Neutrinoless Double-β Decay and QCD Corrections

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We consider one-loop QCD corrections and renormalization group running of the neutrinoless double-β decay amplitude focusing on the short-range part of the amplitude (without the light neutrino exchange) and find that these corrections can be sizeable. Depending on the operator under consideration, there can be moderate to large cancellations or significant enhancements. We discuss several specific examples in this context. Such large corrections will lead to significant shifts in the half-life estimates, which currently are known to be plagued with the uncertainties due to nuclear physics inputs to the physical matrix elements.

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It is now experimentally well established that neutrinos have mass and they mix with each other (see Ref. [1] for the best fit values of the parameters). Being electrically neutral allows the possibility of them to be Majorana particles [2]. The observation of neutrinoless double-β ($0\nu 2\beta$) decay $(A, Z) \to (A, Z+2) + 2e^-$ will establish the Majorana nature and lepton number violation beyond any doubt [3]. Therefore, the search for neutrinoless double-β decay continues to be an important area. Theoretically as well, $0\nu 2\beta$ decay is heralded as a useful probe of physics beyond the standard model (SM); $0\nu 2\beta$ can potentially discriminate between the two hierarchies of the neutrino masses, and this, in turn, can be used to rule out specific models of neutrino mass generation. In the context of models which involve TeV-scale particles, like low-scale seesaw models or low energy supersymmetric models including models with R-parity violation, $0\nu 2\beta$ imposes stringent constraints on the model parameters. The same set of diagrams with appropriate changes in the momentum flow can lead to interesting signatures at the LHC. Constraints from $0\nu 2\beta$, thus, can prove rather useful for phenomenological studies (see, e.g., Ref. [4] for an incomplete list discussing various aspects).

The $0\nu 2\beta$ decay amplitude can be split into the so-called long-range and short-range parts (for a review of the theoretical and experimental issues and the sources of uncertainties and errors, see Ref. [5] and references therein). Here, the long range refers to the fact that there is an intermediate light neutrino involved. This should be contrasted with the short-range part of the amplitude in which the intermediate particles are all much heavier than the relevant scale of the process $\sim O$ (GeV). In such a case, the heavier degrees of freedom can be systematically integrated out leaving behind a series of operators built out of low energy fields weighted by coefficients called Wilson coefficients (denoted by C_i below), which are functions of the parameters of the large mass degrees of freedom that have been integrated out (see, e.g., Ref. [6]). This provides a very convenient framework to evaluate the decay amplitude in terms of short-distance coefficients which encode all the information about the high energy physics one may be trying to probe via a low energy process. This also neatly separates the particle physics input from the nuclear physics part, which enters via the nuclear matrix elements (NMEs) of the quark level operators sandwiched between the nucleon states. In what follows, the discussion will be centered around the short-range part, though we believe that many of the arguments and results may also apply to the long-range part. More care may be needed in the latter case though.

Given a specific model, it is straightforward to write down the amplitude for the quark level $0\nu 2\beta$ process and compute the short-distance coefficient. The complete amplitude then involves NMEs. At present, the biggest source of uncertainty stems from the NMEs, and theoretical predictions show a marked sensitivity on the NMEs used (see Ref. [7] for some of the recent NME calculations and predictions for the $0\nu 2\beta$ rates). On the experimental side, studies have been carried out on several nuclei. Only one of the experiments, the Heidelberg-Moskow (HM) Collaboration [8] has claimed observation of a $0\nu 2\beta$ signal in ^{76}Ge. The half-life at 68% confidence level is $T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr. A combination of the KamLAND-Zen Collaboration [9] and EXO-200 Collaboration [10] results, both using ^{136}Xe, yields a lower limit on the half-life $T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 3.4 \times 10^{25}$ yr, which is at variance with the HM claim. Very recently, the GERDA Collaboration reported the lower limit on the half-life based on the first phase of the experiment [11]: $T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 2.1 \times 10^{25}$ yr. A combination of all the previous limits results in a lower limit $T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 3.0 \times 10^{25}$ yr at 90% confidence level. The new GERDA result (and the combination) is (are) again at odds with the positive claim of the HM Collaboration. The GERDA results have been challenged [12] on account of low statistics and poorer resolution. Very clearly, there is some tension among the experimental results, and higher statistics in the future will shed more light. To reduce the dependence (or sensitivity) on NMEs, predictions for $0\nu 2\beta$ for various nuclei can be compared. Further, it is necessary to establish if the long-range contribution coming from the light neutrino exchange can saturate the experimental limits (or positive claims). This is investigated in Ref. [13], and the conclusion drawn is that the light neutrino exchange falls short of saturating the current limits. Also, for some choices of NMEs, the ^{76}Ge positive result can be consistent with the ^{136}Xe limits when considered individually but not when combined.

