Integrable Deformation of the $AdS_5 \times S^5$ Superstring Action

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(Received 7 October 2013; published 5 February 2014)

An integrable deformation of the type IIB $AdS_5 \times S^5$ superstring action is presented. The deformed field equations, Lax connection, and κ-symmetry transformations are given. The original $\mathfrak{psu}(2,2|4)$ symmetry is expected to become q deformed.

DOI: 10.1103/PhysRevLett.112.051601 PACS numbers: 11.25.Tq, 11.30.Ly

*Introduction.—*Integrability plays a central role in the study of the AdS/CFT correspondence [1] between type IIB superstring theory on the $AdS_5 \times S^5$ background [2] and the maximally supersymmetric Yang-Mills gauge theory in four dimensions (see Ref. [3] for a review). On the anti–de Sitter side of this correspondence, integrability entered the scene with the discovery that the Lagrangian field equations of the $AdS_5 \times S^5$ theory can be recast in the zero curvature form [4]. This implies the existence of an infinite number of conserved quantities.

It is quite natural to seek deformations of the $AdS_5 \times S^5$ superstring that preserve this integrable structure. An important example of such an integrable deformation is the so called β deformation associated with strings on the Lunin-Maldacena background [5]. The integrability of this model was shown in Refs. [6,7] (see also the review of Ref. [8] and references therein). Here, we shall take a more systematic approach to the construction of integrable deformations by demanding the deformed theory to be integrable from the very outset. This requires approaching the problem from the Hamiltonian perspective.

Let us recall that in order to prove integrability in the Hamiltonian formalism, one must show the existence of an infinite number of conserved quantities in involution. More precisely, this follows at once if the Poisson bracket of the Hamiltonian Lax matrix can be shown to take the specific form in Refs. [9,10]. This was achieved in the case of the $AdS_5 \times S^5$ superstring in Ref. [11].

The algebraic structure underpinning this property of the $AdS_5 \times S^5$ superstring was identified in Ref. [12]. By utilization of this structure, an alternative Poisson bracket with the same property was subsequently constructed in Ref. [13]. Moreover, this second Poisson bracket is compatible with the original one, giving rise to a one-parameter family of Poisson brackets sharing the same property [9,10], which ensures integrability.

These features of the superstring theory are in fact shared with bosonic integrable σ models [14]. In this latter context, the two compatible Poisson brackets were used very recently in Ref. [15] as a building block for constructing integrable q deformations of the principal chiral model associated with a compact Lie group and of the σ model on a symmetric space F/G with F compact. In the case of the principal chiral model, the deformation coincides with the Yang-Baxter σ model introduced by Klimčík in Ref. [16]. A key characteristic of this procedure is that the integrability of the deformed theories is automatic since it is used as an input in the construction. Moreover, an interesting output is that the symmetry associated with left multiplication in the original models is deformed into a classical q-deformed Poisson-Hopf algebra.

It is possible to generalize the method developed in Ref. [15] to deform the $AdS_5 \times S^5$ superstring theory. The whole construction is carried out at the Hamiltonian level and will be presented in detail elsewhere. The purpose of this Letter is to present the deformed model and exhibit its key properties, namely, the existence of a Lax connection and the invariance of the action under κ symmetry. Both of these properties are shared with the Metsaev-Tseytlin action [2]. The κ-symmetry invariance is an important property in the context of type IIB supergravity. We will comment on this last point in the Conclusion.

