Complete Next-to-Leading-Order Study on the Yield and Polarization of $\Upsilon(1S, 2S, 3S)$ at the Tevatron and LHC

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Based on the nonrelativistic QCD factorization scheme, we present the first complete next-to-leading-order study on the yield and polarization of $\Upsilon(1S, 2S, 3S)$ hadroproduction. By using the color-octet long-distance matrix elements obtained from fits of the experimental measurements on Υ yield and polarization at the Tevatron and LHC, our results can explain the measurements on the yield very well, and for the polarizations of $\Upsilon(1S, 2S, 3S)$, they are in (good, good, bad) agreement with recent CMS measurement, but still have some distance from the CDF measurement.

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The well-known J/ψ polarization puzzle became obvious when the CDF measurement at the Tevatron [1] was found completely different from the leading-order (LO) theoretical prediction in the framework of nonrelativistic QCD (NRQCD) [2], which was proposed as a factorization approach on heavy quarkonium decay and production [3]. Even with the progress in the next-to-leading-order (NLO) QCD calculation, theoretical studies [4–6] on J/ψ polarization at NLO could not clearly clarify the situation. Early measurements [7,8] on the polarization of Υ at the Tevatron are also in conflict with the corresponding LO NRQCD prediction [9]. Recently, a very important and interesting measurement on the polarization of $\Upsilon(1S, 2S, 3S)$ at the LHC was reported by the CMS Collaboration [10], which employed improved consideration in the measurement [11]. Since the bottom quark is almost three times as heavy as the charm quark, the NRQCD α_s and velocity expansions are of better convergence and the theoretical predictions at QCD NLO are more reliable for Υ than that for J/ψ. Therefore, it is very important to extend the theoretical predication on Υ at QCD NLO to solve or clarify the long-standing polarization puzzle when there are already measurements at the LHC.

In the last six years, there has been some very important progress in the NLO QCD correction calculation. The NLO corrections to color-singlet (CS) J/ψ hadroproduction have been investigated in Refs. [12,13], where its transverse momentum (p_t) distribution is found to be enhanced by 2–3 orders of magnitude at the high p_t region and its polarization changes from transverse to longitudinal at NLO [13]. The results are reproduced at p_t LO in a new factorization scheme for large p_t quarkonium production [14]. The NLO corrections to J/ψ production via S-wave color-octet (CO) states (${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$) are studied in Ref. [15] and the corrections to the p_t distributions of both the J/ψ yield and polarization are small. In Ref. [16], NLO corrections for χ_{cJ} hadroproduction are studied. The complete NLO calculation for prompt J/ψ hadroproduction (with ${}^3P_J^{[8]}$ included) was given by two groups [17,18], and their predictions for p_t distributions agree with the experimental measurements at the Tevatron and LHC. The calculation for polarization of direct J/ψ hadroproduction at NLO QCD was presented by two groups [4,5]. The complete NLO calculation of the polarization for prompt J/ψ hadroproduction was completed by our group [6] last year. It is known that at the large p_t region the logarithm term $\ln(p_t/m_c)$ may ruin fixed order perturbative expansion and it is resummed in the new factorization scheme mentioned above [14]. But it is unclear how large the p_t region is where fixed order calculation works well for the J/ψ case.

For Υ hadroproduction, there are studies on the p_t distribution of yield and polarization for the CS channel at QCD NLO [12,13,19] and at the partial next-to-next-to-leading order [20]. NLO QCD correction to the p_t distribution of the yield and polarization for $\Upsilon(1S, 3S)$ via S-wave CO states is presented in Ref. [21], and NLO QCD correction to the p_t distribution of the yield for $\Upsilon(1S)$ via all the CO states (including ${}^3P_J^{[8]}$) is presented in Ref. [22]. The complete NLO study on polarization of Υ hadroproduction has not yet been achieved since there is more complicated feed-down than in the charmonium case. However, the advantages for studying Υ are also obvious. Since the bottom quark is almost 3 times as heavy as the charm quark, both the QCD coupling constant $\alpha_s(\sqrt{4m_Q^2 + p_t^2})$ and v^2 (v is the velocity of the heavy quark in the meson rest frame) are smaller, and the perturbative calculation is of better convergence in the double expansion of α_s and v^2 on bottomonium than that on charmonium. Furthermore, it is known that fixed order calculation should be good enough at the intermediate p_t region although the logarithm term $\ln(p_t/m_Q)$ needs to be resummed at the large p_t region; hence, the fixed order prediction on Υ hadroproduction will be very good for p_t up to 60 GeV if that on J/ψ is very good for p_t up to 20 GeV, where 20 GeV is a very conservative estimate.

