

Universal Attractor for Inflation at Strong Coupling

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We introduce a novel nonminimal coupling between gravity and the inflaton sector. Remarkably, for large values of this coupling all models asymptote to a universal attractor. This behavior is independent of the original scalar potential and generalizes the attractor in the φ^4 theory with nonminimal coupling to gravity. The attractor is located in the “sweet spot” of parameter values that are preferred by Planck’s recent results.

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Introduction.—The data releases by WMAP9 and Planck2013 [1] attracted attention of cosmologists to two very different cosmological models which, surprisingly, made very similar observational predictions: the Starobinsky model $R + R^2$ [2] and the chaotic inflation model $V(\varphi) \sim \varphi^4$ [3] with nonminimal coupling to gravity $(\xi/2)\varphi^2 R$ [4,5]. For $\xi \gtrsim 0.1$, both of these models predict that, for large number of e -foldings N , the spectral index and tensor-to-scalar ratio are given by

$$n_s = 1 - 2/N, \quad r = 12/N^2. \quad (1)$$

For $N \sim 60$, these predictions are $n_s \sim 0.967$, $r \sim 0.003$ ($n_s \sim 0.964$, $r \sim 0.004$ for $N \sim 55$) which are in the “sweet spot” of cosmological observables that are highlighted by the WMAP9 and Planck2013 data.

Further investigations revealed that many other inflationary theories also predict n_s and r given by (1). In particular, (1) is a universal attractor point for a broad class of theories with spontaneously broken conformal or superconformal invariance [6], and for closely related models with negative nonminimal coupling $\xi < 0$ [7].

However, until now, in the theories with nonminimal coupling $(\xi/2)\varphi^2 R$ with $\xi > 0$, this generality did not extend beyond the models with the potentials $\sim \varphi^4$ studied in [4,5]. In this Letter, we propose a very simple generalization of this class of models, which applies to practically every inflationary potential $V(\varphi)$. This can be achieved by introducing a generalized version of nonminimal coupling to gravity, such as $\xi\sqrt{V(\varphi)}R$, or even a much simpler one, $\xi\varphi R$. We will show that all of these models have the universal set of predictions (1) in the strong coupling limit $\xi \rightarrow \infty$. We will also show exactly how the predictions of the theories with different potentials $V(\varphi)$ depend on ξ and approach the universal attractor point (1) with the growth of ξ .

Nonminimal coupling.—The starting point of many inflationary models is a Lagrangian consisting of the Einstein-Hilbert term for gravity plus a kinetic term and scalar

potential for the inflaton field. The Lagrangian including the generalized nonminimal coupling to gravity reads

$$\mathcal{L}_J = \sqrt{-g} \left[\frac{1}{2} \Omega(\varphi) R - \frac{1}{2} (\partial\varphi)^2 - V_J(\varphi) \right], \quad (2)$$

with (various aspects of generalized nonminimal coupling were studied in [8])

$$\Omega(\varphi) = 1 + \xi f(\varphi), \quad V_J(\varphi) = \lambda^2 f^2(\varphi). \quad (3)$$

Our notation for $V_J(\varphi)$ does not imply any constraint on the scalar potential other than being positive, and is motivated by the superconformal version of the model that will be introduced later. Because of the nonminimal coupling, we will refer to this form of the theory as the Jordan frame. In order to transform to the canonical Einstein frame, one needs to redefine the metric:

$$g_{\mu\nu} \rightarrow \Omega(\varphi)^{-1} g_{\mu\nu}. \quad (4)$$

This brings the Lagrangian to the Einstein-frame form:

$$\mathcal{L}_E = \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} \left\{ \Omega(\varphi)^{-1} + \frac{3}{2} [\log \Omega(\varphi)]^2 \right\} (\partial\varphi)^2 - V_E(\varphi) \right), \quad \text{with} \quad V_E(\varphi) = \frac{V_J(\varphi)}{\Omega(\varphi)^2}. \quad (5)$$

Note that in the absence of nonminimal coupling $\xi = 0$, the distinction between Einstein and Jordan frame vanishes. In this case the inflationary dynamics is fully determined by the properties of the scalar potential $V_J(\varphi) = V_E(\varphi)$. In the presence of a nonminimal coupling, however, one has to analyze the interplay between the different contributions to the inflationary dynamics due to $V_J(\varphi)$ and ξ .

