Two Color Free-Electron Laser and Frequency Beating

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We review the theory of two color high gain free-electron laser emission, derive the integral equation characterizing the evolution of the optical intensities, and provide a description of the relevant dynamics. The characteristic feature of this regime is the existence of a mutual bunching, whose origin and role are discussed.

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The generation of free-electron laser (FEL) radiation with two or more simultaneous colors opens new scenarios in applications [1,2] and in the study of the underlying physics. By means of two color x radiation, it is possible to deepen the fundamental knowledge of the properties of materials and living systems, probing the matter on the atomic scale in time and space. Pairs of colored x-ray pulses can be particularly useful to perform pump-probe experiments of structural dynamics, a class of experiments where the process is excited with a first pulse, and, after a certain time, a second pulse of another color is used to interrogate the state of the system and get the image.

Several schemes have been proposed to produce this type of radiation, whose first realization was achieved with an infrared FEL oscillator [3]. Experiments in the soft x-ray regime [4] and on the generation of two color pulses in the FEL gain modulated regime [5] have been reported more recently. At the SPARC test facility [6,7] double color operation [8] has been carried out by using electron beams constituted by two beamlets with two energy levels and with different split in time, prepared with a method combining laser comb technique and velocity bunching in the linac, extensively used in beam dymamics [9] and FEL radiation [10,11] experiments.

In this Letter we discuss from a theoretical point of view the physics of two color operation. We will derive the small signal high gain integral equation, which rules the evolution of the field amplitudes associated with the colors at small times, and compare the analytical results with the data obtained with the FEL code PROMETEO [12] implemented in such a way as to include the case of electron beams with two energy levels. The role of the mutual bunching between the two waves, which represents a key feature of the process, is analyzed. Furthermore, other mechanisms of generation of two frequency radiation are discussed.

When two color pulses are emitted by two electron bunches presenting different energies and propagating along the undulator independently, the small signal analysis relies on two decoupled integral equations. If, instead, the two colors arise from a single bunch with an energy distribution shaped in two narrow and close bands, the FEL process is described by a single integral equation, with a kernel characterized by a double peak structure.

The equation for the small signal evolution of the laser field driven by an e-beam with two different energy bands equally distributed around the peaks and narrow enough to permit us to neglect the associated inhomogeneous broadening is

$$\frac{da}{d\xi} = \frac{i}{6\sqrt{3}} \int_0^{\xi} \xi' (e^{-i(\tilde{\nu}-\tilde{\nu}_1)\xi'} + e^{-i(\tilde{\nu}-\tilde{\nu}_2)\xi'}) a(\xi - \xi') d\xi'. \quad (1)$$

Here, $a(\xi)$ is Colson's dimensionless amplitude [with initial condition $a(0) = 1$], defined as $a(\xi) = (\chi_1/2k_u\rho^2)E$ [13], with $\xi = z/L_g$ the normalized longitudinal coordinate along an undulator with period $\lambda_u = k_u/2\pi$ and deflection strength K. $L_g = \lambda_u/(4\pi\sqrt{3}\rho)$ represents the gain length, ρ the Pierce parameter [14], E the radiation field, and $\chi_1 = [(eK[JJ])/(2\gamma^2 mc^2)]$ is connected to the Bessel coefficient $[JJ]$ for a planar undulator and to the Lorentz factor γ. The detuning parameters $\tilde{\nu}_\alpha = (\omega_\alpha - \omega_R)/(2\sqrt{3}\rho\omega_R)$ measure the relative distance between the frequency peaks ω_α ($\alpha = 1, 2$) with respect to the gain bandwidth, ω_R denoting the resonant frequency of the mean e-beam energy. Figure 1 shows the gain derived from Eq. (1) as a function of the normalized frequency $\tilde{\nu}$, in the case of close energy bands [1(a)], and well separated energies [1(b)].

Furthermore, by setting $\tilde{\Delta} = \tilde{\nu} - \tilde{\nu}_1$, $\eta = \tilde{\nu}_1 - \tilde{\nu}_2$ and deriving both sides of Eq. (1), we obtain

$$\frac{d^3 a}{d\xi^3} + 2i\tilde{\Delta}\frac{d^2 a}{d\xi^2} - \tilde{\Delta}^2 \frac{da}{d\xi} = \frac{i}{6\sqrt{3}}[(1+e^{-i\eta\xi})a + \Phi], \quad (2)$$

with $\Phi = -\eta e^{-i(\tilde{\Delta}+\eta)\xi}\left[\eta I + 2i\frac{dI}{d\xi}\right]$ and $I = \int_0^{\xi}(\sigma - \xi)\times e^{i\tilde{\Delta}\sigma} a(\sigma) d\sigma$.

