Seeding of Self-Modulation Instability of a Long Electron Bunch in a Plasma

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We demonstrate experimentally that a relativistic electron bunch shaped with a sharp rising edge drives plasma wakefields with one to seven periods along the bunch as the plasma density is increased. The plasma density is varied in the 10^{15}–10^{17} cm^{-3} range. The wakefields generation is observed after the plasma as a periodic modulation of the correlated energy spectrum of the incoming bunch. We choose a low bunch charge of 50 pC for optimum visibility of the modulation at all plasma densities. The longitudinal wakefields creating the modulation are in the MV/m range and are indirect evidence of the generation of transverse wakefields that can seed the self-modulation instability, although the instability does not grow significantly over the short plasma length (2 cm). We show that the seeding provides a phase reference for the wakefields, a necessary condition for the deterministic external injection of a witness bunch in an accelerator. This electron work supports the concept of similar experiments in the future, e.g., SMI experiments using long bunches of relativistic protons.

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Plasma-based acceleration has been proven a very attractive new acceleration technique due to the large acceleration gradient (> 50 GV/m) it has reached [1]. For a future, more compact and more affordable linear electron/positron collider, such an accelerator will need to produce bunches with extremely low emittance and multikilojoules of energy to reach the required luminosity and to access new physics at the energy frontier ($\approx$1 TeV/particle). However, present relativistic electron bunch and laser pulse drivers carry less than 100 J, thereby limiting the energy gain by the accelerated bunch to < 100 J. Reaching high energies would require many acceleration stages to reach the desired energy. Staging would add complexity and length to a plasma-based accelerator and has not yet been demonstrated.

Proton bunches produced in circular accelerators can carry many tens of kilojoules. These bunches with $\sim 10^{11}$ protons are routinely produced, for example, at CERN by the Super Proton Synchrotron (450 GeV) and the Large Hadron Collider (3.5–7 TeV). Proof-of-principle numerical studies [2] showed that a single plasma section driven by a 1 TeV, 100 μm-long bunch with 10^{11} protons could accelerate a 10 GeV incoming electron bunch to $\approx$500 GeV in $\approx$300 m of plasma with density $n_e = 6 \times 10^{14}$ cm^{-3}. However, the CERN proton bunches are $\approx$12 cm long, and high-energy, high-charge, and short proton bunches are not currently available.

In 2010 Kumar, Pukhov, and Lotov [3] proposed that long charged particle bunches traveling in dense plasmas ($L_{\rm beam} \gg \lambda_{\rm pe} \propto n_e^{-1/2}$, $\lambda_{\rm pe}$ is the electron-plasma wavelength) are subject to a transverse, two-stream, self-modulation instability (SMI) and can drive wakefields to large amplitudes [3]. The radius of the long bunch and, thereby, also its density become modulated at the scale of the plasma period by the transverse focusing and defocusing wakefields, providing feedback for the SMI to grow and saturate. Thereafter, the modulated proton bunch resonantly drives the wakefields to amplitudes that can approach the nonrelativistic cold wave breaking field $E_{\rm wb}$ [4]. The instability is convective and grows both along the bunch (ξ) and along the plasma (z) [3,5,6]. Note that all relativistic charged particle bunches (electrons, positrons, protons, etc.) are subject to the SMI. Note also that the self-modulation of a particle bunch is completely analogous to that of a laser pulse or photon beam [7].