*Setting.—*We begin by recalling the necessary ingredients for defining the $AdS_5 \times S^5$ superstring action (see Ref. [17] for more details). Let $\mathfrak{f}$ denote the Grassmann envelope of the superalgebra $\mathfrak{su}(2,2|4)$, namely, the Lie algebra

$$\mathfrak{f} = \mathcal{G}r^{[0]} \otimes \mathfrak{su}(2,2|4)^{[0]} \oplus \mathcal{G}r^{[1]} \otimes \mathfrak{su}(2,2|4)^{[1]},$$

where $\mathcal{G}r$ is a real Grassmann algebra. Introduce the two-dimensional field $g(\sigma,\tau)$ taking value in the Lie group F with Lie algebra $\mathfrak{f}$. The corresponding vector current $A_\alpha = g^{-1}\partial_\alpha g$ belongs to $\mathfrak{f}$. The integrability of the $AdS_5 \times S^5$ superstring action relies heavily on the existence of an order 4 automorphism that induces a $\mathbb{Z}_4$ grading of the superalgebra $\mathfrak{su}(2,2|4)$ and, thus, of $\mathfrak{f}$. We denote by $\mathfrak{f}^{(i)}$ the subspace of $\mathfrak{f}$ with grade $i = 0, \ldots, 3$. The projector on $\mathfrak{f}^{(i)}$ shall be denoted by P_i, and we also write $M^{(i)} = P_i M$ for the projection of $M \in \mathfrak{f}$ on $\mathfrak{f}^{(i)}$. The invariant part $\mathfrak{f}^{(0)}$ is the Lie algebra $\mathfrak{so}(4,1) \oplus \mathfrak{so}(5)$, and the corresponding Lie group is $G = SO(4,1) \times SO(5)$. The supertrace is compatible with the $\mathbb{Z}_4$ grading, which means that $\mathrm{Str}(M^{(m)}N^{(n)}) = 0$ for $m + n \neq 0 \mod 4$.

The extra ingredient needed to specify the deformation is a skew-symmetric solution of the modified classical Yang-Baxter equation on $\mathfrak{f}$. Specifically, this is an $\mathbb{R}$-linear operator R such that, for $M, N \in \mathfrak{f}$,

$$[RM, RN] - R([RM, N] + [M, RN]) = [M, N] \quad (1)$$

and $\mathrm{Str}(MRN) = -\mathrm{Str}(RMN)$. We choose the standard Dynkin diagram of the complexified algebra $\mathfrak{sl}(4|4, \mathbb{C})$ and take R to be the restriction to $\mathfrak{su}(2,2|4)$ of the $\mathbb{C}$-linear operator acting on $\mathfrak{sl}(4|4, \mathbb{C})$ by $-i$ on generators associated with positive roots, $+i$ on generators associated with negative roots, and 0 on Cartan generators (for some choice of positive and negative roots, see forthcoming article for details). Other possible choices of Dynkin diagram and R matrices require further study. We will make use of the operator $R_g = \mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g$ with $\mathrm{Ad}_g(M) = gMg^{-1}$. The operator R_g is also a skew-symmetric solution of Eq. (1). Finally, we define the following linear combinations of the projectors,

$$d = P_1 + \frac{2}{1-\eta^2} P_2 - P_3,$$
$$\tilde{d} = -P_1 + \frac{2}{1-\eta^2} P_2 + P_3.$$

The operator $\tilde{d}$ is the transpose operator of d and, thus, satisfies $\mathrm{Str}(M\, d(N)) = \mathrm{Str}(\tilde{d}(M)N)$. The real variable $\eta \in [0, 1[$ will play the role of the deformation parameter.

Deformed action.—As pointed out in the Introduction, we will restrict ourselves here to presenting the deformed action and summarizing its most important properties. In this section, we shall write down this action and indicate the properties it shares with the undeformed action. Properties that depend on the deformation parameter η are presented in the next section.

Action: The action, which can be obtained by generalizing the method developed in Ref. [15] to the case at hand, reads $S[g] = \int d\sigma d\tau L$ with

$$L = -\frac{(1+\eta^2)^2}{2(1-\eta^2)} P_-^{\alpha\beta}\, \mathrm{Str}\left(A_\alpha d \circ \frac{1}{1-\eta R_g \circ d}(A_\beta)\right). \quad (2)$$

