Isospin Breaking in the Nucleon Mass and the Sensitivity of β Decays to New Physics

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We discuss the consequences of the approximate conservation of the vector and axial currents for the hadronic matrix elements appearing in β decay if nonstandard interactions are present. In particular, the isovector (pseudo)scalar charge $g_{S(P)}$ of the nucleon can be related to the difference (sum) of the nucleon masses in the absence of electromagnetic effects. Using recent determinations of these quantities from phenomenological and lattice QCD studies we obtain the accurate values $g_S = 1.02(11)$ and $g_P = 349(9)$ in the modified minimal subtraction scheme at $\mu = 2$ GeV. The consequences for searches of nonstandard scalar interactions in nuclear β decays are studied, finding for the corresponding Wilson coefficient $\epsilon_S = 0.0012(24)$ at 90% C.L., which is significantly more stringent than current LHC bounds and previous low-energy bounds using less precise g_S values. We argue that our results could be rapidly improved with updated computations and the direct calculation of certain ratios in lattice QCD. Finally, we discuss the pion-pole enhancement of g_P, which makes β decays much more sensitive to nonstandard pseudoscalar interactions than previously thought.

In pure QCD, the charged $d \to u$ transitions induce approximately conserved vector and axial currents

$$\partial_\mu(\bar{u}\gamma^\mu d) = -i(m_d - m_u)\bar{u}d, \tag{1}$$

$$\partial_\mu(\bar{u}\gamma^\mu\gamma_5 d) = i(m_d + m_u)\bar{u}\gamma_5 d \tag{2}$$

with $m_{u,d}$ the respective light-quark masses. These equalities are a particular case of the venerable "conservation of the vector current" (CVC) and "partial conservation of the axial current" (PCAC) relations, which are derived from global-symmetry considerations [1–3] and have become a cornerstone for model-independent approaches to the structure and interactions of hadrons [4,5].

A straightforward application of CVC and PCAC concerns the derivation of relations between different hadronic matrix elements of local quark bilinears and it is indeed customarily used, e.g., in meson decays to reduce the number of independent form factors (see, e.g., Ref. [6] for kaon decays). A well-known application of PCAC to nucleon matrix elements is the Golberger-Treiman relation between the πN coupling and the nucleon axial coupling g_A [7–9].

As shown in the next section, similar relations can be established between the well-known isovector (axial) vector charges $g_{V(A)}$ of the nucleon and their (pseudo)scalar counterparts $g_{S(P)}$ [10]. The latter are needed to describe nuclear and neutron β decays if nonstandard (pseudo)scalar interactions are present [11–14], and they currently are subject to intensive research mainly through lattice QCD (LQCD) calculations [15,16]. These investigations are of crucial importance to assess the implications of precise β-decay measurements to constrain new physics.

To make things more interesting, it turns out that the nucleon mass splitting in the absence of electromagnetism $\delta M_N^{\rm QCD} \equiv (M_n - M_p)_{\rm QCD}$ is a necessary input for the calculation of the scalar charge g_S. Actually, the isospin corrections to the hadron masses, and in particular to the nucleon mass, are starting to receive much attention. While phenomenological determinations are being revised [17], different lattice collaborations have embarked on the *ab initio* computation of these effects in pure QCD [18–20] or even including QED [21–27].

We show how this can be exploited for translating recent calculations of $\delta M_N^{\rm QCD}$ into a precise determination of the scalar charge, which subsequently is used to extract a stringent bound on nonstandard scalar $d \to u$ transitions from β-decay data. Inversely, we discuss the implications that recent LQCD calculations of the scalar charge have on the isospin breaking effects in the nucleon mass. Finally, we study the pion-pole enhancement of g_P and explore its impact on the β-decay phenomenology.