In this Letter, we present the first experimental evidence that a 50 pC, $\sim$1 mm-long relativistic electron bunch does drive wakefields with the periodicity of the relativistic plasma wave or wake. The number of wake periods within the bunch is varied between $\sim$1 and $\sim$7 by varying the plasma density from $\sim 10^{15}$ to $\sim 10^{17}$ cm^{-3}, while keeping other parameters fixed. The bunch is shaped with a sufficiently sharp rising edge [8,9] that drives initial wakefield amplitudes in the MV/m range. These wakefields act as a seed for the SMI [10]. The effect of the longitudinal wakefields on the bunch entering the plasma with a time-energy correlation is observed as periodic modulation of the final energy spectrum. The seeding of the SMI by the sharp rising edge of the bunch is confirmed by the observation that the initial phase of the wakefields is fixed by the bunch rising edge, a condition necessary to deterministically inject a witness bunch in an accelerator experiment. Experimental and simulation results show that with the 50 pC charge the SMI does not grow over the 2 cm-long plasma, and therefore, the above observations are indirect evidence of the seeding of the SMI. However, the theory [11] and simulations of plasma wakefield accelerator (PWFA) indicate the necessary coexistence of longitudinal *and* transverse fields, which lead to the seeding of SMI. Simulations show that with a bunch with higher charge (1 nC) the SMI does reach saturation over the 2 cm plasma length [12]. We use a long electron bunch to study the physics of SMI seeding, but the results apply equally to bunches of other particle species (positrons, protons, etc.).

The experiment is performed at the Brookhaven National Laboratory Accelerator Test Facility (ATF). A radiofrequency (rf) photoinjector and two S-band accelerator sections produce the $\cong 58$ MeV electron bunch with a charge adjustable from a few tens of picocoulombs to ≈ 1 nC. The bunch travels along the accelerator sections off the crest of the sinusoidal rf wave so that it acquires an approximately linear correlated energy spread (chirp) of about 1% of the incoming energy, with the bunch front having the lowest energy. The bunch then enters a magnetic dogleg section where it is dispersed in energy in the horizontal plane. The combination of linear energy chirp and energy dispersion allows for the shaping in time (or equivalently in $\xi = ct - z$ or in energy) of the bunch current profile, as was demonstrated in Refs. [8] and [9] using a variable width slit and a mask. The present experiments use a rectangular mask to produce a bunch with a square current temporal profile.

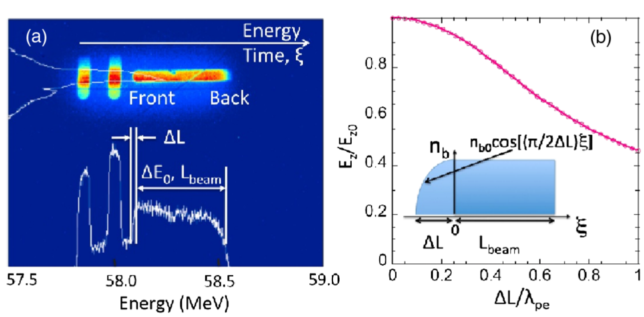

FIG. 1 (color online). (a) Image of the long bunch dispersed in energy a short distance downstream from the mask. (Two low-energy electron bunches not used for these experiments are shown early in time and in the front of the bunch and are to be disregarded.) The experimental traces indicate the sum of the image signal over the vertical and horizontal axes, respectively, and are displayed on the same scale. The energy range selected by the mask is ΔE_0. The bunch length is L_{beam}, and the length scale for the rising edge is ΔL. (b) Ratio of the normalized longitudinal wakefield amplitude ratio $E_z(\Delta L)/E_{z0}$ as a function of the length of front rising edge ΔL. Here, E_{z0} is the result for a step function (when ΔL tends to zero). The profile of the bunch's rising front is modeled as a cosine function over ΔL to the value n_{b0}. All results are calculated with 2D linear theory.