In other words, it is expected that the theoretical predictions on the polarization and yield of Υ at QCD NLO should be in better agreement with experimental measurement up to large p_t than that of charmonium. Therefore, a full study on the polarization and yield on Υ at QCD NLO is a very interesting and important task to fix the heavy quarkonium polarization puzzle while there are already polarization measurement on $\Upsilon(1S, 2S, 3S)$ by the CMS Collaboration. In this Letter, we present the first complete NLO study on the polarization and yield of $\Upsilon(1S, 2S, 3S)$ based on the NRQCD factorization scheme.

According to the NRQCD factorization formalism, the cross section for hadroproduction of H is expressed as

$$d\sigma[pp \to H + X] = \sum_{i,j,n} \int dx_1 dx_2 G_p^i G_p^j \times \hat{\sigma}[ij \to (b\bar{b})_n X]\langle \mathcal{O}_n^H \rangle, \quad (1)$$

where p is either a proton or an antiproton, the indices i, j run over all the partonic species, and n denotes the color, spin, and angular momentum states of the intermediate $b\bar{b}$ pair. It can be ${}^3S_1^{[1]}$, ${}^3S_1^{[8]}$, ${}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$ for Υ, or ${}^3P_J^{[1]}$ and ${}^3S_1^{[8]}$ for χ_{bJ}. The short-distance contribution $\hat{\sigma}$ can be calculated perturbatively, while the long-distance matrix elements (LDMEs) $\langle \mathcal{O}_n^H \rangle$ are fully governed by nonperturbative QCD effects.

The polarization of Υ is described by three parameters, as defined in Ref. [23]:

$$\lambda = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \mu = \frac{\sqrt{2}\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \nu = \frac{2d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}},$$

where $d\sigma_{S_z S_z'}$ is the spin density matrix of Υ hadroproduction. In this work, we focus on the polarization parameter λ in the helicity frame only.

To obtain $d\sigma_{S_z S_z'}$, similar treatment as in Ref. [6] is taken for both direct and feed-down contributions. There are various feed-down contributions in Υ production, while some of them are ignored in our calculation as they are thought to be small. The feed-down contributions included in this work are (1) for $\Upsilon(3S)$, no feed-down contribution is included; (2) for $\Upsilon(2S)$, feed-down contributions from $\Upsilon(3S)$ and $\chi_{bJ}(2P)$ are included; (3) for $\Upsilon(1S)$, feed-down contributions from $\Upsilon(2S, 3S)$ and $\chi_{bJ}(1P, 2P)$ are included.

The newly updated Feynman diagram calculation package [24] is used in our calculation.

In our numerical calculation, the CTEQ6M parton distribution functions [25] and corresponding two-loop QCD coupling constant α_s are used. Branching ratios and masses involving bottomonia can be found in Table I. The mass of the bottom quark is set to $m_b = M_H/2$ as an approximation, while M_H is the mass of the bottomonium H. The CS LDMEs are estimated by using a potential model result [27], which gives $|R_{\Upsilon(1S,2S,3S)}(0)|^2 = 6.477$, 3.234, 2.474 GeV^3, and $|R'_{\chi_b(1P,2P)}(0)|^2 = 1.417$, 1.653 GeV^5, respectively. The renormalization and factorization scales are chosen as $\mu_r = \mu_f = m_T$, with $m_T = \sqrt{(2m_b)^2 + p_t^2}$, while the NRQCD scale is chosen as $\mu_\Lambda = m_b v \approx 1.5$ GeV. The center-of-mass energy is 1.8 and 1.96 TeV for Tevatron run I and run II and 7 TeV for the LHC, respectively. Various rapidity cuts are chosen according to various experimental data, including both the central and the forward rapidity regions. Additionally, a shift $p_t^H \approx p_t^{H'} \times (M_H/M_{H'})$ is used while considering the kinematics effect in the feed-down from higher excited states.