Form factors in β decay.—The theoretical description of neutron β decay within the standard model (SM) requires the calculation of the vector and axial hadronic matrix elements, which can be decomposed as follows [28]:

$$\langle p(p_p)|\bar{u}\gamma^\mu d|n(p_n)\rangle = \bar{u}_p(p_p)\left[g_V(q^2)\gamma^\mu + \frac{\tilde{g}_{T(V)}(q^2)}{2\bar{M}_N}\sigma^{\mu\nu}q_\nu\right.$$
$$\left. + \frac{\tilde{g}_S(q^2)}{2\bar{M}_N}q^\mu\right]u_n(p_n), \qquad (3)$$

$$\langle p(p_p)|\bar{u}\gamma^\mu\gamma_5 d|n(p_n)\rangle = \bar{u}_p(p_p)\left[g_A(q^2)\gamma^\mu + \frac{\tilde{g}_{T(A)}(q^2)}{2\bar{M}_N}\sigma^{\mu\nu}q_\nu\right.$$
$$\left. + \frac{\tilde{g}_P(q^2)}{2\bar{M}_N}q^\mu\right]\gamma_5 u_n(p_n), \qquad (4)$$

where $u_{p,n}$ are the proton and neutron spinor amplitudes, $\bar{M}_N$ is the average nucleon mass, and q is the difference between the neutron and the proton momenta $q = p_n - p_p$. The vector and axial charges g_V and g_A, respectively, are responsible for the leading contributions to the decay rate due to the relatively small energies ($q^2 \simeq 0$) involved in the process. We have $g_V = 1$ up to second order isospin-breaking corrections [29], whereas the axial charge has been accurately measured in β decays, $g_A = 1.2701(25) \times g_V$ [30]. Lastly, the subleading contributions coming from the so-called "induced" form factors $\tilde{g}_i$ are known in the limit of isospin symmetry, a safe approximation at the current level of experimental precision [13,31]. The description of nuclear β decays requires the introduction of the Fermi and Gamow-Teller nuclear matrix elements that play an analogous role to g_V and g_A in the neutron decay.

If nonstandard (pseudo)scalar interactions are present we need to introduce the following matrix elements in the theoretical description:

$$\langle p(p_p)|\bar{u}d|n(p_n)\rangle = g_S(q^2)\bar{u}_p(p_p)u_n(p_n), \qquad (5)$$

$$\langle p(p_p)|\bar{u}\gamma_5 d|n(p_n)\rangle = g_P(q^2)\bar{u}_p(p_p)\gamma_5 u_n(p_n), \qquad (6)$$

whereas a tensor interaction introduces an additional matrix element not relevant for our discussion [13]. Equations (5)–(6) show how the size of the (pseudo)scalar charges modulates the sensitivity of β decay experiments to nonstandard (pseudo)scalar interactions.

Relation between charges and the isospin breaking contribution to the nucleon mass.—Using the CVC result of Eq. (1) in combination with the above-defined form factors it is straightforward to derive

$$g_S(q^2) = \frac{\delta M_N^{\rm QCD}}{\delta m_q}g_V(q^2) + \frac{q^2/2\bar{M}_N}{\delta m_q}\tilde{g}_S(q^2), \qquad (7)$$

where $\delta m_q = m_d - m_u$. Note that the contribution due to electromagnetic effects $\delta M_N^{\rm QED}$ is of the same order of magnitude as $\delta M_N^{\rm QCD}$, and so the experimental value cannot be used. Indeed the inclusion of QED in the analysis would modify the CVC relation given in Eq. (1), introducing a correction proportional to $\alpha_{\rm e.m.}$.

In the limit $q^2 \to 0$ the expression (7) reduces to

$$g_S = \frac{\delta M_N^{\rm QCD}}{\delta m_q}, \qquad (8)$$

up to second order isospin-breaking corrections. Notice that the renormalization-scale and scheme dependence of g_S and the scalar Wilson coefficient ϵ_S [defined precisely in Eq. (14)] is the opposite [13,32], rendering the observable quantity $\epsilon_S g_S$ scale independent. Throughout this Letter we use the modified minimal subtraction scheme at $\mu = 2$ GeV for both the (pseudo)scalar charges and the light quark masses.

