Universal Trimers Induced by Spin-Orbit Coupling in Ultracold Fermi Gases

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In this Letter we address the issue of how synthetic spin-orbit (SO) coupling can strongly affect three-body physics in ultracold atomic gases. We consider a system which consists of three fermionic atoms, including two spinless heavy atoms and one spin-$1/2$ light atom subjected to an isotropic SO coupling. We find that SO coupling can induce universal three-body bound states with a negative s-wave scattering length at a smaller mass ratio, where no trimer bound state can exist if in the absence of SO coupling. The energies of these trimers are independent of the high-energy cutoff, and therefore they are universal ones. Moreover, the resulting atom-dimer resonance can be effectively controlled by SO coupling strength. Our results can be applied to systems like a ^{6}Li and ^{40}K mixture.

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"Universal phenomenon" refers to observations independent of short-range or high energy details, which is one of the most beautiful and charming parts of physics. Universal physics not only emerges in interacting many-body systems but also exists in quantum mechanical few-body problems. A cold atom system, because of its diluteness, is an ideal platform to investigate various intriguing phenomena of few-body systems. For instance, an Efimov trimer with a universal scaling factor [1,2] has been extensively studied experimentally [3–10]. Another type of trimer whose energy is universal has also been predicted by Kartavtsev and Malykh [11].

On the other hand, thanks to fast experimental developments [12–22], synthetic spin-orbit (SO) coupling has recently emerged as one of the most exciting research directions in cold atom physics [23]. Among the many profound effects of SO coupling, one distinct factor is that certain types of SO coupling can dramatically change the two-body physics. For instance, with Rashba-type SO coupling, because the low-energy density of state is enhanced to a finite constant, any small attractive interaction between atoms can support a two-body bound state in three dimensions, and the binding energy increases with the strength of SO coupling [24]. Consequently, this two-body result dramatically changes many-body physics in the scenario of BEC-BCS crossover for spin-$1/2$ fermions [25–27], where the superfluidity is greatly enhanced by SO coupling even in the far BCS side [25].

The dramatic effect of SO coupling in two-body problem and its profound consequence naturally raises the question of whether a similar significant manifestation also exists in a three-body problem. However, so far three-body problems with SO coupling have not been studied in cold atom content, though historically there have been some related studies in investigating the nucleus [28–30]. In this work we study a three-fermion problem which consists of two heavy fermionic α atoms with mass M and one light fermionic β atom with mass m, and the α and β atom interact via a zero-range s-wave interaction in the vicinity of the two-body scattering resonances. The α atom is spinless and the β atom is spin-$1/2$. As the first attempt to demonstrate rich physics of SO coupling in the few-body cold atom system, we consider a simple case where only the β atom is subjected to an isotropic SO coupling [31,32]. This is realistic for a cold atom system, since synthetic SO coupling for atoms is induced by atom-light (or atom-magnetic field) interaction which can be selectively applied to certain species. For instance, we can consider a mixture of two-component ^{6}Li with single component ^{40}K, and the (pseudo)-spin of ^{6}Li is coupled to its momentum [33].

Indeed, we find that SO coupling leads to intriguing new physics in this three-body system. The most significant finding is that when $M/m \gtrsim 5.92$ (satisfied by ^{6}Li and ^{40}K mixture), SO coupling can induce a universal trimer state whose energy is independent of the short-range parameter. Such trimers can exist at the negative scattering length side—a regime where the universal trimer can never exist in the absence of SO coupling. Moreover, the locations of three-body resonances are tunable by the strength of the SO coupling. This result reveals a unique manifestation of SO coupling in dilute quantum gases and also adds a new control knob to the three-body system. Potentially it can also shed light on the few-body system of a nucleus where SO coupling is inevitable.

Before proceeding, we shall first briefly review the known results for such an α-α-β system without SO coupling. Two types of trimer states have been found. First, when $M/m > 13.6$, the Efimov trimer emerges in both sides nearby the resonance. The energy of the Efimov trimer is not universal since it depends on the high-energy cutoff known as the three-body parameter, while the energies of two successive trimers obey a universal scaling behavior [1]. Second, when $8.17 < M/m < 13.6$, there exists another type of trimer named the "Kartavtsev-Malykh" trimer, whose energy is universal (i.e., independent of any high-energy cutoff) [11]. Since the s-wave scattering length a is the only length scale, the trimer energy has to simply scale with the two-body binding energy. Thus, such a universal trimer appears only for positive a when a two-body bound state exists. Because of the antisymmetrization of the two α atoms, both types of trimer states have a total orbital angular momentum $L = 1$.