In view of the immense importance of $0\nu2\beta$, both experimentally and theoretically, it is important to ensure that theoretical calculations are very precise. In this Letter, we consider dominant one-loop QCD corrections and renormalization group (RG) effects to the $0\nu2\beta$ amplitude. To the best of our knowledge, this has not been studied before, and as we show below, QCD corrections can have a significant impact on the $0\nu2\beta$ rate, thereby impacting the constraints on the model parameters.

We begin by recapitulating the essential steps in arriving at the final amplitude for $0\nu2\beta$. Using the Feynman rules for a given model, all possible terms can be easily written. Since the momentum flowing through any of the internal lines is far smaller than the masses of the respective particles and can be neglected, this leads to the low energy amplitude at the quark level. Parts of the amplitude may require Fierz rearrangement (for example, in supersymmetric theories) to express it in color singlet form, which can then be sandwiched between the nucleon states after taking the nonrelativistic limit. This last step results in NMEs. We shall not be concerned with the issue of uncertainties creeping in due to NME calculations here. We shall, rather, choose to work with a particular set of NMEs and focus on the impact of perturbative QCD corrections. As an example, consider a heavy right-handed neutrino and SM gauge group. The resulting amplitude is of the form

$$\begin{aligned}\mathcal{A} &\sim \frac{1}{M_W^4 M_N}\bar{u}\gamma_\mu(1-\gamma_5)d\bar{e}\gamma^\mu\gamma^\nu(1+\gamma_5)e^c\bar{u}\gamma_\nu(1-\gamma_5)d\\ &= \underbrace{\frac{1}{M_W^4 M_N}}_{G}\underbrace{\bar{u}\gamma_\mu(1-\gamma_5)d\bar{u}\gamma^\mu(1-\gamma_5)d}_{\mathcal{J}_{q,\mu}\mathcal{J}_q^\mu}\underbrace{\bar{e}(1+\gamma_5)e^c}_{j_l},\end{aligned} \tag{1}$$

where we used $\gamma_\mu\gamma_\nu = g_{\mu\nu} - 2i\sigma_{\mu\nu}$ and the fact that $\bar{e}\sigma_{\mu\nu}(1+\gamma_5)e^c$ vanishes identically. So does $\bar{e}\gamma_\mu e^c$. This was noted in Ref. [14]. These, thus, restrict the form of the leptonic current. G denotes the analogue of the Fermi constant. The exact form of G will be model dependent. The physical $0\nu2\beta$ amplitude is written as

$$\mathcal{A}_{0\nu2\beta} = \langle f|i\mathcal{H}_{\rm eff}|i\rangle \sim G\underbrace{\langle f|\mathcal{J}_{q,\mu}\mathcal{J}_q^\mu|i\rangle}_{\rm NME}j_l. \tag{2}$$

This clearly illustrates how the short-distance or high energy physics separates from the low energy matrix elements. The effective Hamiltonian for a given model is expressed as a sum

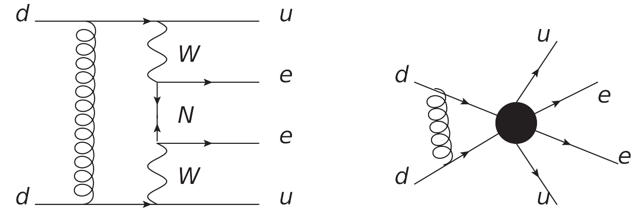

FIG. 1. Representative Feynman diagrams (drawn using the package JAXODRAW [18]) showing one-loop QCD corrections. Left: full theory. Right: effective theory.