Likewise, using PCAC in Eq. (2) one obtains

$$g_P(q^2) = \frac{\bar{M}_N}{\bar{m}_q}g_A(q^2) - \frac{q^2/2\bar{M}_N}{(2\bar{m}_q)}\tilde{g}_P(q^2), \qquad (9)$$

with $\bar{m}_q$ the average light-quark mass and where we have dropped the "QCD" subindex in the average nucleon mass, since, in this case, all isospin breaking contributions represent small corrections and we can just use the experimental value of the nucleon masses. At zero momentum transfer $q^2 \to 0$ this expression reduces to

$$g_P = \frac{\bar{M}_N}{\bar{m}_q}g_A. \qquad (10)$$

Before discussing the phenomenological applications of these relations, let us mention that an isospin-rotated version of them has been discussed previously in the context of electric dipole moments [33–35] and β decays [36]. Notice that the derivation followed in this work does not rely on the use of the isospin symmetry.

Numerical analysis.—The determination of the scalar charge through the above-derived relation requires the knowledge of the light-quark mass difference δm_q, and its contribution to the nucleon mass splitting $\delta M_N^{\rm QCD}$. Interestingly enough, these quantities and the understanding of the interplay between isospin breaking due to the quark masses and electromagnetic effects, have become a topic of very intensive research.

On one hand, the phenomenological determination of the QED contributions to $\delta M_N = M_n - M_p$ using the Cottingham's sum rule [37,38] has been recently updated finding $\delta M_N^{\rm QED} = -1.30(3)(47)$ MeV [17], which combined with the experimental value [30] implies the $\delta M_N^{\rm QCD}$ value shown in Table I. Moreover, the error is expected to be reduced in the future through measurements of the isovector magnetic polarizability of the nucleon [17,39,40].

On the other hand, LQCD collaborations are starting to implement isospin breaking effects [18–20] or even directly simulating QED together with QCD [21–27] (for a recent review see Ref. [42]). In addition to the determination by the NPLQCD Collaboration in 2006 [18], we consider four

TABLE I. Summary of results for $\delta M_N^{\mathrm{QCD}}$, with uncertainties shown as they were presented in the corresponding references. We also show the results obtained using Eq. (8) (CVC) with the FLAG value for δm_q [41] and the g_S calculations of LHPC and PNDME [15,16].

Type	Label	$\delta M_N^{\mathrm{QCD}}$ [MeV]
Pheno	WLCM [17]	2.59 (03)(47)
LQCD	NPLQCD [18]	2.26(57)(42) (10)
LQCD	QCDSF-UKQCD [19]	3.13(15)(16)(53)
LQCD	STY [20]	2.9(4)
LQCD	RM123 ($N_f = 2$) [25]	2.9(6)(2)
LQCD	BMW [26]	2.28(25)(7)(9)
LQCD + CVC	LHPC [16]	2.72(83)
LQCD + CVC	PNDME [15]	1.66(62)

new calculations reported in the last two years (see Table I and Fig. 1). We do not include the results obtained in quenched LQCD by Duncan *et al.* in their seminal work [22], or the determination by Blum *et al.* [23] due to the absence of an estimate of systematic errors.

The weighted average of these determinations, with their respective errors combined in quadrature, is

$$[\delta M_N^{\mathrm{QCD}}]_{av} = 2.58(18) \text{ MeV} \qquad (11)$$

with $\chi^2/\mathrm{DOF} = 0.64$. This average should be taken with care, since it comes from pioneering calculations in a rapidly developing field. The estimate of systematic errors is a very complicated issue and future lattice studies of isospin breaking effects are needed to confirm these first calculations. Similar caveats apply to the recent numerical evaluation of the Cottingham formula [17]. Nonetheless, the average reflects the good agreement between current determinations and we will take it as a reference number whose robustness should improve in the future.