Figure 1(a) shows an image of the bunch dispersed in energy (and therefore in time) a short distance downstream from the mask. The length of the square bunch is inferred from its accelerating phase of $-9°$ relative to the crest of the 2.856 GHz sinusoidal rf wave, its mean energy ($E_0 \cong 58.3$ MeV), and selected energy spread ($\Delta E_0 \cong 0.48$ MeV). Assuming the bunch length is short when compared to the rf wavelength (~ 10 cm), the extracted bunch length is $L_{\text{beam}} \cong 960$ μm. The current rise profile is fitted with cos $[(\pi/2\Delta L)\xi]$ for $-\Delta L \leq \xi \leq 0$ with characteristic width $\Delta L(\cong 0.06 L_{\text{beam}})$, as shown in the inset of Fig. 1(b). Figure 1(b) will be further illustrated in the later text. Upon exiting the dogleg, the bunch propagates towards the plasma and the imaging magnetic spectrometer with a resolution < 0.03 MeV, sufficient to resolve the ≈ 0.1 MeV features observed below. The bunch is focused near the plasma entrance with transverse size $\sigma_r \cong 120$ μm, and the normalized emittance is $\varepsilon_N \cong 13$ mm mrad. The plasma source is a capillary discharge [13] with length $L_p \cong 2$ cm and radius $\cong 500$ μm, similar to the setup in the recent current filamentation instability experiments [14].

As the bunch enters the plasma, its space charge field displaces the plasma electrons and drives the plasma wakefields. The wakefields have both longitudinal and transverse components. The transverse component creates a periodic modulation of the bunch transverse momentum (the SMI seed) and of the bunch radius and charge density if strong enough or the plasma is long enough. The longitudinal component leads to periodic energy loss and gain by particles along the bunch. Since the incoming bunch is relativistic and the energy gain or loss is small (< 1 MeV), no significant dephasing occurs over the short plasma length. However, with energy-position-time correlation of the incoming linear energy chirp, the longitudinal plasma wakefields cause the beam particles to bunch in energy, which can directly lead to observable peaks in the bunch energy spectrum. In our experiment where particles in the front have lower energy, the positions of energy bunching are those corresponding to the odd zeros of the periodic longitudinal wakefields along the bunch with $E_z = 0$ at the sharp front. This means that there is always an energy bunch forming at the front. Therefore, when the plasma density is such that the bunch is l plasma wavelengths long, $l + 1$ peaks form in the energy spectrum (for simplicity l is considered integer here).

For optimum bunching at the plasma exit, the particles' energy gain or loss must be on the order of $\Delta E_l \cong \Delta E_0 / 4l$. Assuming constant wakefields over the plasma length (no SMI growth), the optimum average electric field amplitude is thus $E_z \cong \frac{1}{4}[\Delta E_0/(elL_p)]$. This expression shows that an E_z amplitude decreasing with increasing n_e is necessary to preserve the energy bunching visibility. Here, E_z values of

5.2,3.0,2.0,1.5,1.2,1.0, and 0.9 MV/m are best for integer values $l = 1$ to 7, respectively.

In two-dimensional (2D) cylindrically symmetric linear PWFA theory [15] the on-axis peak electric field amplitude driven by a bunch with charge Q, a $L_{\rm beam}$-long square longitudinal profile and a Gaussian transverse profile with rms radius σ_r is given by $E_z(r=0) = [-lQ/(\varepsilon_0 L_{\rm beam}^2 \sigma_r^2)] \int_0^\infty e^{-r^2/2\sigma_r^2} K_0(2\pi l r/L_{\rm beam}) r dr$. Here, the plasma wave number for the lth mode is taken as $k_p = 2\pi l/L_{\rm beam}$, K_0 is the modified zeroth-order Bessel function of the second kind, and we assume $L_{\rm beam} > \lambda_{\rm pe}$. This expression shows that for $Q = 50$ pC, $E_z(r=0) = 3.7$,3.2, 2.6, 2.1, 1.8, 1.5, and 1.3 MV/m for $l = 1$ to 7, respectively. These values decrease as n_e (or l) increases and are close to the ones for optimum bunching visibility. When entering the plasma the bunch density is $n_b = 3.6 \times 10^{12}$ cm^{-3}, much lower than the range of plasma densities used in the experiment ($n_b/n_e < 3 \times 10^{-3}$, see below). The beam-plasma interaction is, therefore, in the linear wakefield regime. In the linear wakefield theory, the focusing strength decreases more rapidly than E_z [12], and this strongly impacts the growth of the instability [see Fig. 4(a)].