In Ref. [22], feed-down contributions from the ${}^3S_1^{[8]}$ channel of χ_{bJ} are included in the corresponding CO LDMEs of Υ. But when studying the polarization, we have to separate the ${}^3S_1^{[8]}$ channels of Υ and χ_{bJ}, as they have different behavior in polarization. Unfortunately, there is still no experimental data for χ_{bJ} hadroproduction, so there is no direct clue to determine the CO LDMEs of χ_{bJ} production. Thus we take them as extra variables in our fit, and include experimental data for Υ polarization as well. It is known that the double expansion in α_s and v^2 is not good enough in the small p_t regions. Therefore, only data in the region $p_t > 8$ GeV are used in our fit.

In the fit, we have used the experimental data for the p_t distribution of the differential cross section by CDF [7], LHCb [28], CMS [29], and ATLAS [30], and of the polarization by CDF [31] (where early measurements from CDF [7] and D0 [8] conflict with new CDF measurements [31] and are ignored) and CMS [10]. Three fits are performed for $\Upsilon(3S, 2S, 1S)$ hadroproduction step by step. In order to express the uncertainty from the CO LDMEs in theoretical predictions correctly, a covariance-matrix method is performed as in Ref. [6]. Note the following.

(1) $\chi^2/\mathrm{d.o.f.} = 117/37$ is obtained in the fit of $\Upsilon(3S)$.

(2) $\chi^2/\mathrm{d.o.f.} = 88/37$ is obtained in the fit of $\Upsilon(2S)$ with four CO LDMEs, one of which is $\langle \mathcal{O}^{\chi_{b0}(2P)}({}^3S_1^{[8]})\rangle$. $\chi_{bJ}(2P)$ feed-down contributes a fraction of about 35%–76% as p_t increases in $\Upsilon(2S)$ hadroproduction.

TABLE I. Branching ratios and masses of bottomonia are taken from PDG [26].

H	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\chi_{b0}(1P)$	$\chi_{b1}(1P)$	$\chi_{b2}(1P)$	$\chi_{b0}(2P)$	$\chi_{b1}(2P)$	$\chi_{b2}(2P)$
$\mathcal{B}(H \to \mu\mu)$ (%)	2.48	1.93	2.18	⋯	⋯	⋯	⋯	⋯	⋯
$\mathcal{B}[H \to \Upsilon(1S)]$(%)	⋯	26.5	6.6	1.76	33.9	19.1	0.9	10.8	8.1
$\mathcal{B}[H \to \Upsilon(2S)]$ (%)	⋯	⋯	10.6	⋯	⋯	⋯	4.6	19.9	10.6
M_H (GeV)	9.5	10.023	10.355	9.859	9.893	9.912	10.23	10.255	10.269

TABLE II. The obtained CO LDMEs for bottomonia production (in units of 10^{-2} GeV^3).

H	$\langle \mathcal{O}^H({}^1S_0^{[8]})\rangle$	$\langle \mathcal{O}^H({}^3S_1^{[8]})\rangle$	$\langle \mathcal{O}^H({}^3P_0^{[8]})\rangle/m_b^2$
$\Upsilon(1S)$	11.15 ± 0.43	-0.41 ± 0.24	-0.67 ± 0.00
$\Upsilon(2S)$	3.55 ± 2.12	0.30 ± 0.78	-0.56 ± 0.48
$\Upsilon(3S)$	-1.07 ± 1.07	2.71 ± 0.13	0.39 ± 0.23
$\chi_{b0}(2P)$	⋯	2.76 ± 0.67	⋯
$\chi_{b0}(1P)$	⋯	1.27 ± 0.16	⋯

(3) In the fit of $\Upsilon(1S)$, we have also included data for the fraction of $\Upsilon(1S)$ from $\chi_{bJ}(1P)$ feed-down by LHCb [32] and obtained the four CO LDMEs with $\chi^2/\text{d.o.f.} = 107/63$. Furthermore, the obtained $\chi_b(1P, 2P)$ fraction in $\Upsilon(1S)$ is consistent with the CDF measurement [33].