Model.—Our system is described by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{U}$,

$$\hat{H}_0 = \frac{\mathbf{p}_1{}^2}{2M} + \frac{\mathbf{p}_2{}^2}{2M} + \frac{(\mathbf{p}_3 - \lambda\hat{\sigma})^2}{2m}, \tag{1}$$

$$\hat{U} = [g\delta(\mathbf{r}_1 - \mathbf{r}_3) + g\delta(\mathbf{r}_2 - \mathbf{r}_3)]\mathbf{I}, \tag{2}$$

in which $\mathbf{p}_{1,2}(\mathbf{r}_{1,2})$ refers to the momentum (position) of two α atoms, and $\mathbf{p}_3(\mathbf{r}_3)$ is for the β atom. $\hat{\sigma}$ is the spin of the β atom, which couples to its momentum via a three-dimensional isotropic SO coupling $\lambda\mathbf{p}\cdot\hat{\sigma}$ where $\mathbf{p} = (p_x, p_y, p_z)$ and $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Without loss of generality, we take $\lambda > 0$. Proposals for realizing such a SO coupling have been presented in Refs. [31,32]. The s-wave contact interaction $\hat{U}$ only takes place between the β atom and α atom, and the interaction strength is assumed to be independent of the spin-index of the β atom, where $\mathbf{I}$ in $\hat{U}$ denotes the identity operator acting on the spin space of the β atom. g is related to a by the renormalization equation

$$\frac{1}{g} = \frac{Mm}{2\pi(M+m)a} - \frac{1}{\Omega}\sum_{\mathbf{k}}\frac{2Mm}{(M+m)k^2}, \tag{3}$$

where Ω is the volume. It has been shown that this relation will not be changed by SO coupling, as long as $1/\lambda$ is much larger than the range of interatomic potential [35–37].

To address the three-body bound state, we should first solve the two-body problem with one α and one β atom to determine the atom-dimer threshold, which can be carried out quite straightforwardly with the Lippman-Schwinger equation [38]. Although our case differs from previous studies of the two-body problem with SO coupling [24,31,32,35–37,39–42] where both two atoms are subjected to SO coupling, the results are quite similar to previous cases with Rashba or three-dimensional isotropic SO coupling, i.e., for any mass ratio M/m and for all a, a two-body bound state with zero center-of-mass momentum exists [24,25,31,35]. The physical reason is also attributed to the enhancement of the density of state of the β atom, which diverges at the threshold scattering energy.

For the same three-body system without SO coupling, the total orbital angular momentum $\mathbf{L}$ is a good quantum number and most previous calculations focus on the lowest bound states in the $L = 1$ channel. After introducing spin degrees of freedom for the β atom, these bound states are always sixfold degenerate. In the presence of SO coupling, these states would split into two channels with a different total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. They are two states with $J = 1/2$ and four states with $J = 3/2$.

Solving the three-body problem.—Generally, we assume the three-body wave function (with total momentum $\mathbf{K}_0$) as

$$|\Psi\rangle = \sum_{\mathbf{p},\mathbf{q},\sigma}\Psi_\sigma(\mathbf{q}, \mathbf{K}_0 - \mathbf{p}, \mathbf{p} - \mathbf{q})\hat{\alpha}^\dagger_{\mathbf{q}}\hat{\alpha}^\dagger_{\mathbf{K}_0-\mathbf{p}}\hat{\beta}^\dagger_{\sigma,\mathbf{p}-\mathbf{q}}|0\rangle, \tag{4}$$

where $\hat{\alpha}^\dagger$ and $\hat{\beta}^\dagger$ are creation operators for the α atom and β atom, respectively, and $\sigma = \uparrow, \downarrow$ is the spin index of the β atom. Introducing an auxiliary function $f_\sigma(\mathbf{p}) = g\sum_{\mathbf{q}}\Psi_\sigma(\mathbf{q}, \mathbf{K}_0 - \mathbf{p}, \mathbf{p} - \mathbf{q})$, we can reach the following integral equation for $f_\sigma(\mathbf{q})$:

$$f_\sigma(\mathbf{k}) = g\sum_{\mathbf{p},\sigma'}G_{\sigma\sigma'}(E; \mathbf{p}, \mathbf{K}_0 - \mathbf{k}, \mathbf{k} - \mathbf{p}) \times [f_{\sigma'}(\mathbf{k}) - f_{\sigma'}(\mathbf{K}_0 - \mathbf{p})], \tag{5}$$

where

$$G_{\sigma\sigma'}(E; \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \left\langle \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \sigma \left| \frac{1}{E - H_0} \right| \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \sigma' \right\rangle$$

is the Green's function in momentum space [43–45]. The nonzero solution of Eq. (5) determines the energy of the trimer states, $E = E_3$. To get physical solutions for E_3, the renormalization equation (3) can be used to eliminate the ultraviolet divergence of $\sum_{\mathbf{p}}G_{\sigma\sigma}$ in Eq. (5) [38].