In view of the previous paragraph results, a bunch charge of 50 pC is therefore chosen in the experiment so that the overall visibility of the energy spectra modulation is maintained as n_e is varied. Figure 2 shows a typical experimental energy spectrum with no plasma [Fig. 2(a) and seven energy spectra Figs. 2(b)–2(h)] acquired with n_e increasing from $\approx 2 \times 10^{15}$ to $\approx 8 \times 10^{16}$ cm^{-3}. The non-integer values of $L_{\rm beam}/\lambda_{\rm pe}$ labeled in the figures are calculated as the ratio of the maximum initial energy spread (ΔE_0) and the average energy difference between the neighboring peaks $\overline{\Delta E_l}$ on Figs. 2(b)–2(h). The spectra reveal that energy self-modulation generates two to seven peaks along the bunch, corresponding to ≈1.4 to ≈7.5 plasma periods. In addition to the evident energy modulation, the spectra of Fig. 2 also show a considerable loss of charge toward the trailing end of the bunch that is at present not understood. This leads to the observed number of peaks in Figs. 2(c)–2(h) to be l rather than $l+1$, where l is the integer part of the number of the plasma periods (see Fig. 3 for the missing peak in the yellow region). However, this does not change the conclusions reached here. In all figures, the self-modulation peaks are clearly visible, suggesting that the energy gain and loss by the particles remain close to that for optimum bunching at all plasma densities. This indicates that as predicted by linear theory, the wakefield amplitude does decrease with increasing n_e, and its average values along the plasma are comparable to those estimated here above. It is also consistent with the wakefields not growing over the plasma length but remaining close to their initial value. Note that the apparent transverse modulation suggested by the spectra of Fig. 2 really only originates from the energy modulation and is not from the radial modulation due to SMI. Figure 2(i) is obtained from simulation and will be discussed later in the Letter.

Figure 3 shows the energies at which the density peaks appear in the spectra of Fig. 2 as a function n_e. The plasma

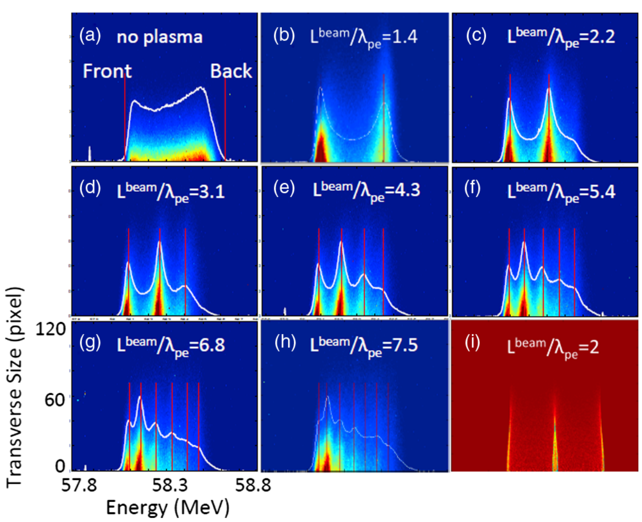

FIG. 2 (color online). Energy spectra obtained at various plasma densities. Spectra (a) with $n_e = 0$ (no plasma) and (b)–(h) with increasing plasma densities between 2×10^{15} and 8×10^{16} cm^{-3} obtained in the experiment. The white lines indicate the sum of the images over the vertical image dimension (no dispersion), and the red lines show the positions of the density peaks as identified for Fig. 3. Figure (i) shows a simulated energy spectrum for the case $L_{\rm beam}/\lambda_{\rm pe} = 2$, very similar to the experimental case of (c). The color tables are deliberately chosen different to avoid possible confusion between experimental and simulation results.