The other ingredient needed to calculate the scalar charge is δm_q. From the light quark masses results by FLAG [41] and the PDG [30], we obtain
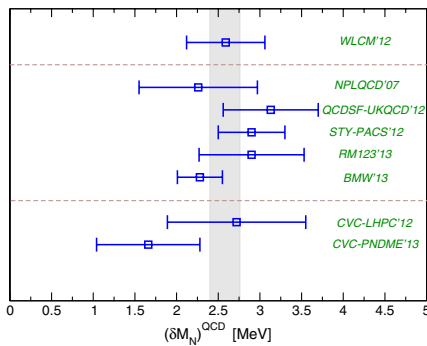

FIG. 1 (color online). Representation of the $\delta M_N^{\mathrm{QCD}}$ results summarized in Table I, along with our average (gray shaded band).

$\delta m_q = 2.52(19)$ MeV and $2.55(25)$ MeV respectively. Combining the FLAG result with the above-given average for $\delta M_N^{\mathrm{QCD}}$, the CVC relation given in Eq. (8) yields

$$g_S = 1.02(8)_{\delta m_q}(7)_{\delta M_N} = 1.02(11). \qquad (12)$$

It is worthwhile stressing that this result has been obtained ignoring possible correlations between the numerator and denominator of Eq. (8), and between the m_u and m_d determinations. These assumptions would be unnecessary in a direct calculation of the ratio $\delta M_N^{\mathrm{QCD}}/\delta m_q$, which should be fairly simple to implement in future LQCD analyses.

This determination of g_S is significantly more precise than direct LQCD calculations available in the literature. The LHPC finds $g_S = 1.08(32)$ [16], whereas the PNDME Collaboration has recently published the result $g_S = 0.66(24)$ [15], which supersedes their original preliminary estimate $g_S = 0.8(4)$ [13].

Inversely, using again Eq. (8) these calculations provide independent determinations of $\delta M_N^{\mathrm{QCD}}$, as shown in Table I and Fig. 1. We see that these results are starting to have an accuracy close to the direct calculations of $\delta M_N^{\mathrm{QCD}}$ and that the PNDME determination marginally disagrees with the average in Eq. (11).

Likewise, the application of PCAC through Eq. (10) yields the following result for the pseudoscalar charge:

$$g_P = 349(9), \qquad (13)$$

where the error is entirely dominated by the error in $\bar{m}_q = 3.42(9)$ MeV, ($N_f = 2 + 1$ FLAG average [41]). Notice the large enhancement experienced by this form factor, due to a charged pion pole in the coupling of a pseudoscalar field to the du vertex in QCD at low energies. In fact, this result is equivalent to the Goldberger-Treiman relation in which the pseudoscalar current serves as an interpolator of the pion field and $g_P(q^2)$ is expressed as a function with a pole at $q^2 = M_\pi^2$, whose residue is defined as the strong pion-nucleon coupling [8,9].

Implications for new-physics searches in β decays.— Given the V-A structure of the weak interaction, the (pseudo)scalar hadronic matrix elements of Eqs. (5)–(6) do not appear in the SM description of β decays. However the contribution due to new physics, like the coupling of a heavy charged scalar to first generation fermions, would require the calculation of these matrix elements. Such nonstandard interactions can be described by a low-energy effective Lagrangian for $d \to ue\nu$ transitions [43,44], where the scalar and pseudoscalar interactions are described by

$$\mathcal{L}_{d\to ue\nu} = \mathcal{L}_{d\to ue\nu}^{SM} - \frac{G_F V_{ud}}{\sqrt{2}} [\epsilon_S \bar{e}(1-\gamma_5)\nu_e \cdot \bar{u}d$$
$$- \epsilon_P \bar{e}(1-\gamma_5)\nu_e \cdot \bar{u}\gamma_5 d] + \text{H.c.} \qquad (14)$$

Here ν_e, e, u, d denote the electron neutrino, electron, and up- and down-quark mass eigenfields, whereas $\epsilon_{S,P}$ are the Wilson coefficients generated by some unspecified nonstandard dynamics. Moreover, and for the sake of simplicity, we will assume in this work that the Wilson coefficients $\epsilon_{S,P}$ are real, corresponding to CP-conserving interactions.

