctuations are negligible, $\varepsilon_2\{4\} = \varepsilon_2\{2\}$. Our results show that fluctuations dominate only for the most central and most peripheral bins.

We conclude that the power-law distribution of Eq. (4) is a very good approximation to the distribution of fluctuation-driven eccentricities, irrespective of the details of the model. This could be checked explicitly with other initial-state models [29,39].

Applications.—We now discuss applications of our result. The distribution of ε_n is completely determined by the parameter α in Eq. (4). This parameter can be obtained directly from experimental data. Assuming that anisotropic flow is proportional to eccentricity in the corresponding harmonic $v_n \propto \varepsilon_n$, which is proven to be a very good approximation for $n = 2, 3$ [22], one obtains

$$\frac{v\{4\}}{v\{2\}} = \frac{\varepsilon\{4\}}{\varepsilon\{2\}} = \left(\frac{2}{2+\alpha}\right)^{1/4}. \tag{10}$$

The first equality has already been checked against Monte Carlo models and experimental data [40,41]. The second equality directly relates the parameter α in Eq. (4) to the measured ratio $v\{4\}/v\{2\}$.

This in turn gives a prediction for ratios of higher-order flow cumulants, which scale like the corresponding ratios of eccentricity cumulants. These predictions are displayed in Fig. 4. One can also directly obtain the rms eccentricity $\varepsilon\{2\}$, which is a property of the initial state.

The ratio $v_3\{4\}/v_3\{2\}$ in Pb-Pb is close to 0.5 in midcentral collisions [6,41]. We thus predict $v_3\{6\}/v_3\{4\} \simeq 0.84$ and $v_3\{8\}/v_3\{6\} \simeq 0.94$ in the same centrality. We also obtain $\varepsilon_3\{2\} \simeq 0.17$, which is a typical prediction

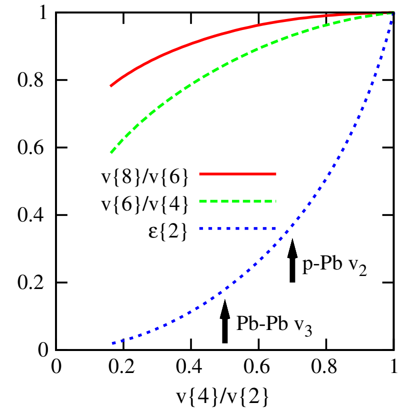

FIG. 4 (color online). Predictions of the model for ratios of higher-order cumulants and $\varepsilon\{2\}$ as a function of the measured $v\{4\}/v\{2\}$ value. Typical values for v_3 in Pb-Pb [6,41] and v_2 in p-Pb collisions [15] are indicated by arrows.

from Monte Carlo models in the 10%–20% or 20%–30% centrality range [42].

Similarly, the ratio $v_2\{4\}/v_2\{2\} \sim 0.7$ measured in p-Pb collisions [14,15] implies $v_2\{6\}/v_2\{4\} \simeq 0.93$ and $v_2\{8\}/v_2\{6\} \simeq 0.98$, that is, almost degenerate higher-order cumulants. We obtain $\varepsilon_2\{2\} \simeq 0.37$, in agreement with Monte Carlo Glauber models [20].

Conclusions.—We have proposed a new parametrization of the distribution of the initial anisotropy ε_n in proton-proton, proton nucleus and nucleus-nucleus, which unlike previous parametrizations takes into account the condition $\varepsilon_n < 1$. This new parametrization is found in good agreement with results of Monte Carlo simulations when ε_n is created by fluctuations of the initial geometry. Our results explain the observation, in these Monte Carlo models, that cumulants of the distribution of ε_n quickly converge as the order increases. This is because the Fourier transform of the distribution of ε_n has a zero at a finite value of the conjugate variable k. This, in turn, is a consequence of the fact that the probability distribution of ε_n has compact support (that is, $\varepsilon_n < 1$).

The consequence of this universality is that while the rms ε_n is strongly model dependent [42], the probability distribution of ε_n is fully determined once the rms value is known—in particular, the magnitudes of higher-order cumulants such as $\varepsilon_n\{4\}$. Assuming that anisotropic flow v_n is proportional to ε_n in every event, we have predicted the values of $v_3\{6\}$ and $v_3\{8\}$ in Pb-Pb collisions and the values of $v_2\{6\}$ and $v_2\{8\}$ in p-Pb collisions.

If future experimental data confirm our prediction, these results will strongly support the picture that the long-range correlations observed in proton-nucleus and nucleus-nucleus collisions are due to anisotropic flow, which is itself proportional to the anisotropy in the initial state. This picture, furthermore, will be confirmed irrespective of the details of the initial-state model.

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*li.yan@cea.fr
†jean-yves.ollitrault@cea.fr

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