Quantum Spectral Curve for Planar $\mathcal{N}=4$ Super-Yang-Mills Theory

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We present a new formalism, alternative to the old thermodynamic-Bethe-ansatz-like approach, for solution of the spectral problem of planar $\mathcal{N}=4$ super Yang-Mills theory. It takes a concise form of a nonlinear matrix Riemann-Hilbert problem in terms of a few Q functions. We demonstrate the formalism for two types of observables—local operators at weak coupling and cusped Wilson lines in a near Bogomol'nyi-Prasad-Sommerfield limit.

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Introduction.—The spectrum of anomalous dimensions in the planar $\mathcal{N}=4$ super Yang-Mills theory (SYM) theory was successfully studied in the last decade, to great extent due to the ideas of AdS/CFT correspondence and integrability [1]. A conventional form of solution to the spectral problem is given by an infinite set of nonlinear integral thermodynamic-Bethe-ansatz (TBA) equations [2–4] for the functions of the spectral parameter $Y_{a,s}(u)$

$$\log Y_{as}(u) = \delta_s^0 iLp_a(u) + \int dv K_{as}^{a's'}(u,v)\log(1+Y_{a's'}(v))$$

where the sum over a', s' in the rhs goes along the internal nodes of the lattice (T hook) in Fig. 1. The momentum p_a and the kernels $K_{as}^{a's'}$ are explicit but rather complicated functions of the spectral parameters u, v [3]. Their important analytic feature is the presence of cuts, parallel to $\mathbb{R}$, with fixed branch points at $u, v \in \pm 2g + i\mathbb{Z}$ or $u, v \in \pm 2g + i(\mathbb{Z}+\frac{1}{2})$ where $g \equiv \sqrt{\lambda}/(4\pi)$ and λ is the 't Hooft coupling. This TBA system completely fixes the Y functions and, hence, the dimension of a particular operator specified by certain poles and zeros incorporated into the driving terms [3]. It was successfully used for the weak and strong coupling analysis [5–7] as well as for the first successful numerical computations of dimensions of Konishi [8,9] and similar operators [10,11]. However, this TBA system has very complex analyticity properties, which limits in practice its applications and obscures the long anticipated beauty of the whole problem.

An obvious sign of this hidden beauty is the direct equivalence of the TBA system to the AdS/CFT Y system, originally proposed as a solution of the AdS/CFT spectral problem in Ref. [12], with additional analyticity conditions [13]. It is a universal set of equations equivalent, by the substitution $Y_{a,s} = (\mathbb{T}_{s+1,a}\mathbb{T}_{s-1,a})/(\mathbb{T}_{a+1,s}\mathbb{T}_{a-1,s})$, to the Hirota discrete bilinear equation (T system) [14,15]

$$\mathbb{T}_{a,s}^{+}\mathbb{T}_{a,s}^{-} = \mathbb{T}_{a+1,s}\mathbb{T}_{a-1,s} + \mathbb{T}_{a,s+1}\mathbb{T}_{a,s-1}, \quad (1)$$

which is integrable in its turn. With the use of this integrability, the general solution of the T system can be explicitly parametrized in terms of Wronskians built from only eight independent Q functions [16,17]. The Q functions are the most elementary constituents of the whole construction with the analyticity properties much simpler than those of Y or T functions [18]. With a savvy choice of the basic Q functions, we managed in Ref. [18] to close a finite system of nonlinear integral equations (FINLIE). It appeared to be a very efficient tool in multiloop weak coupling computations [19,20]. But it was clear that the somewhat bulky form of that FINLIE [18] hides a much more beautiful and simple formulation, with a clear insight into the full analytic structure of the underlying functions.

We formulate in this note a new, much more transparent and concise system of the planar AdS_5/CFT_4 spectral equations of the Riemann-Hilbert type. It might represent the ultimate simplification of this spectral problem.