However, in general, solving the coupled three-dimensional integral equation is highly nontrivial. Nevertheless, great simplification can be obtained in the subspace with $\mathbf{K}_0 = 0$. As shown in the Supplemental Material [38], for a quantum state labeled by $(J, J_z) = (j + 1/2, m + 1/2)$ (where j and m are integers), $f_\sigma(\mathbf{k})$ satisfies

$$\begin{aligned} f_\uparrow(\mathbf{k}) &= C^0_\uparrow f_0(k)Y^m_j(\Omega_{\mathbf{k}}) + C^1_\uparrow f_1(k)Y^m_{j+1}(\Omega_{\mathbf{k}}), \\ f_\downarrow(\mathbf{k}) &= C^0_\downarrow f_0(k)Y^{m+1}_j(\Omega_{\mathbf{k}}) + C^1_\downarrow f_1(k)Y^{m+1}_{j+1}(\Omega_{\mathbf{k}}), \end{aligned} \tag{6}$$

where $k = |\mathbf{k}|$ is the magnitude of $\mathbf{k}$ and f_0, f_1 are functions that only depend on k, C^0_σ, C^1_σ are Clebsch-Gordan coefficients,

$$C^\delta_\sigma = \left\langle j + \delta, m - \sigma; \frac{1}{2}, \sigma \middle| j + \frac{1}{2}, m + \frac{1}{2} \right\rangle, \tag{7}$$

with $\delta = 0,1$ and $\sigma = \pm\frac{1}{2}$. After substituting Eq. (6) into Eq. (5), Eq. (5) is reduced to two coupled one-dimensional

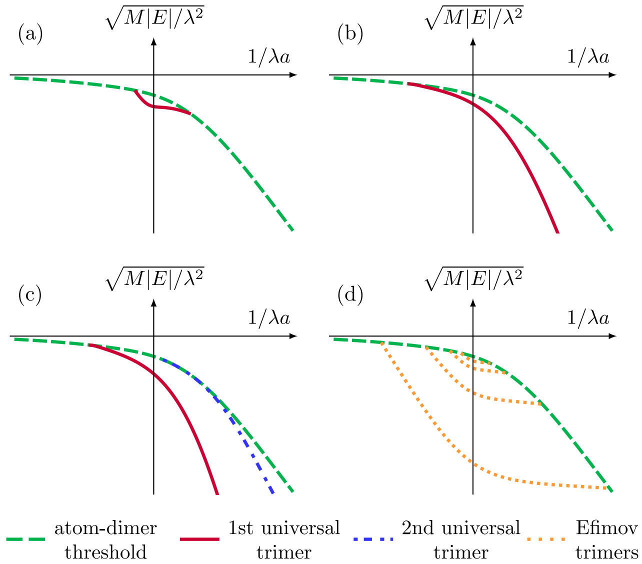

FIG. 1 (color online). Schematic of the atom-dimer threshold (green dashed line) and trimer energy in the presence of SO coupling for $6.5 < M/m < 8.17$ (a), $8.17 < M/m < 12.9$ (b), $12.9 < M/m < 13.6$ (c) and $13.6 < M/m$ (d). Red solid line in (a)–(c) represents the universal trimer with lowest energy. Blue dashed-dotted line in (c) represents the second universal trimer, and yellow dotted lines in (d) represent Efimov trimers. This is a schematic plot in order to highlight main features. The actual numbers are shown in Fig. 2.