All of the fitted CO LDMEs can be found in Table II. With these LDMEs, our predictions for the differential cross section of Υ hadroproduction are shown in Fig. 1, while those for the polarization are shown in Fig. 2. The uncertainty bands in the figures come from errors of the LDMEs. Our calculations show that there is about 25%–10%(35%–20%) uncertainty for the differential cross section with the factorization (renormalization) scale changing as 0.5–$2m_T$, and it decreses as p_t increases. And it is smaller for the polarization distribution. We do not include these uncertainties in our fitting and final plots since it requires much more computing time resources.

From the figures we see that the predictions on the yield of Υ hadroproduction can explain the experimental data very well with very small uncertainty in a wide range of p_t at the LHC and Tevatron, while for the polarization, things are quite different. For $\Upsilon(3S)$, the production is dominated by the ${}^3S_1^{[8]}$ channel, which results in a transverse polarization in the high p_t region and makes the theoretical predictions far, far away from the experimental data as p_t increases. And it is obvious that the polarization cannot be explained at LO in v^2 and NLO in α_s if unknown feed-down contribution from higher excited bottomonia is negligible. For $\Upsilon(1S, 2S)$, the predictions for polarizations can explain the CMS data well, but there is still some distance from the measurement by CDF. From the measurement at the LHC, it is easy to see that the p_t distribution for $\Upsilon(1S)$ is of steepest slope, and that for $\Upsilon(2S)$ is of steeper slope than that for $\Upsilon(3S)$. For the CO contribution, we find that the p_t distribution of ${}^1S_0^{[8]}$ is the steepest one and that of