$P\mu$ system for the spectrum.—We will demonstrate our new approach on the most important example of the left-right symmetric states for which $\mathbb{T}_{a,s} = \mathbb{T}_{a,-s}$ (in the appropriate gauge described in Ref. [18]). To start with, all T and Y functions can be expressed in terms of $4+4$ Q functions [17]. Let us exemplify this relation for T functions of the right band (see Fig. 1), where we have for $s > 0$ [18]

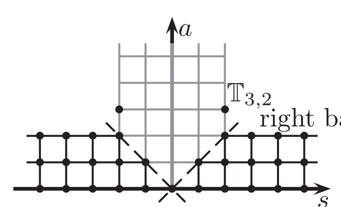

FIG. 1. T hook: lattice for the AdS/CFT T system.

$$\mathbb{T}_{1,s} = \mathbf{P}_1^{[+s]}\mathbf{P}_2^{[-s]} - \mathbf{P}_2^{[+s]}\mathbf{P}_1^{[-s]}, \tag{2}$$

where the symbol $\mathbf{P}$ is used to denote the Q functions in the right band, to avoid a clash with other notations existing in the literature. An important feature of this parametrization is that $\mathbf{P}$'s have only one single cut between $-2g$ and $2g$, otherwise being analytic in the whole complex plane [18]. This property is tightly related to what we refer to as $\mathbb{Z}_4$ symmetry [18,21,22].

Ideally, we would like to reduce the whole problem to a single spectral curve or a Riemann surface on which all Q functions are defined. For that we need to know in particular the analytic continuations of $\mathbf{P}_1$ and $\mathbf{P}_2$ through the cut which we denote as $\tilde{\mathbf{P}}_1$, $\tilde{\mathbf{P}}_2$. Quite expectedly, $\tilde{\mathbf{P}}_1$, $\tilde{\mathbf{P}}_2$ have an infinite "ladder" of cuts, with branch points at $\pm 2g + in$ for any integer n. To describe completely the Riemann surface, one should know the analytic continuation through any of those new cuts and so on. One of the main results of this note is that this complicated cut structure has a stunningly simple algebraic description.

Namely, after inspecting the properties of Q functions of Ref. [18], we managed to construct [23] two additional functions $\mathbf{P}_3$ and $\mathbf{P}_4$, again with only one single cut, such that after the analytic continuation the four functions $\tilde{\mathbf{P}}_a$, $a = 1, 2, 3, 4$ can be expressed as linear combinations of the initial $\mathbf{P}$'s

$$\tilde{\mathbf{P}}_a = -\mu_{ab}\chi^{bc}\mathbf{P}_c \tag{3}$$

where χ is the antisymmetric constant 4×4 matrix with the only nonzero entries $\chi^{23} = \chi^{41} = -\chi^{14} = -\chi^{32} = 1$ and μ_{ab} is a 4×4 antisymmetric matrix constrained by

$$\mu_{12}\mu_{34} - \mu_{13}\mu_{24} + \mu_{14}^2 = 1, \qquad \mu_{23} = \mu_{14}, \tag{4}$$

$$\tilde{\mu}_{ab}(u) = \mu_{ab}(u + i). \tag{5}$$

Equation (5) is a very peculiar pseudoperiodicity condition (see Fig. 2. It tells us that if we define a function μ such that it coincides with μ in the strip $0 < \mathrm{Im}u < 1$ but has all its cuts going to infinity then μ is a truly i-periodic function: $\mu_{ab}(u+i) = \mu_{ab}$.

To close the system of equations on $\mathbf{P}$, μ we have to find a condition on μ_{ab} similar to Eq. (3). An important part of it is already dictated by Eq. (3): since the branch points are quadratic, we have $\tilde{\tilde{\mathbf{P}}}_a = \mathbf{P}_a$, which leads to $\mathbf{P} = -\tilde{\mu}\chi\tilde{\mathbf{P}}$. This, together with Eq. (3), gives a set of linear equations fixing the discontinuity of the matrix μ up to a single unknown factor $e(u)$: $\tilde{\mu}_{ab} - \mu_{ab} = e(u)(\mathbf{P}_a\tilde{\mathbf{P}}_b - \tilde{\mathbf{P}}_a\mathbf{P}_b)$. We argue below that $e(u) = 1$ and, hence,

$$\tilde{\mu}_{ab} - \mu_{ab} = \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a. \tag{6}$$