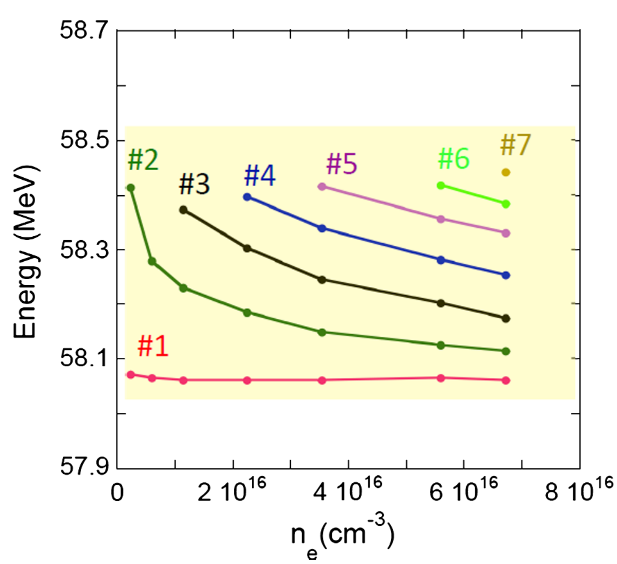

FIG. 3 (color online). Energy of the peaks identified by the red lines on Figs. 2(b)–2(h) as a function of plasma density. The yellow zone corresponds to the FWHM of the incoming bunch energy and suggests that peaks are missing in the back, high-energy part of the bunch for $n_e > 0.5 \times 10^{16}$ cm^{-3}.

density is deduced from the value of $L_{\text{beam}}/\lambda_{\text{pe}}$ as mentioned above [$n_e \propto \lambda_{\text{pe}}^{-2} = (\Delta E_0/\overline{\Delta E_l} L_{\text{beam}})^2$]. They are in excellent agreement with those determined from the Stark broadening of the H_α line [16]. Note that for this negative incoming energy chirp, the fact that the peak labeled No. 1 does not shift in energy with varying n_e confirms that this particular peak is indeed at the front of the bunch. The sharp-rising bunch front, therefore, acts as a starting point or phase reference for the wakefields along the bunch. This is particularly important in an accelerator experiment in order to deterministically inject a witness bunch into the accelerating and focusing wake phase.

Numerical simulations with the experimental beam and plasma parameters are performed in 2D cylindrically symmetric geometry using the code OSIRIS [17] to validate the experimental results. A step function current rise at the bunch front is assumed. Simulations are performed with densities varying from 1.2×10^{15} to 5.9×10^{16} cm^{-3} corresponding to integer values of $l = L_{\text{beam}}/\lambda_{\text{pe}} = 1$–7, a range similar to that of the experiments; see Fig. 3. Figure 4 shows the evolution of maximum accelerating field E_z along the bunch as a function of propagation distance z for $l = 1$–7 with the bunch charge of $Q = 50$ pC in Fig. 4(a) and for one plasma density ($l = 2$) with $Q = 1$ nC in Fig. 4(b) [12]. First, as suggested by Fig. 2, Fig. 4(a) confirms that over $z = 2$ cm, the experimental plasma length, there is no significant SMI growth at any plasma density considered here. Second, it shows that the initial values $E_z(z = 0)$ decrease from ~4 to ~1 MV/m as n_e increases. Both the $E_z(z = 0)$ amplitudes and their trend [see Fig. 4(a)] are in excellent agreement with those from 2D PWFA linear theory (< 7% difference) and from the experimental observations. Energy spectra generated from these simulations exhibit remarkable similarities with the measured ones of Fig. 2 (e.g., Fig. 2(i) for $l = 2$). As mentioned above, a lower transverse force leads to a decreasing SMI growth rate when l increases from 2 to 7, as visible on Fig. 4(a). Note that the $l = 1$ case has smaller growth rate at larger z values (e.g., when compared to the to $l = 2$ case) despite its larger focusing force. This is because there is no feedback for the SMI to grow in this short bunch case [3,5,6].