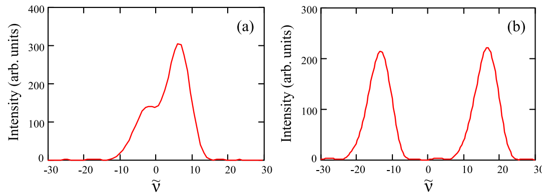

FIG. 1 (color online). Gain curve versus the detuning parameter. The gain function is calculated with a Pierce parameter $\rho = 10^{-3}$ and with a number of undulator periods $N = 400$. Two cases are shown for different energy separation (a) $\tilde{\nu}_2 - \tilde{\nu}_1 = 10$ and (b) $\tilde{\nu}_2 - \tilde{\nu}_1 = 30$. The gain processes of the two waves can be considered mutually independent if $\tilde{\nu}_2 - \tilde{\nu}_1 > 20$ (more than 3 MeV in the case of the SPARC parameters).

In the case of bands completely overlapping, with $\eta = 0$, Eq. (2) reduces to the ordinary FEL third-order high gain equation [14]. If the bunches are separated, the signals carrying the two colors are distinct entities that can grow independently. A certain level of interaction may, however, occur due to partial phase locking or nonlinear beating wave mechanisms, with interference measurable effects. The FEL field can be considered, therefore, as the superposition of two components oscillating at different frequencies and experiencing a mutual competition.

A further possibility is that the FEL interaction occurs between two waves oscillating at different but close frequencies ω_m ($m = 1, 2$) and an electron beam having an energy distribution with two levels resonating with them.

Colson's picture is based on the development of the Maxwell nonlinear equations driven by single particle currents [13]. In the generalization to the case of two waves of different wave numbers k_m, the electron phases in the mth ponderomotive potential $\zeta_m = (k_u + k_m)z - \omega_m t$ satisfy the pendulum equations:

$$\frac{d^2\zeta_n}{d\tau^2} = c_n \sum_m \left(1 + \frac{1}{2}\delta_{nm}\right) |a_m| \cos(\zeta_m + \varphi_m) \quad m \neq n, \tag{3}$$

where $c_n = (K/\sqrt{2})L_u^2(k_n/\gamma_{0n}^4)[1 + (K^2/2)]$ and $\delta_{nm} = (\gamma_{0n}^2/\gamma_0^2)(k_R/k_m) - 1$. Furthermore, $L_u = N\lambda_u$ is the total length and K the strength of the undulator, $\tau = (L_g/L_u)\xi$, k_R is the mean wave number, γ_0 is the mean Lorentz factor of the beam, and γ_{0n} are the Lorentz factors corresponding to each peak. The dimensionless fields $a_n = |a_n|e^{i\phi_n}$ evolve in time according to

$$\frac{da_n}{d\tau} = -j_n\langle e^{-i\zeta_n}\rangle, \tag{4}$$

where $j_n = (f_{Bn}^2/\gamma_{0n})\sqrt{2}\pi(e/mc^2)\tilde{J}_n N\lambda_u K$, f_{Bn} are the Bessel coupling terms, and $\tilde{J}_n$ the current density of each beamlet. The average in Eq. (4) is taken over the initial phase distributions:

$$\langle\cdots\rangle = \frac{1}{4\pi^2}\int_0^{2\pi} d\zeta_1^0 \int_0^{2\pi} d\zeta_2^0(\cdots)f(\zeta_1^0, \zeta_2^0),$$
$$f(\zeta_1^0, \zeta_2^0) = \sum_{p,q=0}^{\infty} b_{pq}^{12} e^{ip\zeta_1^0 + iq\zeta_2^0} \tag{5}$$

where $\zeta_m^0 = \zeta_m(\tau = 0)$, b_{p0}^{12} and b_{0q}^{12} denote the pth and qth harmonic bunching coefficient of the waves 1 and 2, respectively, while b_{pq}^{12} are the p-q beating wave harmonic coefficients.