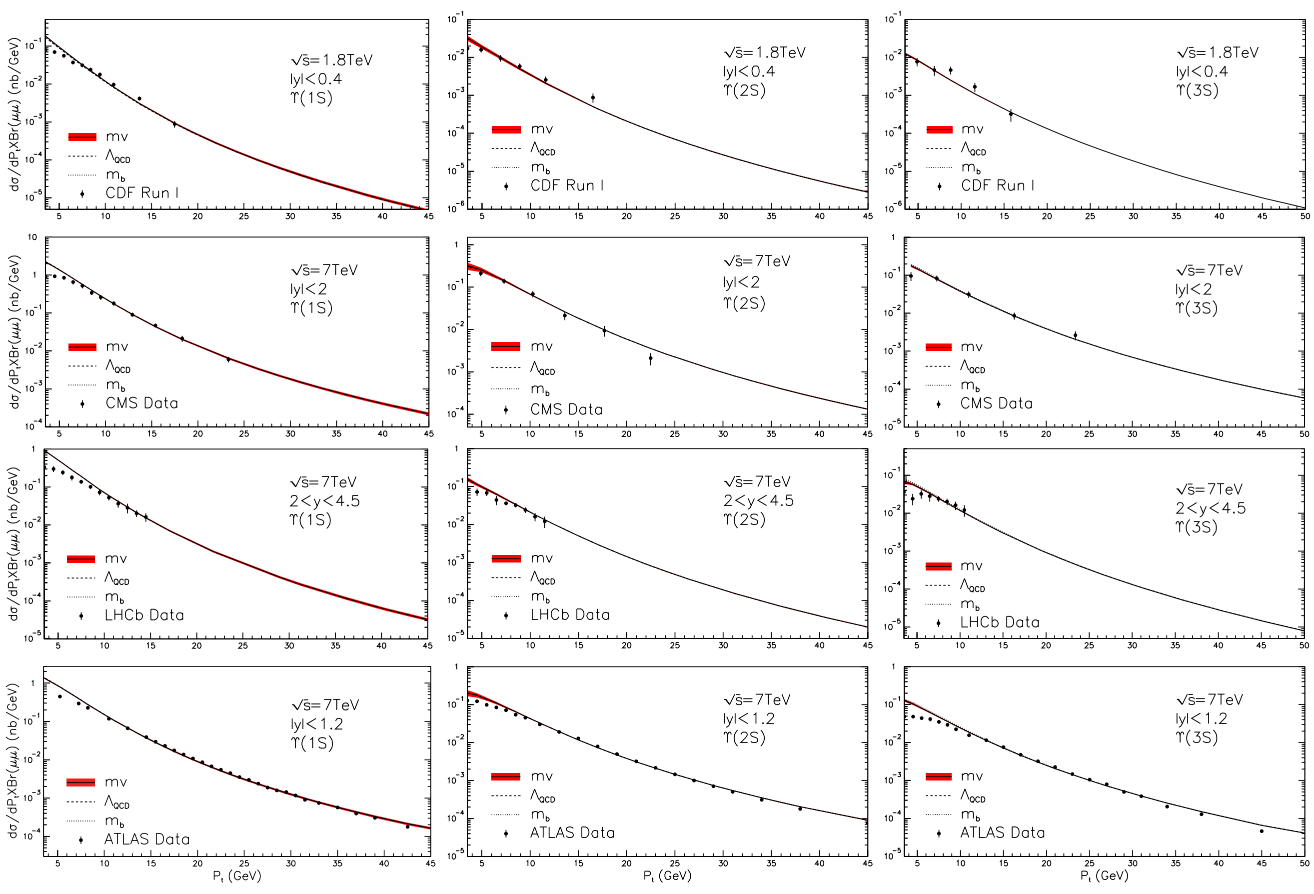

FIG. 1 (color online). Differential cross section for Υ hadroproduction at the Tevatron and LHC. From left to right: $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. Rows from top to bottom correspond to different experimental conditions of CDF run I, CMS, LHCb, and ATLAS. The experimental data are taken from Refs. [7,28–30].