of operators O_i weighted by the Wilson coefficients C_i: $\mathcal{H}_{\rm eff} = G_iC_iO_i$, where we have allowed for more than one G for more complicated theories. In the above case, there is only one operator $O_1 = \mathcal{J}_{q,\mu}\mathcal{J}_q^\mu j_l = \bar{u}_i\gamma_\mu(1-\gamma_5)d_i\bar{u}_j\gamma^\mu(1-\gamma_5)d_j\bar{e}(1+\gamma_5)e^c$ (i, j denoting the color indices) and the corresponding Wilson coefficient $C_1 = 1$. In other models like supersymmetry (SUSY) with R-parity violation [15] or leptoquarks [16], Fierz transformations have to be employed to bring the operators in the color matched form. The specific NME that finally enters the $0\nu2\beta$ rate depends on the Lorentz and Dirac structure of the quark level operator involved.

This is not the entire story. From the effective field theory point of view, the integrating out of the heavier degrees of freedom happens at the respective thresholds, and then the obtained effective Lagrangian or Hamiltonian has to be properly evolved down to the relevant physical scale of the problem [$\sim O$ (GeV) in the present case]. This is similar to what happens in nonleptonic meson decays (see, for example, Ref. [17]). For simplicity, we assume that the heavy particles are all around the electroweak (EW) scale, and in obtaining the numerical values, we shall put M_W as the scale for all. This facilitates one step integrating out of all the heavy degrees of freedom. Therefore, the above statement about C_1 being unity should now be written as $C_1(M_W) = 1$. Next, consider one-loop QCD corrections. The full amplitude is evaluated with one gluon exchange [$O(\alpha_s)$] and matched with the amplitude at the same order in α_s in the effective theory. Figure 1 shows representative diagrams in the full and effective theory. This has two effects: (i) C_1 gets corrected and reads $C_1(M_W) = 1 + (\alpha_s/4\pi)\mathcal{N}\ln(M_W^2/\mu_W^2)$, where μ_W is the renormalization scale and $\mathcal{N}$ is a calculable quantity. This coefficient is then evolved down to the O (GeV) using the RG equations; (ii) QCD corrections induce the color mismatched operator $O_2 = \bar{u}_i\gamma_\mu(1-\gamma_5)d_j\bar{u}_j\gamma^\mu(1-\gamma_5)d_i\bar{e}(1+\gamma_5)e^c$ with coefficient $C_2 = (\alpha_s/4\pi)\mathcal{N}'\ln(M_W^2/\mu_W^2)$. When evaluating the quark level matrix element in the effective theory, both the operators contribute and, in fact, lead to mixing. This approach is a consistent one and also reduces the scale dependence of the physical matrix elements. Without following the above steps, the short-distance coefficient would have been evaluated at the high scale while the physical matrix elements at a low scale, leading to large-scale dependence, which is not a physical effect but rather an artifact of the calculation.

Armed with this machinery, we now consider specific examples to bring out the impact of QCD corrections. As mentioned above, to simplify the discussion, we assume all the heavy particles beyond the SM to be around the EW scale. The technical details and explicit expressions for some of the models leading to neutrinoless double-β decay and related phenomenology will be presented elsewhere. Here we provide approximate numerical values of the Wilson coefficients of the operators considered. For the time being, we neglect the mixing of operators under renormalization. This can have a large impact on some of the coefficients, but their inclusion is beyond the scope of the present work.

First, we consider a left-right symmetric model and focus our attention on operators generated due to W_L and W_R exchange:

$$\begin{aligned} O_1^{LL} &= \bar{u}_i\gamma_\mu(1-\gamma_5)d_i\bar{u}_j\gamma^\mu(1-\gamma_5)d_j\bar{e}(1+\gamma_5)e^c, O_2^{LL} \\ &= \bar{u}_i\gamma_\mu(1-\gamma_5)d_j\bar{u}_j\gamma^\mu(1-\gamma_5)d_i\bar{e}(1+\gamma_5)e^c, O_1^{RR} \\ &= \bar{u}_i\gamma_\mu(1+\gamma_5)d_i\bar{u}_j\gamma^\mu(1+\gamma_5)d_j\bar{e}(1+\gamma_5)e^c, O_2^{RR} \\ &= \bar{u}_i\gamma_\mu(1+\gamma_5)d_j\bar{u}_j\gamma^\mu(1+\gamma_5)d_i\bar{e}(1+\gamma_5)e^c, O_1^{LR} \\ &= \bar{u}_i\gamma_\mu(1-\gamma_5)d_i\bar{u}_j\gamma^\mu(1+\gamma_5)d_j\bar{e}(1+\gamma_5)e^c, O_2^{LR} \\ &= \bar{u}_i\gamma_\mu(1-\gamma_5)d_j\bar{u}_j\gamma^\mu(1+\gamma_5)d_i\bar{e}(1+\gamma_5)e^c. \end{aligned} \tag{3}$$