Equations (3), (5), and (6) represent our main result—a complete nonlinear system of Riemann-Hilbert equations for the AdS/CFT spectral problem. They allow us to walk through the cuts to any sheet (out of infinite number) of the Riemann surface of the functions $\mathbf{P}$ and μ. In this sense, they give the full description of the spectral curve of the problem. Indeed, by means of Eqs. (3) and (5) it is easy to walk through the central cut in Fig. 2. The other cuts are present only in μ. To define the analytic continuation through them, we use a combination of Eqs. (3), (5), and (6),

$$\mu_{ab}(u+i) = \mu_{ab} - \mathbf{P}_a\mathbf{P}_e\chi^{ec}\mu_{cb} - \mathbf{P}_b\mathbf{P}_e\chi^{ec}\mu_{ac}, \tag{7}$$

which allows us to recursively express $\mu_{ab}(u+in)$ through $\mu_{ab}(u)$ and shifted $\mathbf{P}$'s—the quantities with known monodromies. We refer to this new formulation of the spectral problem, given by Eqs. (3)–(6), as the $\mathbf{P}\mu$ system.

Let us argue now that $\mathbf{P}$ and μ contain the complete information about the initial Y system. Indeed, from Eq. (2) we restore $\mathbb{T}_{1,s}$ for $s > 0$. Furthermore, $\mathbb{T}_{2,s} = \mathbb{T}_{1,1}^{[+s]}\mathbb{T}_{1,1}^{[-s]}$, $\mathbb{T}_{0,s} = 1$, and with a help of one extra relation $\mathbb{T}_{3,2} = \mathbb{T}_{2,3}\mu_{12}$ (see Ref. [18], where $\mathcal{F}^- = \mu_{12}$) we have just enough information to recover any $\mathbb{T}_{a,s}$ using solely the Hirota Eq. (1), for any left-right symmetric state. It is just a matter of elementary algebra to write any Y function explicitly through $\mathbf{P}$, μ. In particular, we find

$$Y_{11}Y_{22} = 1 + \frac{\mathbf{P}_1\tilde{\mathbf{P}}_2 - \mathbf{P}_2\tilde{\mathbf{P}}_1}{\mu_{12}} = \frac{\mu_{12}(u+i)}{\mu_{12}(u)}. \tag{8}$$

We note that the first equality holds for any $e(u)$, but imposing [18] $\tilde{Y}_{11}\tilde{Y}_{22} = 1/(Y_{11}Y_{22})$ we fix $e(u) = 1$.

Asymptotics and charges.—The quantity of Eq. (8) is known to contain the energy/dimension Δ of the state in its large u asymptotics [18]: $\log Y_{11}Y_{22} \simeq i(\Delta - L)/u$. Similarly, the large u asymptotics of $\mathbf{P}$ and μ contain the information about other conserved charges of the state. In fact, $\mathbf{P}_a^+/\mathbf{P}_a^-$ is the exact quantum analogue of the S^5 eigenvalues of the monodromy matrix [22] of classical strings moving in $AdS_5 \times S^5$ and, thus, $\mathbf{P}_a^+/\mathbf{P}_a^- \simeq 1 + M_a/(2iu)$, where M_a are integer charges of the global SO(6) symmetry. For instance, in the $\mathfrak{sl}_2$ sector, i.e., for spin S twist L operators of the type $\mathrm{Tr}Z\nabla_+^S Z^{L-1}$ dual to the string which is pointlike in S^5 and moves there with the angular momentum L, one has the following asymptotics

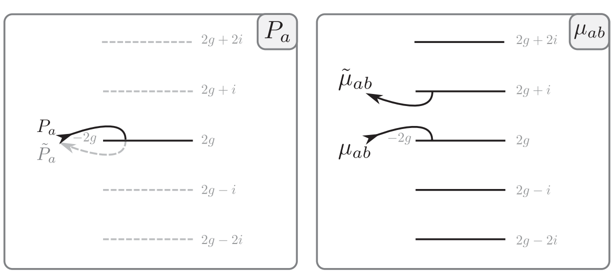

FIG. 2. Cut structure of $\mathbf{P}$ and μ.

$$\mathbf{P}_a \simeq (A_1 u^{-L/2}, A_2 u^{-(L+2)/2}, A_3 u^{L/2}, A_4 u^{(L-2)/2})_a. \quad (9)$$

Note that at odd values of L, $\mathbf{P}_a$ have a sign ambiguity.