Behavior at weak coupling.—We first analyze the effect of the nonminimal coupling for small ξ . At linear order, the kinetic terms in (5) give rise to the following definition of the canonical scalar field ϕ :

$$\frac{\partial\phi}{\partial\varphi} = 1 - \frac{\xi}{2}f(\varphi), \quad (6)$$

where we are suppressing higher-order terms. A similar approximation can be made to the Einstein-frame potential,

$$V_E = \lambda^2 f(\varphi)^2 [1 - 2\xi f(\varphi)]. \quad (7)$$

Remarkably, this implies that the number of e -foldings,

$$N = \int_{\varphi_{\text{end}}}^{\varphi_N} \left(\frac{\partial\phi}{\partial\varphi} \right)^2 \frac{V_E}{\partial V_E / \partial\varphi} d\varphi, \quad (8)$$

has no linear corrections. There will be corrections to N due to changes to the field value φ_{end} when inflation breaks down since ϵ or η become of order 1. However, these will be subdominant as N generically receives the largest contribution from the first phase of the inflationary trajectory, where $\partial V / \partial\varphi$ is small. At first approximation, there are therefore no changes to N at linear order. The only corrections to the slow-roll parameters follow from the explicit expressions for these quantities,

$$\begin{aligned} \epsilon &= \frac{1}{2} \left(\frac{1}{V_E} \frac{\partial V_E}{\partial\varphi} \frac{\partial\phi}{\partial\varphi} \right)^2 = [1 - \xi f(\varphi_N)] \epsilon_J, \\ \eta &= \frac{1}{V_E} \frac{\partial}{\partial\varphi} \left(\frac{\partial V_E}{\partial\varphi} \frac{\partial\phi}{\partial\varphi} \right) \frac{\partial\phi}{\partial\varphi} = \eta_J - \frac{5}{2} \xi f(\varphi_N) \epsilon_J, \end{aligned} \quad (9)$$

evaluated at the same point in field space φ_N as for the original scalar potential $V_J(\varphi)$. Given a value (r_0, n_{s0}) for the cosmological observables of any inflationary model without nonminimal coupling, at small coupling these will transform in the following universal way:

$$\begin{aligned} n_s &= 1 + 2\eta - 6\epsilon = n_{sJ} + \frac{\xi}{16} f(\varphi) r_J, \\ r &= 16\epsilon = r_J - \xi f(\varphi) r_J. \end{aligned} \quad (10)$$

Therefore, all models at first will move along parallel lines with a slope of -16 in the (n_s, r) plane.

Behavior at strong coupling.—Next we turn to the strong coupling limit of inflation, where ξ becomes very large. We will later quantify how large ξ needs to be for this limit. First, we will present two arguments for a universal attractor behavior in the limit of infinite ξ . The first argument follows the line of reasoning above, but considers an expansion for large ξ instead. The number of e -foldings in this case reads

$$N = \int_{\varphi_{\text{end}}}^{\varphi_N} \left(\frac{3}{4} \xi f(\varphi)' + \frac{f(\varphi)}{2f(\varphi)'} - \frac{3f(\varphi)'}{4f(\varphi)} \right) d\varphi. \quad (11)$$

Without specifying the function $f(\varphi)$, the first term can be integrated in a model-independent way; this would not be

possible when including next-to-linear order terms. Here we assume that we are away from the extrema of f where $f' = 0$ so that the second term in (11) blows up. Moreover, one can neglect the contribution from the end of inflation in the large- N limit (this is also true at strong coupling). We therefore obtain

$$N = \frac{3}{4} \xi f(\varphi_N). \quad (12)$$

Note that this requires $f(\varphi_N)$ to asymptote to zero in the strong coupling limit; one zooms in on the region where the scalar potential vanishes. In this limit one obtains the simple formula for the spectral index and tensor-to-scalar ratio (1). This analysis demonstrates that the values of n_s and r for all positive scalar potentials $V_J(\varphi)$ with a Minkowski minimum asymptote to (1) in the strong coupling limit.