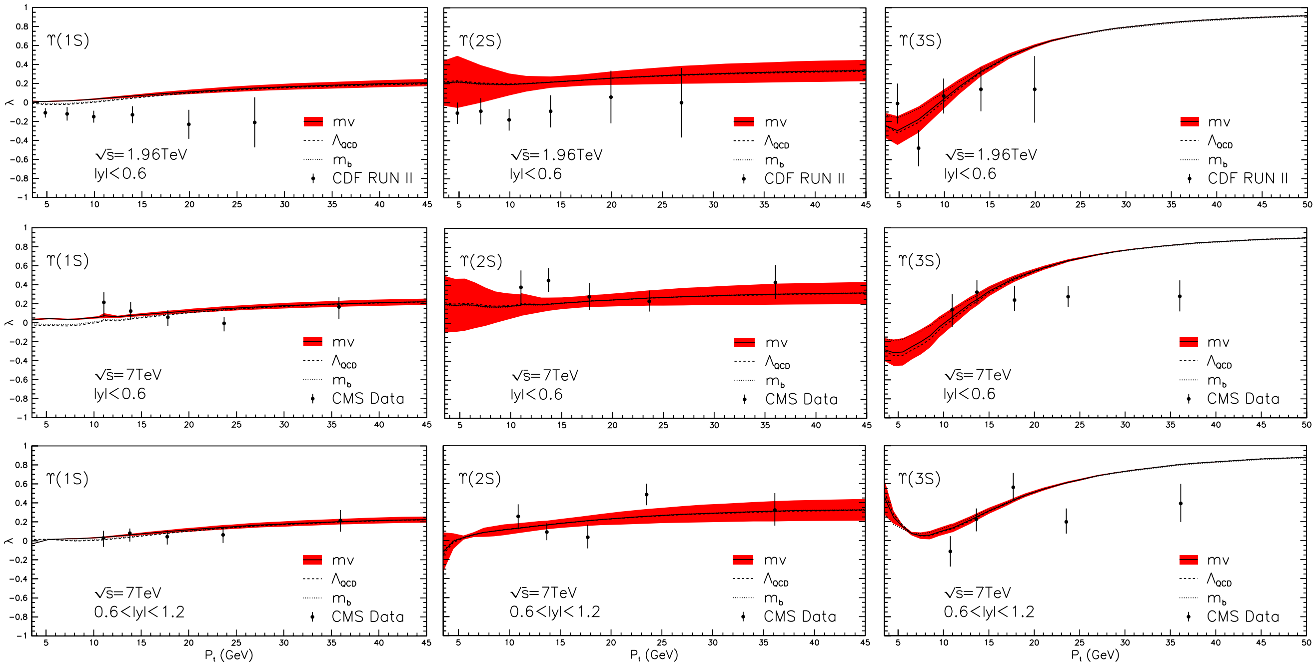

FIG. 2 (color online). Polarization parameter λ for Υ hadroproduction at the Tevatron and LHC. From left to right: $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. Rows from top to bottom correspond to different experimental conditions of CDF run II, CMS ($|y| < 0.6$), and CMS ($0.6 < |y| < 1.2$). The CMS and CDF data are taken from Refs. [10,31].

${}^3S_1^{[8]}$ is the flattest one, while that of ${}^3P_J^{[8]}$ is sensitive to the choice of the NRQCD factorization scale μ_Λ. From the numerical results, we obtained that the χ_{bJ} feed-down contribution in $\Upsilon(1S)$ becomes dominant as p_t increases, and polarization via this channel is slightly transverse polarized. Combined with the fact that the direct part is dominated by the ${}^1S_0^{[8]}$ channel at the small p_t region, we find that $\Upsilon(1S)$ is almost unpolarized at all p_t range. The situation for $\Upsilon(2S)$ is similar to $\Upsilon(1S)$, but with more χ_{bJ} feed-down contribution in the small p_t range. Therefore, the χ_{bJ} feed-down contribution is very important to explain the experimental measurement on polarization. Although the experimental measurement on the fraction of $\chi_{bJ}(1P)$ feed-down in $\Upsilon(1S)$ is already used in the fit, it is preliminary with large errors.

It is believed that final physical results are independent of the NRQCD factorization scale μ_Λ, but the dependence does exist when theoretical calculation is truncated at fixed order in the perturbative expansion. And this dependence can be found when the detailed arrangement of the NRQCD factorization formula is taken in the calculation with the P-wave intermediate state involved. So a better way to present the final results is to take the μ_Λ dependence into the consideration of uncertainty. In both figures, we have also shown the results with $\mu_\Lambda = m_b$ and $\mu_\Lambda = \Lambda_{\mathrm{QCD}}$. It is found that the μ_Λ dependence is quite small for the p_t distribution of the Υ yield and polarization due to the small contribution of $\Upsilon({}^3P_J^{[8]})$ and $\chi_{bJ}({}^3P_J^{[1]})$.

In summary, we present the first complete NLO study on the polarization and yield of $\Upsilon(1S, 2S, 3S)$ hadroproduction. Based on the calculation of the polarization and yield for both direct and feed-down contributions, 11 CO LDMEs are obtained by fitting the experimental data at the Tevatron and LHC step by step for $\Upsilon(3S, 2S, 1S)$. With different choices of the NRQCD factorization scale μ_Λ, we find that μ_Λ dependence is very small in the p_t distribution of the yield and polarization for Υ even though it could be quite large for J/ψ where the P-wave component contributions are very large. For the p_t distribution of the Υ yield, the experimental measurements at the Tevatron and LHC can be explained very well in a wide range of p_t. For $\Upsilon(3S)$, the polarization cannot be explained at LO in v^2 and NLO in α_s if unknown feed-down contribution from higher excited bottomonia is negligible. For $\Upsilon(1S, 2S)$, the predictions for polarization can explain the CMS data well, but still have some distance from the CDF data.