Next, we also have to specify the asymptotics of μ. Assuming its powerlike behavior, we immediately get from Eq. (8) $\mu_{12} \simeq u^{\Delta-L}$. To deduce the asymptotics of the remaining μ's, we consider $\tilde{\mathbf{P}}_1 = -\mu_{14}\mathbf{P}_1 + \mu_{13}\mathbf{P}_2 - \mu_{12}\mathbf{P}_3$ and assume that all the terms in the rhs scale in the same way. This gives, e.g., $\mu_{13} \sim \mu_{12} u^{L+1} \sim u^{\Delta+1}$ and, similarly, $(\mu_{14}, \mu_{24}, \mu_{34}) \sim (u^{\Delta}, u^{\Delta-1}, u^{\Delta+L})$. This strategy allows one to easily determine the asymptotics for any state even outside of the $\mathfrak{sl}_2$ sector.

Finally, let us fix the coefficients A_i in Eq. (9). Note that Eq. (7) becomes at large u value a homogeneous differential equation on the five independent components of μ_{ab}. By plugging into this equation the asymptotics for μ_{ab} and $\mathbf{P}_a$, we get a fifth-order algebraic equation on Δ. Its roots are of the form $(\pm\alpha, \pm\beta, 0)$ where α, β are functions of A_i. The root $\alpha = \Delta$ reproduces the correct asymptotics of μ_{ab}, whereas one can show (see a motivation in the Discussion) that $\beta + 1 = S$ is the Lorentz spin of the state. By inverting these relations one gets

$$A_2 A_3 = \frac{[(L-S+2)^2 - \Delta^2][(L+S)^2 - \Delta^2]}{16iL(L+1)},$$
$$A_4 A_1 = \frac{[(L+S-2)^2 - \Delta^2][(L-S)^2 - \Delta^2]}{16iL(L-1)}. \quad (10)$$

Note that Δ enters Eq. (10) only as Δ^2, which suggests that the function $S(\Delta)$ is even, as claimed in Ref. [24]. Interestingly, A_i enter only through the products (10), due to the rescaling symmetry: $\mathbf{P}_a \to c_a \mathbf{P}_a$, $\mu_{ab} \to c_a c_b \mu_{ab}$ with constants c_a obeying $c_1 c_4 = 1$, $c_2 c_3 = 1$.

Regularity condition.—$\mathbf{P}$-s and μ-s do not have poles on their defining sheet and, hence, due to Eqs. (3), (5), and (7), on the whole Riemann surface. The regularity condition singles out physical solutions of the $\mathbf{P}\mu$ system. In practice, one can identify the physical solutions in the one-loop approximation, as demonstrated below for the $\mathfrak{sl}(2)$ sector, and then develop the perturbation theory.

Weak coupling in the SL(2) sector.—Since $\Delta = L + S + \mathcal{O}(g^2)$ in this sector, we see from Eq. (10) that $A_2 A_3 = \mathcal{O}(g^2)$, which also suggests that $\mathbf{P}_2\mathbf{P}_3 = \mathcal{O}(g^2)$. The rescaling symmetry allows us to make a convenient choice $\mathbf{P}_2 = \mathcal{O}(g^2)$ for which Eq. (7) simplifies considerably at the leading order: Since $\mathbf{P}_2 \simeq 0$, equations for μ_{12} and μ_{24} decouple from the rest. Excluding μ_{24} we get a second-order difference equation

$$TQ + Q^{[-2]}/(\mathbf{P}_1^-)^2 + Q^{[+2]}/(\mathbf{P}_1^+)^2 = 0, \quad (11)$$

where

$$T \equiv \frac{\mathbf{P}_4^+}{\mathbf{P}_1^+} - \frac{\mathbf{P}_4^-}{\mathbf{P}_1^-} - \frac{1}{(\mathbf{P}_1^-)^2} - \frac{1}{(\mathbf{P}_1^+)^2} \quad \text{and}$$
$$\mu_{12}^+ \equiv Q + \mathcal{O}(g^2).$$