The second argument starts from the kinetic term in Einstein frame (5). In the large- ξ limit, the two contributions to the kinetic terms scale differently under ξ . Retaining only the leading term, the Lagrangian becomes

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{1}{2} R - \frac{3}{4} [\partial \log(\Omega(\varphi))]^2 - \lambda^2 \frac{f(\varphi)^2}{\Omega(\varphi)^2} \right]. \quad (13)$$

Remarkably, the canonically normalized field ϕ involves the function $\Omega(\varphi)$ of the scalar potential itself:

$$\phi = \pm \sqrt{\frac{3}{2}} \log(\Omega(\varphi)). \quad (14)$$

Therefore, in terms of ϕ , the theory has lost all reference to the original scalar potential; it has the universal form. In case of odd $f(\varphi)$, we choose the same sign in (14) for both signs of ϕ and find

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda^2}{\xi^2} (1 - e^{-\sqrt{2/3}\phi})^2 \right], \quad (15)$$

which is the scalar formulation of the Starobinsky model [2]. In case the function $f(\varphi)$ is even in φ , we choose opposite signs and find the following attractor action,

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda^2}{\xi^2} (1 - e^{\sqrt{2/3}\phi})^2 \right], \quad (16)$$

symmetric under $\phi \rightarrow -\phi$.

The crucial assumption in the above derivation was that the kinetic term is dominated by the second contribution. In other words, we require

$$\Omega(\varphi) \ll \frac{3}{2} \Omega(\varphi)^{1/2}. \quad (17)$$

In terms of our original scalar potential and the associated slow-roll parameter ϵ_J , this translates into

$$\frac{1 + \xi f(\varphi)}{\xi^2 f(\varphi)^2} \ll \frac{3}{4} \epsilon_J(\varphi). \quad (18)$$

Interestingly, this implies that models with a flatter scalar potential require a stronger coupling in order to reach the vicinity of the attractor. In contrast, for less fine-tuned models with larger values of ϵ_J , the system reaches the attractor for a lower value of the coupling ξ . It is important to point out that even models with a scalar potential that does not support inflation still asymptote to (15) or (16) at strong coupling and have the same observables (1).

The amplitude normalization of the power spectrum constrains the overall coefficient of the scalar potentials. For $\xi = 0$ this depends on the coefficient λ of the original scalar potential. For large ξ , the Planck normalization of the power spectrum requires $\lambda/\xi \approx 10^{-5}$. For intermediate values there is an interplay between the coefficients λ and ξ , which can always be satisfied by suitable choice of λ . For the specific case of the φ^4 theory, this was discussed in detail in [5].

Supergravity embedding.—The nonminimal coupling can be embedded in supergravity. We follow the setup of [9], which introduces two chiral multiplets with scalar fields Φ and S . The former will contain the inflaton while the latter is responsible for supersymmetry breaking. We thus take the sGoldstini to be orthogonal to the inflaton, allowing for an arbitrary scalar potential and avoiding the restrictions of [10]. While the original proposal has a specific Kähler potential and an arbitrary function in the superpotential, we take the Kähler potential to depend on $\Omega(\sqrt{2}\Phi)$, which will be related to the scalar potential. Our final expressions are

$$K = -3 \log \left\{ \frac{1}{2} [\Omega(\sqrt{2}\Phi) + \Omega(\sqrt{2}\bar{\Phi})] - \frac{1}{3} S\bar{S} + \frac{1}{6} (\Phi - \bar{\Phi})^2 + \xi \frac{(S\bar{S})^2}{\Omega(\sqrt{2}\Phi) + \Omega(\sqrt{2}\bar{\Phi})} \right\}, \quad (19)$$

$$W = \lambda S f(\sqrt{2}\Phi),$$

where $\Omega(\sqrt{2}\Phi) = 1 + \xi f(\sqrt{2}\Phi)$ and $f(\sqrt{2}\Phi)$ is a real holomorphic function. This leads exactly to the bosonic model discussed above upon identifying $\Phi = \varphi/\sqrt{2}$ while $S = 0$. It can easily be seen that this is a consistent truncation.

The superconformal version of this model explains the simplicity of the Jordan frame potential in these models: in a gauge where the conformon is fixed, the superconformal potential is given by $\mathcal{W} = \lambda S f(\sqrt{2}\Phi)$ (in the notation of [11,12]). This implies that the Jordan frame potential at $S = 0$, $\Phi = \varphi/\sqrt{2}$, is given by

$$V_J = \lambda^2 \left| \frac{\partial \mathcal{W}}{\partial S} \right|^2 = \lambda^2 f^2(\varphi). \quad (20)$$

This model generalizes the supersymmetric embedding of the φ^4 theory considered in [12] to arbitrary scalar

potentials. In that specific case, one could interpolate between a canonical Kähler potential depending on $\Phi\bar{\Phi}$ and a shift-symmetric one depending on $(\Phi - \bar{\Phi})^2$ by means of ξ , but this is not possible in the general case.