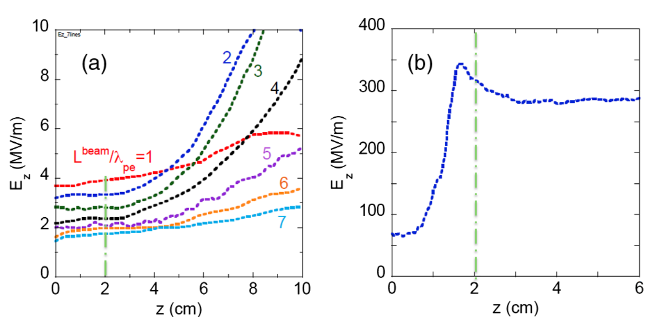

FIG. 4 (color online). (a) Peak accelerating field along the bunch versus propagation distance in the simulation for the electron bunch with 50 pC charge and plasma densities such that $L_{\text{beam}}/\lambda_{\text{pe}} = 1$ (red), 2 (blue), 3 (green), 4 (black), 5 (purple), 6 (orange), and 7 (cyan line). The initial $E_z(z = 0)$ values are 3.7, 3.2, 2.7, 2.2, 1.9, 1.6, 1.3 MV/m for $l = 1$ to 7, respectively. (b) Similar figure for the 1 nC bunch for $L_{\text{beam}}/\lambda_{\text{pe}} = 2$. The vertical dotted green lines indicate the length of the plasma in the experiment $L_p = 2$ cm.

Figure 4(b) shows the evolution of maximum accelerating field E_z in the case of a $Q = 1$ nC bunch and $l = 2$. The initial E_z amplitude is much larger (≈ 67 MV/m $\propto n_b \propto Q$), as expected. More importantly, since the SMI growth rate also increases with bunch density ($\propto n_b^{1/3} \propto Q^{1/3}$), a much larger saturated field is reached (~350 MV/m) *within* the experimental plasma length of 2 cm. However, because of experimental limitations, no evidence of SMI growth can be inferred from the measured energy spectrum.

The results presented here show that the electron bunch generated in the experiment seeds the wakefield generation. We now estimate the effect of the experimental bunch current rise length on the initial longitudinal wakefield amplitude ($E_{z,\max}$) using 2D linear PWFA theory. We consider the ramp profile to be a cosine function of ΔL as mentioned earlier [see Fig. 1(b) inset]. With $L_{\text{beam}}/\lambda_{\text{pe}}$ varying from 1 to 7 in our experiment, the ratio $\Delta L/\lambda_{\text{pe}}$ ranges approximately from 0.06 to 0.42. Figure 1(b) shows that over such a range the effect of the ramp length is small, i.e., $E_{z,\max}/E_{z0} > 0.8$, where E_{z0} is the longitudinal wakefield amplitude excited by an infinitely sharp bunch ($\Delta L = 0$). Therefore, the rising edge of the bunch is sharp enough to effectively seed the SMI, in agreement with the experimental results.

The observation of the bunch energy self-modulation in Fig. 2 is the direct evidence of the generation of the longitudinal component of plasma wakefields with multiple plasma periods along the bunch. Wakefield theory implies the existence of corresponding transverse focusing and defocusing components [11]. The energy modulation observation is, hence, indirect evidence of the seeding of the SMI. However, over the 2 cm plasma length, the transverse wakefield components lead to a very significant periodic modulation of the bunch radial momentum. Further numerical calculation of the ballistic propagation of the particles in vacuum downstream from the plasma shows radial modulation of the bunch (not shown here). The radial modulation could be measured using, for example, a transverse deflecting cavity with subpicosecond time resolution.

In summary, we have demonstrated experimentally for the first time that energy modulation is indirect but nonetheless important evidence for the seeding of the transverse self-modulation instability by the sharp-rising edge of an electron bunch. A number of SMI experiments on a much larger scale are contemplated or planned at major facilities (CERN, Fermi National Laboratory, SLAC National Accelerator Laboratory, DESY, etc.) [18,19]. All of them will rely on seeding to observe the instability, some to deterministically inject external electrons in the wakefields or to mitigate the occurrence of the hose instability [20,21]. The results presented here are an important seed for these major experiments.

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