We have introduced the projectors $P_\pm^{\alpha\beta} = \frac{1}{2}(\gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$ where $\gamma^{\alpha\beta}$ is the world sheet metric with $\det \gamma = -1$ and $\epsilon^{01} = 1$. World sheet indices are lowered and raised with the two-dimensional metric. The operator $1 - \eta R_g \circ d$ is invertible on $\mathfrak{f}$ for all values of the deformation parameter $\eta \in [0, 1]$. As in the undeformed case, there is an Abelian gauge invariance $g(\sigma, \tau) \to g(\sigma, \tau) e^{i\theta(\sigma,\tau)}$ under which the vector field A_α transforms as $A_\alpha \to A_\alpha + \partial_\alpha \theta.\mathbf{1}$. Indeed, this leaves the action associated with Eq. (2) invariant because $\mathrm{Str}(\mathbf{1}.M) = 0$ for any M in $\mathfrak{su}(2,2|4)$. This invariance means that physical degrees of freedom do not belong to the whole group F but rather to the projective group PF. From now on, the commutators that will appear should be considered as commutators of the projective algebra $\mathfrak{pf}$, and the adjoint action of g Ad_g is that of the projective group PF. This peculiarity already appears in the undeformed case, and the reader is referred, for instance, to the review in Ref. [17] for more details.

Original Metsaev-Tseytlin action: The undeformed action corresponds to $\eta = 0$. Indeed, when η vanishes, the Lagrangian (2) simply becomes

$$L|_{\eta=0} = -\frac{1}{2} P_-^{\alpha\beta}\, \mathrm{Str}(A_\alpha d|_{\eta=0}(A_\beta)),$$
$$= -\frac{1}{2}\, \mathrm{Str}(\gamma^{\alpha\beta} A_\alpha^{(2)} A_\beta^{(2)} + \epsilon^{\alpha\beta} A_\alpha^{(1)} A_\beta^{(3)}).$$

One, therefore, recovers at $\eta = 0$ the type IIB superstring action on the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background. This is the celebrated Metsaev-Tseytlin action [2] (see also the reviews in Refs. [17,18]).

$SO(4,1) \times SO(5)$ gauge invariance: The action corresponding to Eq. (2) has a gauge invariance $g(\sigma, \tau) \to g(\sigma, \tau) h(\sigma, \tau)$ where the function $h(\sigma, \tau)$ takes values in the subgroup G. This can be easily shown using the corresponding transformations

$$A_\alpha \to h^{-1}\partial_\alpha h + \mathrm{Ad}_h^{-1}(A_\alpha),$$
$$d(A_\alpha) \to \mathrm{Ad}_h^{-1} \circ d(A_\alpha),$$
$$R_g \to \mathrm{Ad}_h^{-1} \circ R_g \circ \mathrm{Ad}_h.$$

This gauge transformation does not depend on the deformation parameter η.

Properties of the deformed action.—To present the properties of the action of Eq. (2), we will follow the approach presented in the review [17] for the undeformed case.

Equations of motion: The equations of motion are most conveniently written in terms of the vectors

$$J_\alpha = \frac{1}{1-\eta R_g \circ d}(A_\alpha),$$
$$\tilde{J}_\alpha = \frac{1}{1+\eta R_g \circ \tilde{d}}(A_\alpha)$$

and their projections $J_-^\alpha = P_-^{\alpha\beta} J_\beta$ and $\tilde{J}_+^\alpha = P_+^{\alpha\beta} \tilde{J}_\beta$. In the following, we shall often use the fact that the components J_-^0 and J_-^1 are proportional to each other. One has in particular $[J_-^\alpha, J_-^\beta] = 0$ (and similarly for $\tilde{J}_+^\alpha$). The equations of motion arising from the Lagrangian (2) are given by $\mathcal{E} = 0$ where

$$\mathcal{E} := d(\partial_\alpha J_-^\alpha) + \tilde{d}(\partial_\alpha \tilde{J}_+^\alpha) + [\tilde{J}_{+\alpha}, d(J_-^\alpha)] + [J_{-\alpha}, \tilde{d}(\tilde{J}_+^\alpha)].$$

It is easy to check that the projection $\mathcal{E}^{(0)}$ of $\mathcal{E}$ onto $\mathfrak{f}^{(0)}$ vanishes, in accordance with the gauge invariance of the action described above.