Further study needs to be considered. The relativistic corrections to J/ψ hadroproduction [34] are negative and large in the small p_t range, and this infers that the relativistic corrections to $\Upsilon(3S)$ are the largest among $\Upsilon(1S, 2S, 3S)$ and detailed study may change the result of the fit. The uncertainty from the poorly known fraction of χ_{bJ} feed-down in the fits for $\Upsilon(1S, 2S)$ could be large, which is not presented in the plots. With the feed-down contribution of $\chi_{bJ}(3P)$, the polarization of $\Upsilon(3S)$ may be explained as well. Therefore, a further precise measurement on the fraction of χ_{bJ} feed-down or on direct Υ production will be very helpful to fix the polarization puzzle.

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[1] T. Affolder *et al.* (CDF Collaboration), Phys. Rev. Lett. **85**, 2886 (2000).
[2] M. Beneke and I. Z. Rothstein, Phys. Lett. B **372**, 157 (1996).
[3] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995).
[4] M. Butenschoen and B. A. Kniehl, Phys. Rev. Lett. **108**, 172002 (2012).
[5] K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, and Y.-J. Zhang, Phys. Rev. Lett. **108**, 242004 (2012).
[6] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, Phys. Rev. Lett. **110**, 042002 (2013).
[7] D. E. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **88**, 161802 (2002).
[8] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **101**, 182004 (2008).
[9] E. Braaten and J. Lee, Phys. Rev. D **63**, 071501 (2001).
[10] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **110**, 081802 (2013).
[11] P. Faccioli, C. Lourenco, and J. Seixas, Phys. Rev. Lett. **105**, 061601 (2010); P. Faccioli, C. Lourenco, J. Seixas, and H. K. Wohri, Phys. Rev. Lett. **102**, 151802 (2009).
[12] J. Campbell, F. Maltoni, and F. Tramontano, Phys. Rev. Lett. **98**, 252002 (2007).
[13] B. Gong and J.-X. Wang, Phys. Rev. Lett. **100**, 232001 (2008).
[14] Z.-B. Kang, J.-W. Qiu, and G. Sterman, Phys. Rev. Lett. **108**, 102002 (2012).
[15] B. Gong, X. Q. Li, and J.-X. Wang, Phys. Lett. B **673**, 197 (2009).
[16] Y.-Q. Ma, K. Wang, and K.-T. Chao, Phys. Rev. D **83**, 111503 (2011).
[17] Y.-Q. Ma, K. Wang, and K.-T. Chao, Phys. Rev. Lett. **106**, 042002 (2011).
[18] M. Butenschön and B. A. Kniehl, Phys. Rev. Lett. **106**, 022003 (2011).
[19] B. Gong and J.-X. Wang, Phys. Rev. D **78**, 074011 (2008).
[20] P. Artoisenet, J. M. Campbell, J. P. Lansberg, F. Maltoni, and F. Tramontano, Phys. Rev. Lett. **101**, 152001 (2008).
[21] B. Gong, J.-X. Wang, and H.-F. Zhang, Phys. Rev. D **83**, 114021 (2011).
[22] K. Wang, Y.-Q. Ma, and K.-T. Chao, Phys. Rev. D **85**, 114003 (2012).
[23] M. Beneke, M. Kramer, and M. Vanttinen, Phys. Rev. D **57**, 4258 (1998).
[24] J.-X. Wang, Nucl. Instrum. Methods Phys. Res., Sect. A **534**, 241 (2004).
[25] J. Pumplin, D. R. Stump, J. Huston, H.-L. Lai, P. Nadolsky, and W.-K. Tung, J. High Energy Phys. 07 (2002) 012.
[26] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[27] E. J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).
[28] R. Aaij *et al.* (LHCb Collaboration), Eur. Phys. J. C **72**, 2025 (2012).
[29] V. Khachatryan *et al.* (CMS Collaboration), Phys. Rev. D **83**, 112004 (2011).
[30] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. D **87**, 052004 (2013).
[31] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **108**, 151802 (2012).
[32] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 11 (2012) 031.
[33] T. Affolder *et al.* (CDF Collaboration), Phys. Rev. Lett. **84**, 2094 (2000).
[34] G.-Z. Xu, Y.-J. Li, K.-Y. Liu, and Y.-J. Zhang, Phys. Rev. D **86**, 094017 (2012).