It is reasonable to assume that singularities from collision of branch points $u = \pm 2g + i\mathbb{Z}$ do not manifest themselves at one loop. Hence, let us choose the maximally analytic ansatz: $\mathbf{P}_4/\mathbf{P}_1$, $1/\mathbf{P}_1^2$, and μ_{12} should be polynomials (of degree $L-1$, L, and S correspondingly). Leaving the detailed explanation of this assumption for further publication [25], let us demonstrate that it allows us to reproduce the full spectrum of the $\mathfrak{sl}(2)$ sector. Note that polynomiality of $1/\mathbf{P}_1^2$ actually means that $\mathbf{P}_1 = A_1 u^{-L/2} + \mathcal{O}(g^2)$; otherwise, $\mathbf{P}_1$ would be an infinite Laurent series that cannot be a regular function on other Riemann sheets at finite coupling. Then Eq. (11) is nothing but the Baxter equation for the Heisenberg spin chain. This equivalence can be confirmed further as the zeros of μ_{12}^+ are indeed *exact* Bethe roots [18].

By standard arguments, regularity implies that zeros of Q satisfy Bethe equations, which singles out a discrete set of possible Q's and, hence, of solutions of the $\mathbf{P}\mu$ system corresponding to the states from the $\mathfrak{sl}(2)$ sector. For AdS/CFT, we have an additional zero-momentum constraint $Q(+i/2)/Q(-i/2) = 1$, which is due to the cyclicity of trace. The $\mathbf{P}\mu$ system also encodes this constraint. Indeed, in the limit $g \to 0$, it is nothing but Eq. (5) evaluated at the branch point $u = 2g$ where we used the analyticity condition $\tilde{\mu}_{ab}(2g) = \mu_{ab}(2g)$.

To compute the one-loop energy, we have to compute the large u asymptotics of μ_{12} to the next order. From $\mu_{12}/Q \sim u^{\Delta-S-L} \simeq 1 + (\Delta - S - L)\log u + \mathcal{O}(g^2)$ we see that we have to find the prefactor of the $\log u$ term. Such large u behavior clearly shows that at the next order μ_{12} can no longer be a polynomial. Instead, μ_{12} develops singularities at the collapsing branch cuts $u = in$, $n \in \mathbb{Z}$ in addition to a modified polynomial part. We denote the singular part of μ_{12}^+ by R. To separate the regular and singular parts we write μ_{12} in the following way:

$$\mu_{12} = \left(\frac{\mu_{12} + \mu_{12}^{++}}{2}\right) + \sqrt{u^2 - 4g^2}\left[\frac{\mu_{12} - \mu_{12}^{++}}{2\sqrt{u^2 - 4g^2}}\right],$$

where, due to Eq. (5), both expressions inside the brackets have a trivial monodromy on the cut $[-2g, 2g]$ thus being very smooth near the origin. The singularity comes solely from the square root factor whose small g expansion reads $\sqrt{u^2 - 4g^2} = u - 2g^2/u + \cdots$, which allows us to fix $R^- \simeq g^2 r/u$ with $r \equiv Q'(i/2) - Q'(-i/2)$ in the vicinity of $u = 0$. Additionally using Eq. (5), we get $R^+ \simeq -g^2 r/u$.

In the vicinity $u \sim i(n + 1/2)$ of the other singularities, R must satisfy the same Baxter equation as Q, up to some regular terms. With the poles of $R(u)$ defined above at $u = \pm i/2$, the solution is unique and is given by

$R(u) = ig^2 r[\psi(\frac{1}{2} - iu) + \psi(\frac{1}{2} + iu)]Q(u)/Q(i/2)$. Now we can expand R at large u to get $R(u)/Q(u) \simeq [2irg^2/Q(i/2)]\log u$ from where we immediately get $\Delta = L + S + 2irg^2/Q(i/2)$, thus reproducing the well-known expression for the one-loop dimension $\Delta = L + S + 2ig^2\partial_u \log(Q^+/Q^-)|_{u=0}$.