Following the general steps outlined above, the Wilson coefficients can be evaluated at the high scale and run down to $\mu \sim O$ (GeV) (see, also, Ref. [19]). Their approximate values read

$$C_1^{LL,RR} \sim 1.3, \qquad C_2^{LL,RR} \sim -0.6, \qquad C_1^{LR,RL} \sim 1.1, \qquad C_2^{LR,RL} \sim 0.7. \tag{4}$$

To evaluate the physical matrix elements, the color mismatched operators O_2^{AB} have to be Fierz transformed. Under Fierz rearrangement, $(V-A)\otimes(V-A)$ and $(V+A)\otimes(V+A)$ retain their form while $(V-A)\otimes(V+A)\to -2(S-P)\otimes(S+P)$. With this rearrangement, the LL, RR operators effectively yield $C_1^{LL,RR}+C_2^{LL,RR}$ as the effective couplings with the same NMEs involved, implying substantial cancellation (by about a factor of 2). The LR operator Fierz transformed brings in a different combination of NMEs. Explicitly following, for example, the last reference in [5], we have the following (not showing the lepton current explicitly):

$$\langle \mathcal{J}^{(V\pm A)}\mathcal{J}_{(V\pm A)}\rangle \propto \frac{m_A}{m_P m_e}(\mathcal{M}_{GT,N}\mp\alpha_3^{SR}\mathcal{M}_{F,N}), \tag{5}$$

where $|\mathcal{M}_{GT,N}| \sim (2\text{–}4)|\mathcal{M}_{F,N}|$ for all the nuclei considered, and $\alpha_3^{SR}\sim 0.63$. Thus, to a good accuracy, the above matrix element is essentially governed by $\mathcal{M}_{GT,N}$. In the above equation, the relative negative sign between the two terms on the right-hand side corresponds to $(V+A)\otimes(V+A)$ and $(V-A)\otimes(V-A)$ structures on the left-hand side, while for the $(V+A)\otimes(V-A)$ structure, the relative sign is positive.

On the other hand,

$$\langle \mathcal{J}^{(S\pm P)}\mathcal{J}_{(S\pm P)}\rangle \propto -\alpha_1^{SR}\mathcal{M}_{F,N}, \tag{6}$$

with $\alpha_1^{SR}\sim 0.145(m_A/m_P m_e)$. Clearly, the Fierz transformed operator in this case turns out to be subdominant. This simple exercise illustrates the large impact and importance of QCD corrections in the context of $0\nu 2\beta$. As obtained above, QCD corrections can lead to a substantial shift in the $0\nu 2\beta$ rate for specific models, thereby changing the limits on the model parameters significantly.

As our next example, we consider theories where the interactions are $S\pm P$ form, like SUSY with R-parity violation or leptoquarks, etc. In such cases, the operators have the structure

$$\begin{aligned} O_1^{SP\pm\pm} &= \bar{u}_i(1\pm\gamma_5)d_i\bar{u}_j(1\pm\gamma_5)d_j\bar{e}(1+\gamma_5)e^c, O_2^{SP\pm\pm} \\ &= \bar{u}_i(1\pm\gamma_5)d_j\bar{u}_j(1\pm\gamma_5)d_i\bar{e}(1+\gamma_5)e^c, O_1^{SP+-} \\ &= \bar{u}_i(1+\gamma_5)d_i\bar{u}_j(1-\gamma_5)d_j\bar{e}(1+\gamma_5)e^c, O_2^{SP+-} \\ &= \bar{u}_i(1+\gamma_5)d_j\bar{u}_j(1-\gamma_5)d_i\bar{e}(1+\gamma_5)e^c. \end{aligned} \tag{7}$$