Regarding the stability of the truncation to the inflationary trajectory, where three scalars are truncated out, the masses of the four fields are given by $m_{\text{Re}\Phi}^2 = \eta V$, $m_{\text{Im}\Phi}^2 = (\frac{4}{3} + 2\epsilon - \eta)V$, $m_S^2 = (-\frac{2}{3} + 6\zeta + \epsilon)V$. Up to slow-roll corrections, one can thus stabilize all three truncated fields with the choice $\zeta > \frac{1}{9}$.

This supergravity embedding goes some way towards an understanding of the symmetries underlying the attractor behavior. In particular, for $\xi = 0$ there is symmetry enhancement in the Kähler potential: it has a shift symmetry in the real part of Φ and hence does not depend on the inflaton. The same holds for any value of ξ when choosing the function $f(\sqrt{2}\Phi)$ to be a constant. Any deviations from this will introduce a spontaneous breaking of this symmetry.

Chaotic inflation.—In this section we illustrate the universal attractor behavior for chaotic inflation [3], with the scalar potential

$$V_J(\varphi) = \lambda^2 M_{\text{Pl}}^{4-n} \varphi^n. \quad (21)$$

Without nonminimal couplings, these have the following cosmological observables:

$$n_{sJ} = 1 - \frac{2+n}{2N}, \quad r_J = \frac{4n}{N}, \quad (22)$$

at large N . These are specific cases of the most general $1/N$ dependence derived in [13]. The attractor behavior for this class is depicted in Fig. 1. The crossover behavior between the two regimes spans a number of decades of the nonminimal coupling ξ , and in addition is model dependent. Indeed, models with a larger $\epsilon_J(\varphi)$ require a smaller coupling to approach the attractor. However, the attractor is always reached before $\xi = 100$.

Generalized strong coupling attractor.—In the previous investigation, we assumed that $\Omega(\varphi) = 1 + \xi f(\varphi)$, and $V_J(\varphi) = \lambda^2 f^2(\varphi)$, but one may also consider a more general possibility,

$$\Omega(\varphi) = 1 + \xi g(\varphi), \quad V_J(\varphi) = \lambda^2 f^2(\varphi), \quad (23)$$

where we introduce an additional functional freedom in the definition of $\Omega(\varphi)$, disconnecting it from $V_J(\varphi)$. Once we do so, $V_E(\varphi) = V_J(\varphi)/\Omega^2(\varphi)$ no longer approaches a constant at large φ . Does it mean that our previous results become inapplicable?

Note that when the field rolls to the minimum of its potential, $f(\varphi)$ is supposed to vanish, or at least become incredibly small to account for the incredible smallness of the cosmological constant $\sim 10^{-120}$. As in the previous analysis, we will assume that the same is true for the