Scalar interaction—The most stringent limits on nonstandard scalar interactions arise from the contribution of the Fierz interference term to the $\mathcal{F}t$ values of superallowed pure Fermi transitions [45], namely. $b_F = -2g_S\epsilon_S = -0.0022(43)$ (at 90% C.L.). Alternative bounds on scalar interactions can be obtained from the measurement of the $\beta\nu$ angular correlation a in pure Fermi transitions. Although several on-going and planned experiments will improve the current measurements of a, it seems unlikely that they will be able to improve the above-given bound in the near future [46]. On the other hand, the Fierz term in neutron β decay is also sensitive to scalar interactions, although the level of precision required to compete with the bounds from nuclear decays looks also quite challenging, at least for the current generation of experiments [13].

Given the experimental value of b_F and the determination of the scalar charge derived in the previous section, we can determine the current bound on ϵ_S from β decays. Following Refs. [13,44,46] we calculate the confidence interval on ϵ_S using the R-fit method [47], which treats all values inside $0.91 \leq g_S \leq 1.13$ [from Eq. (12)] on an equal footing, whereas values outside the interval are not permitted. Note that the bound on ϵ_S depends only on the lower limit of the scalar form factor, as long as b_F is compatible with zero at 1σ.

In this way we obtain the following limit on CP-conserving scalar interactions

$$\epsilon_S = 0.0012(23), \qquad (90\% \text{ C.L.}), \qquad (15)$$

which, as it is shown in Fig. 2, improves significantly the bound obtained in Ref. [13] using $g_S = 0.8(4)$. Figure 2 shows also the ϵ_S bound that we obtain using more recent LQCD calculations of g_S [15,16]. It is worth mentioning that if we abandoned the R-fit scheme and treated g_S as a normally distributed variable we would obtain $\epsilon_S = 0.0011(21)$ at 90% C.L., in good agreement with the R-fit result of Eq. (15).

The LHC searches can also be used to set bounds on $\epsilon_{S,P}$. This can be done in a model-independent way if the new degrees of freedom that produce the effective scalar interaction in β decays are too heavy to be produced on shell at the LHC, since in that case it is possible to study collider observables using a high-energy $SU(2)_L \times U(1)_Y$-invariant effective theory that can be connected to the low-energy effective theory of Eq. (14).

In Fig. 2 we show the most stringent bound on ϵ_S from LHC searches, obtained in Ref. [46] studying the channel
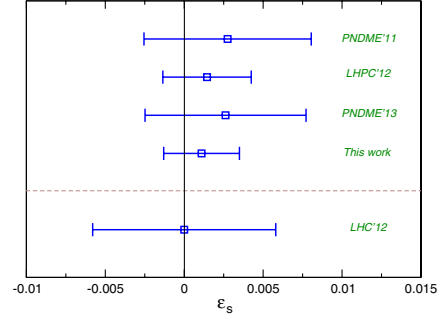

FIG. 2 (color online). Ninety percent C.L. bounds on ϵ_S from the measurement of b_F in superallowed pure Fermi transitions [45], using different values for the scalar charge g_S [13,15,16]. We show also the bound obtained from the analysis of LHC data carried out in Ref. [46].

$pp \to e + \text{MET} + X$, where MET stands for missing transverse energy. More specifically, a CMS search with 20 fb^{-1} of data recorded at $\sqrt{s} = 8$ TeV [48], was used to obtain $|\epsilon_{S,P}| < 5.8 \times 10^{-3}$ at 90% C.L.