The Wilson coefficients of the color mismatched operators are about 0.1–0.5 times those of the color allowed operators in magnitude. This could be argued from the $1/N_c(\sim 0.3 \text{ for } N_c=3)$ counting rules for the color mismatched structures, up to factors of order unity. Following the same chain of arguments, the color mismatched operators need to be Fierz transformed before computing the physical matrix elements. Under Fierz transformations, we have $(S+P)\otimes(S-P)\to\frac{1}{2}(V+A)\otimes(V-A)$ implying that the color mismatched operator, after Fierz transformation, may provide the dominant contribution [see Eqs. (5) and (6)]. Consequently, the amplitudes, and, therefore, the limits on the parameters may change by a factor of 5 or so. That the color mismatched operator can provide a large contribution is again something we are familiar with from $K\to\pi\pi$ decays, where the QCD (and electroweak) penguin operator after Fierz transformation gives the dominant contribution, though QCD and electroweak penguin contributions tend to cancel each other in this case.

The most interesting and the largest effect in the examples considered above comes about when considering the $O^{SP++,--}$ operators. $(S\pm P)\otimes(S\pm P)\to\frac{1}{4}[2(S\pm P)\otimes(S\pm P)-(S\pm P)\sigma_{\mu\nu}\otimes\sigma^{\mu\nu}]$ under Fierz rearrangement. The tensor-pseudotensor structure yields the following NME:

$$\langle \mathcal{J}^{\mu\nu}\mathcal{J}_{\mu\nu}\rangle \propto -\alpha_2^{SR}\mathcal{M}_{GT,N}, \tag{8}$$

with $\alpha_2^{SR} \sim 9.6(m_A/m_P m_e)$, which is about 200 times larger than $\langle \mathcal{J}^{(S\pm P)}\mathcal{J}_{(S\pm P)}\rangle$. Conservatively taking the corresponding Wilson coefficient to be 0.1 of the color allowed operator, the relative contributions are

$$\left|\frac{O_2^{SP++}}{O_1^{SP++}}\right| \geq 10. \tag{9}$$

The above discussion makes it very clear that the QCD corrections to $0\nu 2\beta$ are rather important and should be included systematically. These corrections can be as large as, or, in fact, larger than, in most cases, the uncertainty due to NMEs and are independent of the particular set of NMEs considered. As eluded to above, we have considered only pairs of operators O_1^{AB}, O_2^{AB} while obtaining the approximate values of $C's$ at the low scale. The effect of mixing with other operators has been ignored at this stage. This could further lead to significant corrections for some of the operators. We plan to systematically investigate these issues elsewhere. This (and the shift above) is rather large and can completely change the phenomenological constraints. In theories with many contributions to $0\nu 2\beta$, it is essential to understand the interplay between different competing amplitudes to set limits on the couplings and masses of the particles. In such cases, the discussion above becomes even more important. Low (TeV) scale models appear to be attractive due to plausible signatures at the LHC, where QCD corrections will be inevitable. It is, therefore, important to include the dominant QCD corrections at the very least in order to set meaningful limits on model parameters.

In this Letter, we have investigated the impact of one-loop QCD corrections to the $0\nu 2\beta$ amplitude. This, to the best of our knowledge, is the first time this issue has been discussed. We found that QCD corrections can have a large impact ranging from near cancellation to a huge enhancement of the $0\nu 2\beta$ rate. Since $0\nu 2\beta$ is an important process to search experimentally and has the potential to link seemingly unrelated processes, particularly in the context of TeV-scale models, it is rather important to ensure that theoretical predictions are precise enough to be compared to the experimental results. As such, the calculations suffer from large uncertainties due to the choice of NMEs, which are nonperturbative in nature. What we have found is that even perturbative corrections have the potential to shift the predictions by a large amount. This by itself is a rather important aspect, and such corrections need to be systematically computed for various models of interest. The shift in the limits on model parameters also implies that the related phenomenology at, say, the LHC (in specific models), will also get modified. There are other issues related to operator mixing which have not been incorporated here. These may also become important in the context of specific theories and should be consistently included. Furthermore, QCD corrections need to be evaluated for the light neutrino exchange contribution as well. As mentioned in the beginning, the light neutrino contribution is unable to saturate the present experimental limits. It remains to be seen if including the radiative corrections eases out this tension and to what extent [20].

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