Rewriting the Maurer-Cartan equation: We now wish to address the question of integrability of the theory defined by Eq. (2). Recall that in the undeformed case, in deriving the Lax connection one makes use of the Maurer-Cartan equation $\mathcal{Z}=0$ satisfied by A_α, where

$$\mathcal{Z} := \frac{1}{2}\epsilon^{\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha + [A_\alpha, A_\beta]).$$

To find a Lax connection, we therefore start by rewriting $\mathcal{Z}$ in terms of J_-^α and $\tilde{J}_+^\alpha$. The resulting expression is a quadratic polynomial in η. Using Eq. (1) for the operator R_g, one can rewrite the coefficient of η^2 of this polynomial to obtain

$$\mathcal{Z} = \partial_\alpha \tilde{J}_+^\alpha - \partial_\alpha J_-^\alpha + [J_{-\alpha}, \tilde{J}_+^\alpha] + \eta^2[d(J_{-\alpha}), \tilde{d}(\tilde{J}_+^\alpha)] + \eta R_g(\mathcal{E}).$$

Anticipating the result, let us note here that choosing R to be a nonsplit solution of the modified classical Yang-Baxter equation is essential in order to preserve integrability as we deform the theory. Before constructing the Lax connection, let us remark that the field equations in the odd sector $P_{1,3}(\mathcal{E})=0$ may be greatly simplified by considering the combinations

$$P_1 \circ (1-\eta R_g)(\mathcal{E}) + P_1(\mathcal{Z}) = -4[\tilde{J}_{+\alpha}^{(2)}, J_-^{\alpha(3)}], \tag{3a}$$

$$P_3 \circ (1+\eta R_g)(\mathcal{E}) - P_3(\mathcal{Z}) = -4[J_{-\alpha}^{(2)}, \tilde{J}_+^{\alpha(1)}]. \tag{3b}$$

As a consequence, one can take as field equations in the odd sector

$$[\tilde{J}_{+\alpha}^{(2)}, J_-^{\alpha(3)}] = 0, \qquad [J_{-\alpha}^{(2)}, \tilde{J}_+^{\alpha(1)}] = 0,$$

which have the same form as those of the undeformed model written in terms of ordinary currents.

Lax connection: We define the two vectors

$$L_+^\alpha = \tilde{J}_+^{\alpha(0)} + \lambda\sqrt{1+\eta^2}\tilde{J}_+^{\alpha(1)} + \lambda^{-2}\frac{1+\eta^2}{1-\eta^2}\tilde{J}_+^{\alpha(2)} + \lambda^{-1}\sqrt{1+\eta^2}\tilde{J}_+^{\alpha(3)},$$

$$M_-^\alpha = J_-^{\alpha(0)} + \lambda\sqrt{1+\eta^2}J_-^{\alpha(1)} + \lambda^2\frac{1+\eta^2}{1-\eta^2}J_-^{\alpha(2)} + \lambda^{-1}\sqrt{1+\eta^2}J_-^{\alpha(3)},$$

where λ is the spectral parameter. Then, the whole set of equations of motion $\mathcal{E}=0$ and zero curvature equations $\mathcal{Z}=0$ is equivalent to

$$\partial_\alpha L_+^\alpha - \partial_\alpha M_-^\alpha + [M_{-\alpha}, L_+^\alpha] = 0. \tag{4}$$

One may define an unconstrained vector

$$\mathcal{L}_\alpha = L_{+\alpha} + M_{-\alpha},$$

in terms of which the Eq. (4) becomes an ordinary zero curvature equation

$$\partial_\alpha \mathcal{L}_\beta - \partial_\beta \mathcal{L}_\alpha + [\mathcal{L}_\alpha, \mathcal{L}_\beta] = 0.$$

The existence of this Lax connection shows that the dynamics of the deformed action admits an infinite number of conserved quantities.