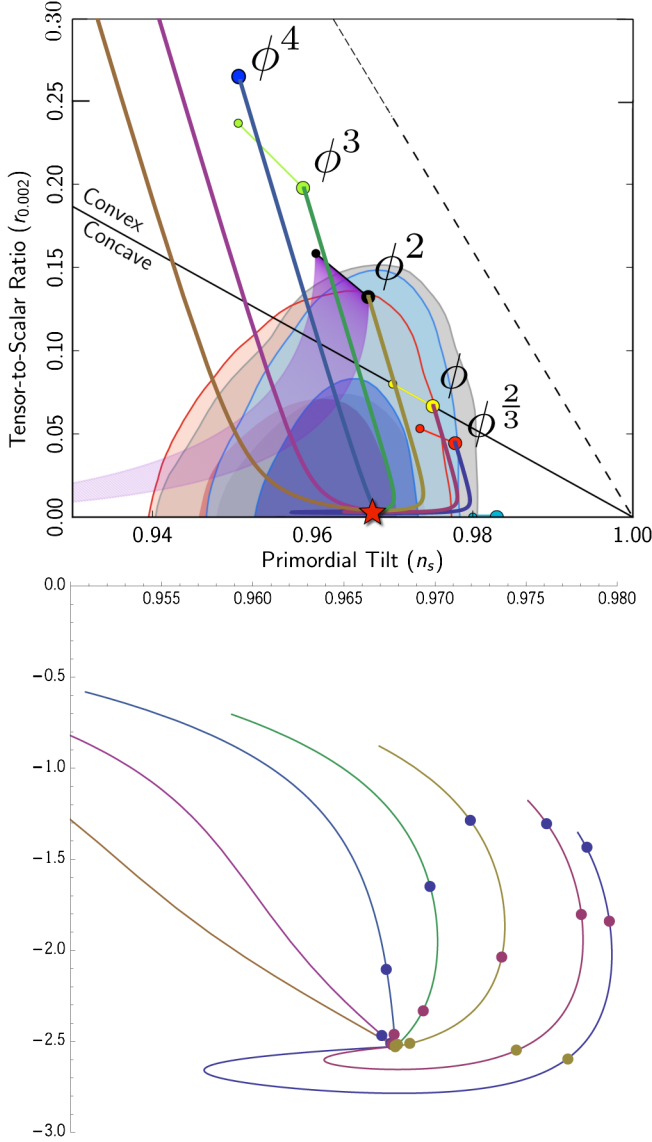


FIG. 1 (color online). The ξ dependence of (n_s, r) on a linear and a logarithmic scale for different chaotic models with $n = (2/3, 1, 2, 3, 4, 6, 8)$, from right to left, for 60 e -foldings (upper panel). The points on the logarithmic scale (lower panel) correspond to $\log(\xi) = (-1, \dots, 1)$, from top down. The convergence to the attractor point occurs almost instantly for $n \geq 4$.

function $g(\varphi)$. Therefore, we will expand both functions in a Taylor series in φ , assuming that they vanish at some point (which can be taken as $\varphi = 0$ by a field redefinition) and that they are differentiable at this point:

$$f(\varphi) = \sum_{n=1}^{\infty} f_n \varphi^n, \quad g(\varphi) = \sum_{n=1}^{\infty} g_n \varphi^n. \quad (24)$$

By rescaling λ and ξ , one can always redefine $f_1 = g_1 = 1$ without loss of generality.

Let us first ignore all higher-order corrections, i.e., take $f(\varphi) = g(\varphi) = \varphi$. In this case our investigation is reduced

to the one performed earlier, and Eq. (12) yields $\varphi_N = (4N)/(3\xi)$. This result implies that for $\xi \gg N$ one has $\varphi_N \ll 1$.

If one now adds all higher-order terms and makes an assumption that the coefficients f_n and g_n are $O(1)$, one finds that in the large coupling limit $\xi \gg N$, these corrections are suppressed by the powers of $(4N)/(3\xi)$, so one can indeed ignore these terms. This means that in the large ξ limit the potential $V(\phi)$ in terms of the canonically normalized inflaton field ϕ coincides with the potential (15), and all observational predictions of this new class of theories coincide with the predictions (1). This universality is similar to the universality of predictions of the broad class of theories found in [6,7].

In this analysis we assumed that the Taylor series begins with the linear term. However, if the theory is symmetric with respect to the change $\varphi \rightarrow -\varphi$, as is the case, e.g., in the φ^4 theory, then the expansion for $f(\varphi)$ and $g(\varphi)$ begins with the quadratic terms. The rest follows just as in the case discussed above: For $\xi \gg N$, higher-order corrections do not affect the description of the observable part of the Universe; therefore, we have the same observational predictions (1) as before, but now the relevant part of the potential is even with respect to the field ϕ and its large ξ limit is given by (16).

Discussion.—In this Letter we have demonstrated that there is a universal attractor for all inflationary models when introducing a specific nonminimal coupling term correlated with the choice of the potential. Upon taking its coefficient ξ large enough, all models asymptote to a spectral index and tensor-to-scalar ratio that are indistinguishable from (1), and hence are in perfect agreement with the Planck results. How large ξ needs to be in order to reach the attractor is model dependent, but in all examples we have found that $\xi = 100$ is sufficient. Moreover, the initial approach to the attractor proceeds in a parallel fashion; upon turning on ξ , the different models move in identical directions in the (n_s, r) plane. The resulting image in Fig. 1 resembles that of a comb. The straight line towards the attractor for the φ^4 theory is a coincidence between the slope of the lines and the location of that particular theory; other models do not start moving in the direction of the attractor.

The new class of cosmological attractors (2) can be generalized in many different ways. We discussed its supergravity or superconformal generalization, as well as the possibility to use the function Ω not related to $f(\varphi)$. This additional universality appears because in the large ξ limit the description of the last N e -foldings of inflation requires knowledge of a very limited range of values of $f(\varphi)$ and φ , where the simplest linear or quadratic approximations may be sufficient. Zooming to this limited range of variation of φ is accompanied by the effective stretching of the potential in terms of the canonical inflaton field ϕ . This stretching allows the existence of an inflationary regime even in the theories where the original potential

$V_J(\varphi)$ is very steep. The resulting Einstein frame potential acquires the form (15) and (16), which leads to the universal observational predictions (1) for this new class of theories.

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