Cusp anomalous dimension.—It was shown in Refs. [26,27] that the Wilson line with a cusp of an angle ϕ can be described by essentially the same system of TBA equations. As a consequence, it can be also studied via the $\mathbf{P}\mu$ system that turns out to be a very efficient approach, as we are going to demonstrate. We consider a particular limit of small ϕ. For a more general case, with more details of the derivation, see Ref. [28].

Whereas $\mathbf{P}\mu$ equations remain unaltered, it is the large u behavior that distinguishes this case from the case of local operators. In particular, one finds that $\mathbf{P}_a \simeq (A_1 u^{-L+1/2}, A_2 u^{-L-1/2}, A_3 u^{+L+3/2}, A_4 u^{+L+1/2})$ instead of Eq. (9). Even though Eq. (10) is not fully applicable now, it appears to capture correctly the behavior of $\mathbf{P}_a$ at small ϕ: For the case of the vacuum state $S = 0$ and $\Delta = L + \mathcal{O}(\phi^2)$. We see that at $\phi = 0$ $A_2 A_3 \simeq A_4 A_1 \to 0$ suggesting that to the leading order $\mathbf{P}_a = 0$. Hence, one gets from Eq. (6) $\tilde{\mu}_{ab} = \mu_{ab}$; i.e., μ_{ab} has no cuts; it is then just a periodic function as follows from Eq. (5).

Another specific feature of this case is that Y functions have poles that originate from the boundary dressing phase. In particular, the product (8) has simple poles at $u = in/2$ for any integer $n \neq 0$ [27]. By requiring the regularity of the $\mathbf{P}\mu$ system, we see that in Eq. (8) the poles can only originate from zeros of μ_{12}. Hence, μ_{12} is a periodic entire function with simple zeros at $in/2$. In addition, as Y functions are even for the vacuum, each μ_{ab} has a certain parity with respect to u: for instance, μ_{12} is odd and, hence, it has the form $\mu_{12} = C\sinh(2\pi u)$.

We have no physical reason to introduce infinite sets of zeros for other μ_{ab}'s, and we assume from their periodicity that they are just constants that are further constrained by the parity $\mu_{13} = \mu_{24} = 0$ because they are odd. Then $\mu_{34} = 0$ and $\mu_{14} = \pm 1$, in order to satisfy Eq. (4). A consistent choice of the sign is $\mu_{14} = -1$. Then, Eq. (3) gives

$$\tilde{\mathbf{P}}_1 - \mathbf{P}_1 = -C\sinh(2\pi u)\mathbf{P}_3, \qquad \tilde{\mathbf{P}}_3 + \mathbf{P}_3 = 0, \tag{12}$$

$$\tilde{\mathbf{P}}_2 + \mathbf{P}_2 = -C\sinh(2\pi u)\mathbf{P}_4, \qquad \tilde{\mathbf{P}}_4 - \mathbf{P}_4 = 0. \tag{13}$$

In what follows, for simplicity we consider the case $L = 0$. The generalization to arbitrary L can be done very similarly. First, we notice that in order to cancel the pole in the denominator of Eq. (8) at $u = 0$ we have to assume $\mathbf{P}_1\mathbf{P}_2 = 0$ at $u = 0$. If we "split" this zero between all $\mathbf{P}$'s by introducing a $\sqrt{u}$ factor into each of them, we also ensure half-integer asymptotics of $\mathbf{P}$'s. From Eq. 13 we see that $\mathbf{P}_4/\sqrt{u}$ should have no cut and behaves as u^0 at infinity, so it is simply $\mathbf{P}_4 = A_4\sqrt{u}$. On the other hand, $\mathbf{P}_3/\sqrt{u}$ should flip its sign when crossing the cut $[-2g, 2g]$ and, thus, $\mathbf{P}_3 = A_3\sqrt{u}\sqrt{u^2 - 4g^2}$. $\mathbf{P}_2$ is given from Eq. 13 by the Hilbert transform of $\sinh(2\pi u)$:

$$\frac{-\mathbf{P}_2}{CA_4\sqrt{u}} = \oint_{-2g}^{2g} \frac{\sqrt{u^2 - 4g^2}}{\sqrt{v^2 - 4g^2}} \frac{\sinh(2\pi v)}{4\pi i(v - u)} = \sum_{n=1}^{\infty} \frac{I_{2n-1}(4\pi g)}{x^{2n-1}},$$

where $x(u)$ is defined by $x + 1/x = u/g$, so we have to set $A_2 = -gCA_4 I_1(4\pi g)$. Finally, the solution for $\mathbf{P}_1$ is $\mathbf{P}_1 = -(A_3/A_4) \times \sqrt{u^2 - 4g^2}\mathbf{P}_2 + (A_1 + A_3 A_2/A_4)\sqrt{u}$. Now, we introduce ϕ by requiring that $1 + Y_{11} \simeq -\varphi^2/2$ for $u \to \infty$ and find the energy from $Y_{11}Y_{22} - 1 \simeq 2i\Delta/u$ (note an extra 2 in this equation, which is due to the open boundary conditions). We notice that to match these expansions we should first assume $A_1 A_4 = A_2 A_3$ as otherwise $Y_{11}Y_{22} - 1$ would grow linearly. Then the first condition gives $-\varphi^2/2 = iA_1 A_4/2$ while the second condition gives $\Delta = -\varphi^2 g^2 (1 - I_3(4\pi g)/I_1(4\pi g))$, which is the same result as that found from localization in Refs. [29,30] or using the TBA/FINLIE approach in Ref. [31].

Discussion.—In this Letter, we formulated the $\mathbf{P}\mu$ system, which provides a new conceptual insight into the AdS/CFT integrability. In particular, the present $\mathbf{P}\mu$ system, with $\mathbf{P}_a^+/\mathbf{P}_a^-$ corresponding to the S^5 eigenvalues of the quasi-classical monodromy matrix, is the perfect counterpart of the $\mathbf{Q}\omega$ system to be described in Ref. [23]: the four fundamental fermionic Q functions $\mathbf{Q}_{\hat{a}}$ have only one long cut $(-\infty, -2g] \cup [2g, \infty)$ and their monodromies are expressed through a 4×4 matrix ω (periodic on the sheet with short cuts). We believe that $\mathbf{Q}_{\hat{a}}^+/\mathbf{Q}_{\hat{a}}^-$ correspond to the AdS_5 eigenvalues of Ω.

These two systems are related by linear relations of the type $\mu_{ab} = \mathbf{Q}_{ab\hat{a}\hat{b}}\omega^{\hat{a}\hat{b}}$, which allowed us to make explicit the Lorenz spin S dependence of the coefficients in Eq. (10), using the idea that S enters the asymptotics of $\mathbf{Q}_{\hat{a}}$ and, thus, to close the $\mathbf{P}\mu$ system on itself [23]. In addition, the symmetry between these two systems would *a priori* allow us to interchange the role of $\mathbf{P}_a$ and $\mathbf{Q}_{\hat{a}}$. One interesting application is the possibility to construct the "physical T hook"—where the Y and T systems have the same algebraic formulation as in the original mirror T hook, but all cuts are short instead. At weak coupling, short cuts collapse and we expect the one-loop physical T functions to be the eigenvalues of transfer matrices of the $\mathfrak{psu}(2,2|4)$ XXX spin chain [32]. The exact physical T functions seem to represent the eigenvalues of, yet to be constructed, T operators of an all-loop $\mathcal{N} = 4$ SYM spin chain.

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[14] We denote $f^{\pm} = f(u \pm i/2)$ and $f^{[\pm k]} = f(u \pm ki/2)$.
[15] With the present denition of $\mathbb{T}_{a,s}$, Eq. 1 holds when $|\mathrm{Im}(u)| < s - a$, and an analytic continuation is necessary to make Eq. 1 hold everywhere by imposing long cuts for $\mathbb{T}_{a,s}$.
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