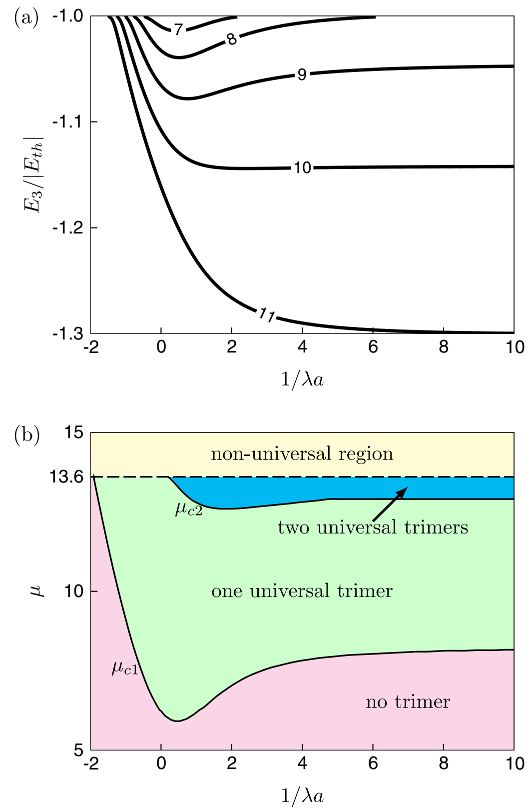

FIG. 2 (color online). (a) The ratio between the $J = 3/2$ trimer energy E_3 and atom-dimer threshold energy $|E_{\rm th}|$, $\gamma = E_3/|E_{\rm th}|$, as a function of $1/\lambda a$ for different mass ratios M/m labeled in the curve. (b) The "phase diagram" for $J = 3/2$ trimer in terms of $1/\lambda a$ and mass ratio $\mu = M/m$.

integral equations, whose explicit forms are given in the Supplemental Material [38] and can be solved numerically to determine the trimer energy E_3.

Results.—With SO coupling, the energies of the $J = 1/2$ channel and $J = 3/2$ channel will split, and we find that in most regions of interest, the $J = 1/2$ channel has a higher energy than that of the $J = 3/2$ channel. In the following we will summarize the results for the ground state $J = 3/2$ channel, while the results for $J = 1/2$ will be presented elsewhere [46].

(1) When $5.92 \lesssim M/m < 8.17$, there is no trimer state if there is no SO coupling. We find that with SO coupling, a trimer state will be induced in the vicinity of two-body resonance. It emerges from the atom-dimer threshold at $a < 0$ side and then merges into the atom-dimer threshold at the $a > 0$ side, as shown in Fig. 1(a). The energy of such a trimer state is independent of any high energy cutoff; thus, similar to the universal "Kartavtsev-Malykh" trimer, the ratio between trimer energy (E_3) and atom-dimer threshold energy ($E_{\rm th}$) $\gamma = E_3/|E_{\rm th}|$ is a universal function of $1/\lambda a$, as plotted in Fig. 2(a). $\gamma < -1$ means that the trimer energy is below the atom-dimer threshold.

(2) When $8.17 < M/m < 13.6$, there exists at least one universal "Kartavtsev-Malykh" trimer at the positive a side if there is no SO coupling. We find that with SO coupling, the lowest trimer starts to appear at the $a < 0$ side. This trimer energy is also universal. The ratio γ plotted in Fig. 2(a) shows that $\gamma < -1$ from a certain point with negative a and saturates to a constant (the same value as predicted by Kartavtsev and Malykh for the case without SO coupling) for large $1/\lambda a$. When $12.9 \lesssim M/m < 13.6$, an second trimer emerges at $a > 0$ side.

(3) When $M/m > 13.6$, the system enters the nonuniversal regime with trimer energies sensitively depending on the short-range parameter [47]. Without SO coupling, there are an infinite number of Efimov trimers whose spectra exhibit a discrete scaling property [2]. When the strength of SO coupling increases, the binding energies of these trimers decrease, and finally these trimers merge into an atom-dimer continuum and disappear one after the other. In addition, because SO coupling introduces an additional length scale, these trimers no longer obey the discrete scaling symmetry even at resonance [46].

With the results above, a "phase diagram" for the $J = 3/2$ trimer is constructed in terms of dimensionless interaction parameter $1/\lambda a$ and mass ratio $\mu = M/m$, as shown in Fig. 2(b), where μ_{c1} (μ_{c2}) is the critical mass ratio for the emergence of the first (second) universal trimer. It is interesting to note that μ_{c1} is a nonmonotonic function of $1/\lambda a$, which reaches its minimum when $1/\lambda a$ is close to zero.

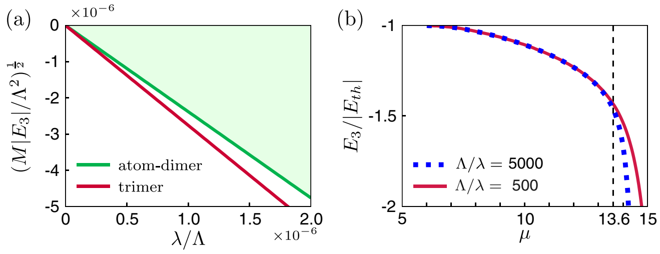

FIG. 3 (color online). (a) Trimer energy (in unit of high energy cutoff $\hbar^2\Lambda^2/M$) as a function of λ/Λ for a given mass ratio $M/m = 12$. (b) The lowest trimer energy E_3 (in unit of atom-dimer threshold energy $|E_{\text{th}}|$) as a function of mass ratio $\mu = M/m$ for two different high energy cutoff Λ's. Both are plotted at two-body resonance $a = \infty$.