Virasoro constraints: It is clear that each term in the Lagrangian (2) is proportional either to the metric $\gamma^{\alpha\beta}$ or to $\varepsilon^{\alpha\beta}$. The part of the action proportional to the metric takes the form

$$S_\gamma = -\frac{1}{2}\left(\frac{1+\eta^2}{1-\eta^2}\right)^2 \int d\sigma d\tau \gamma^{\alpha\beta}\mathrm{Str}(J_\alpha^{(2)}J_\beta^{(2)}), \tag{5a}$$

$$= -\frac{1}{2}\left(\frac{1+\eta^2}{1-\eta^2}\right)^2 \int d\sigma d\tau \gamma^{\alpha\beta}\mathrm{Str}(\tilde{J}_\alpha^{(2)}\tilde{J}_\beta^{(2)}). \tag{5b}$$

To obtain this result, the skew symmetry of R_g has been used. The Virasoro constraints are then found to be

$$\mathrm{Str}(\tilde{J}_+^{\alpha(2)}\tilde{J}_+^{\beta(2)}) \approx 0, \qquad \mathrm{Str}(J_-^{\alpha(2)}J_-^{\beta(2)}) \approx 0.$$

Kappa symmetry: The invariance under κ symmetry is a characteristic of the Green-Schwarz formulation. We now want to show that the kappa invariance is essentially unchanged after deformation. To do this, consider an infinitesimal right translation of the field, $\delta g = g\epsilon$, where the parameter ε takes the form

$$\epsilon = (1-\eta R_g)\rho^{(1)} + (1+\eta R_g)\rho^{(3)}.$$

The fields $\rho^{(1)}$ and $\rho^{(3)}$, whose expressions will be determined shortly, respectively take values in $\mathfrak{f}^{(1)}$ and $\mathfrak{f}^{(3)}$. Then the variation of the action with respect to g reads

$$\delta_g S = \frac{(1+\eta^2)^2}{2(1-\eta^2)}\int d\sigma d\tau \mathrm{Str}(\rho^{(1)}P_3\circ(1+\eta R_g)(\mathcal{E}) + \rho^{(3)}P_1\circ(1-\eta R_g)(\mathcal{E})).$$

We may then use Eq. 3 to write this variation as

$$\delta_g S = -2\frac{(1+\eta^2)^2}{(1-\eta^2)}\int d\sigma d\tau\, \mathrm{Str}(\rho^{(1)}[J_{-\alpha}^{(2)}, \tilde{J}_+^{\alpha(1)}] + \rho^{(3)}[\tilde{J}_{+\alpha}^{(2)}, J_-^{\alpha(3)}]).$$

In full analogy with the undeformed case (see Ref. [17]), we take the following ansatz for $\rho^{(1)}$ and $\rho^{(3)}$:

$$\rho^{(1)} = i\kappa^{(1)}_{+\alpha} J^{\alpha(2)}_{-} + J^{\alpha(2)}_{-} i\kappa^{(1)}_{+\alpha},$$
$$\rho^{(3)} = i\kappa^{(3)}_{-\alpha} \tilde{J}^{\alpha(2)}_{+} + \tilde{J}^{\alpha(2)}_{+} i\kappa^{(3)}_{-\alpha},$$

where $\kappa^{(1)}_{+}$ and $\kappa^{(3)}_{-}$ are constrained vectors of respective gradings 1 and 3. Note that we are using the standard convention for the real form $\mathfrak{su}(2,2|4)$ (see for instance appendix C of Ref. [19]). Then, a short calculation leads to

$$\mathrm{Str}(\rho^{(1)}[J^{(2)}_{-\alpha}, \tilde{J}^{\alpha(1)}_{+}]) = \mathrm{Str}(J^{\alpha(2)}_{-} J^{\beta(2)}_{-} [\tilde{J}^{(1)}_{+\alpha}, i\kappa^{(1)}_{+\beta}]),$$
$$\mathrm{Str}(\rho^{(3)}[\tilde{J}^{(2)}_{+\alpha}, J^{\alpha(3)}_{-}]) = \mathrm{Str}(\tilde{J}^{\alpha(2)}_{+} \tilde{J}^{\beta(2)}_{+} [J^{(3)}_{-\alpha}, i\kappa^{(3)}_{-\beta}]).$$