Pseudoscalar interaction—In the study of the effect of nonstandard interactions in nuclear and neutron β decays, it is common lore that the pseudoscalar terms can be safely neglected in the analysis because the associated hadronic bilinear $\bar{u}_p\gamma_5 u_n$ is of order $q/\bar{M}_N$, which represents a suppression of order $\sim 10^3$ [49]. However, we showed in the previous section how the application of PCAC yields $g_P = 348(11)$, reducing considerably the suppression from the pseudoscalar bilinear. This result means that, modulo numerical factors of order 1, β decays with a nonzero Gamow-Teller component are as sensitive to pseudoscalar interactions as they are to scalar and tensor couplings.

As a representative example we show here the leading contribution of a nonzero pseudoscalar interaction to the electron energy spectrum in the β decay of an unpolarized neutron

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2 |V_{ud}|^2 (1+3\lambda^2)}{2\pi^3} p_e E_e (E_0 - E_e)^2 (1 + \delta_P), \quad (16)$$

where E_e and p_e denote the electron energy and the modulus of the three-momentum, $E_0 = \delta M_N - (\delta M_N^2 - m_e^2)/(2M_n)$ is the electron endpoint energy, and m_e is the electron mass. (For subleading SM effects see Refs. [13,50]). The nonstandard contribution δ_P coming from a nonzero effective coupling ϵ_P is given by

$$\delta_P = -\frac{\lambda}{1+3\lambda^2} g_P \epsilon_P \frac{E_0 - E_e}{M_n} \frac{m_e}{E_e}, \qquad (17)$$

where the factor $(E_0 - E_e)/M_n$ represents the above-discussed suppression from the pseudoscalar bilinear. For comparison we show now the (well-known) correction stemming from a scalar coupling

$$\delta_S = \frac{2}{1+3\lambda^2} g_S \epsilon_S \frac{m_e}{E_e}. \qquad (18)$$

In the best case $E_e = m_e$ we have $\delta_P \approx -0.06\epsilon_P$ and $\delta_S \approx 0.36\epsilon_S$, and the pseudoscalar contribution is only a factor 6 smaller than the scalar one.

This is certainly an interesting result that deserves more detailed studies, in particular related to the sensitivity of current and future β-decay measurements to ϵ_P. We hasten to add that the this coupling is very strongly constrained by the helicity-suppressed ratio $R_\pi \equiv \Gamma(\pi \to e\nu[\gamma])/\Gamma(\pi \to \mu\nu[\gamma])$ [51–53]. It should be noticed, however, that there are some possible loopholes in the bound from leptonic pion decays, like the cancellation of effects between the electron and muon channel in R_π due to an ϵ_P coupling proportional to the lepton masses, or cancellations between linear and quadratic terms originated from flavor non-diagonal contributions or interactions with right-handed neutrinos [11,13,54].

Conclusions.—In summary, we have discussed the application of the CVC and PCAC relations of QCD to connect different form factors describing β decays in the SM and beyond.

On one hand, CVC relates the scalar charge g_S to the vector charge $g_V \approx 1$, the isospin breaking of the nucleon mass in pure QCD, and the quark mass splitting δm_q. Using a set of recent phenomenological and LQCD determinations of these quantities we found a value for g_S with an uncertainty much smaller than the one reported by direct LQCD calculations of this form factor, cf. Eq. (12). In turn, we discussed the consequences of this novel determination to the bounds set on nonstandard scalar interactions from β decays, finding the limit on ϵ_S given in Eq. (15), which is much stronger than in previous analyses of β decays or than those currently obtained from the LHC.

On the other hand, PCAC relates the pseudoscalar charge g_P to the axial-vector one g_A, and the sums of the nucleon and quark masses. We found that g_P is enhanced by the pion pole, counterbalancing the suppression from the associated pseudoscalar quark bilinear. This finding opens the possibility of using nuclear and neutron β decay experiments to study new-physics scenarios with effective pseudoscalar couplings that are unconstrained by leptonic pion decays.

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