In Fig. 3, we show that the trimer energy is indeed universal when $M/m < 13.6$. At resonance, if the trimer energy is universal, $1/\lambda$ becomes the only length scale in the problem and the trimer energy has to scale with $\hbar^2\lambda^2/M$. We introduce a high-momentum cutoff Λ for the atom-dimer motion, or equivalently, to the argument of the f_σ function in Eq. (5). This scaling behavior is shown in Fig. 3(a). In Fig. 3(b), we plot the lowest trimer energy at resonance as a function of mass ratio, with two different high-energy cutoffs Λ. It clearly shows that for $M/m < 13.6$ the energy is independent of the cutoff while it is not for $M/m > 13.6$. The scenario of how universal "Kartavtsev-Malykh" trimers cross over to the Efimov trimer is similar to what has been discussed in Ref. [48] for the case without SO coupling.

Among the above results 1–3, 1 and 2 are the most significant ones. It means that SO coupling favors trimer formation; i.e., universal trimer can now exist for a smaller mass ratio and also at the $a < 0$ side. Another way to view it is that, once $M/m \gtrsim 5.92$, the trimer state can always be induced by increasing the strength of SO coupling, even for the system at weak interaction regime.

We attribute the reason that SO coupling favors trimer formation to the lifting of the ground state degeneracy. If there is no SO coupling, all the bound states are highly degenerate, while SO coupling mixes different orbital angular momentum channels, which breaks such degeneracy and lowers the ground-state energy according to the second perturbation theory. For example, in Fig. 4, we show a case with $M/m > 8.17$ at $a > 0$ side. The dashed line represents the energy of the "Kartavtsev-Malykh" trimer without SO coupling, where $J = 3/2$ and $J = 1/2$ states are degenerate. With SO coupling, it is found that the splitting between $J = 3/2$ and $J = 1/2$ increases the energy of the $J = 1/2$ trimer but lowers the energy of $J = 3/2$ trimers. Consequently, the $J = 3/2$ trimers can exist for a smaller mass ratio and also at the $a < 0$ side. Furthermore, because the mixing of different orbital angular momentum channels is an intrinsic effect of SO coupling, we anticipate that our results qualitatively hold for a general type of SO coupling.

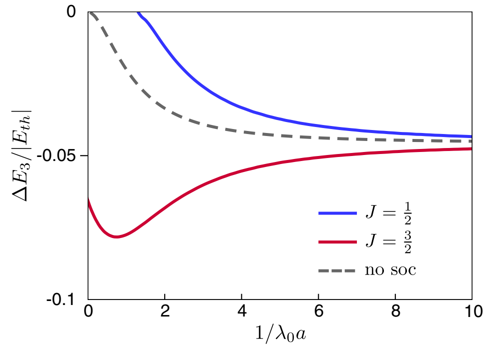

FIG. 4 (color online). The trimer binding energy ΔE_3 (in unit of atom-dimer threshold energy $|E_{\text{th}}|$ with a given λ_0) as a function of interaction strength $1/\lambda_0 a$ for a given mass ratio $M/m = 12$. Dashed line represents the case without SO coupling $\lambda = 0$ (sixfold degenerate). Two solid lines represent the cases for $J = 3/2$ trimers (fourfold degenerate) and $J = 1/2$ trimers (twofold degenerate), respectively, for a given SO coupling strength λ_0.

We would also like to point out a phenomenological analogy between two-body physics and three-body physics. Without SO coupling, both the two-body bound state and three-body universal trimer only exist in the region with positive a_s, while with SO coupling, they both extend to the negative a_s side. In this sense, the few-body physics in the two-body sector and three-body sector are modified by SO coupling in a similar way.

Final remark.—Our results can potentially influence many-body physics. When the trimer energies touch the atom-dimer threshold, it will lead to an atom-dimer resonance where the atom-dimer scattering length will change dramatically. Without SO coupling, usually the resonance position of the Efimov trimer is controlled by the three-body parameter $1/\Lambda a$, which is not tunable for a given mixture. While with SO coupling, the resonance position of the universal trimer is controlled by $1/\lambda a$, which can be tuned quite flexibly by the SO coupling strength λ. Thus, this introduces a new way to manipulate a strongly interacting quantum many-body system.

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