At this point, we use the standard property (see Ref. [17]) that the square of an element of grade 2 only contains a term proportional to $W = \mathrm{diag}(\mathbf{1}_4, -\mathbf{1}_4)$ and a term proportional to the identity that does not play a role in the case at hand. We finally obtain

$$\delta_g S = -\frac{(1+\eta^2)^2}{4(1-\eta^2)} \int d\sigma d\tau \big(\mathrm{Str}(J^{\alpha(2)}_{-} J^{\beta(2)}_{-}) \times \mathrm{Str}(W[\tilde{J}^{(1)}_{+\alpha}, i\kappa^{(1)}_{+\beta}]) + \mathrm{Str}(\tilde{J}^{\alpha(2)}_{+} \tilde{J}^{\beta(2)}_{+}) \mathrm{Str}(W[J^{(3)}_{-\alpha}, i\kappa^{(3)}_{-\beta}])\big).$$

This expression comes from the variation of the field g in the action. It may be compensated by another term coming from the variation of the metric γ. To determine this variation we use the result of Eq. (5). We are then led to choose

$$\delta\gamma^{\alpha\beta} = \frac{1-\eta^2}{2} \mathrm{Str}(W[i\kappa^{\alpha(1)}_{+}, \tilde{J}^{\beta(1)}_{+}] + W[i\kappa^{\alpha(3)}_{-}, J^{\beta(3)}_{-}])$$

for the transformation of the metric in order to ensure κ symmetry.

Conclusion.—The Lagrangian of Eq. (2) is a semisymmetric space generalization of the one obtained in Ref. [15] by deforming the symmetric space σ model on F/G. In the latter case, it was shown that the original F_L symmetry is deformed to a Poisson-Hopf algebra analogue of $U_q(\mathfrak{f})$. The same fate is confidently expected for the $\mathfrak{psu}(2,2|4)$ symmetry of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring. Hence, the q deformation proposed here generalizes the situation that holds for the squashed sphere σ model [20,21].

As mentioned in the Introduction, the construction of the deformed theory relies on the existence of a second compatible Poisson bracket. The latter is known to be related [13] to the Pohlmeyer reduction of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring [19,22]. In fact, one motivation for deforming the superstring action comes from the q-deformed S matrix appearing in this context [23–25], built from the q-deformed R matrix of Ref. [26], and in terms of which the corresponding thermodynamic Bethe ansatz equations are constructed [27,28]. It would, therefore, be very interesting to make contact between these two deformations.

The geometric background of the undeformed action consists of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric, a constant dilaton and a nontrivial self-dual 5-form. This constitutes a maximally supersymmetric background of type IIB supergravity [29–32]. We have shown that the deformed action is also invariant under κ symmetry. However, type IIB supergravity backgrounds ensure the existence of κ symmetry [33]. It is, therefore, desirable to explicitly determine the whole deformed geometry and check that it satisfies the equations of type IIB supergravity.

Let us end on a more conjectural note by commenting on the limit $\eta \to 1$ of the deformed model. The analogous limit in the case of the deformed $SU(2)/U(1)$ σ model corresponds to a $SU(1,1)/U(1)$ σ model [15]. If such a property were to generalize to the case at hand, we expect that the cosets $\mathrm{AdS}_5 \simeq SO(4,2)/SO(4,1)$ and $\mathrm{S}^5 \simeq SO(6)/SO(5)$ would respectively be replaced in this limit by $SO(5,1)/SO(4,1) \simeq \mathrm{dS}_5$ and $SO(5,1)/SO(5) \simeq \mathrm{H}^5$. Such cosets have already been considered in Ref. [34]. This point certainly requires closer investigation, and we will come back to it from the Hamiltonian point of view elsewhere.

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