Figure 2 gives the coupled evolution of the signals for different values of the e-beam peak energies. The unidimensional FEL code PROMETEO [12] has been modified to provide the solution of the pendulum and field equations (3) and (4) with bichromatic electron beams. The initial phases have been distributed uniformly with the same values for the two beamlets, equivalent to an initial phase locking. The fields have been assumed to be distinct, with the wavelength resonating at the electron energy peaks. The field intensity growths are characterized by a "trembling" behavior associated with an exchange of energy between the two fields, which increases in amplitude and period with the decrease of the difference in frequency of the waves. Though the field equations appear to be completely symmetric, the growth of the amplitudes associated with the two waves is not equal, because the longer wavelength, having a shorter gain length, is amplified at a larger level and drives the emission of the other one, while the oscillations are similar (the difference in visibility depends on the logarithmic scale).

An analysis of the wave interaction can be done in the small signal approximation. In this limit, the electron

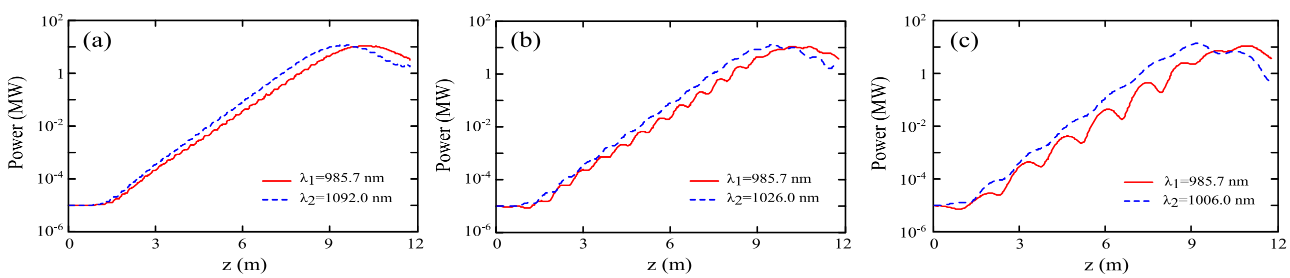

FIG. 2 (color online). Field power evolution (PROMETEO simulation) of the two peaks, along the physics undulator coordinate $z(m)$, for different detuning conditions: (a) $\lambda_2 - \lambda_1 = 107.3$ nm, (b) $\lambda_2 - \lambda_1 = 30.3$ nm, (c) $\lambda_2 - \lambda_1 = 10.3$ nm.

phases, developed at first order, are $\zeta_m = \zeta_m^0 + \nu_{0m}\tau + \delta\zeta_m$, with $\nu_{0m} = (d\zeta/d\tau)|_{\tau=0}$. Since $\delta\zeta_m$ is assumed small, the right-hand side of Eq. (4) can be linearized, thus finding

$$\frac{da_m}{d\tau} = \frac{i}{2} C_m j_m \sum_k \langle e^{-i(\zeta_m^0 - \zeta_k^0)} \times \int_0^\tau d\tau' (\tau - \tau') e^{-i(\nu_{0m}\tau - \nu_{0k}\tau')} a_k(\tau') \rangle, \tag{6}$$

where all frequencies ν are defined as $\nu = 8\sqrt{3}\pi N \rho \tilde{\nu}$. After the evaluation of the average in the right-hand side, the following integral equation is obtained as a generalization of the ordinary high gain integral [Eq. (1)]:

$$\frac{da_1}{d\tau} = \frac{i}{2} C_1 j_1 \left[\int_0^\tau (\tau - \tau') e^{-i\nu_{01}(\tau - \tau')} a_1(\tau')\, d\tau' + \overline{b_{1,2}} \int_0^\tau (\tau - \tau') e^{-i(\nu_{01}\tau - \delta\nu_{12}\tau')} a_2(\tau') d\tau' \right] \tag{7}$$

(and the same for a_2), with $\delta\nu_{12} = -\delta\nu_{21} \approxeq 2\pi N([\lambda_2 - \lambda_1)/\lambda_1]$. The first term on the right-hand side determines the high gain evolution, while the second addend accounts for the wave interaction through the phenomenological factor $\overline{b_{12}} = \overline{b_{21}}$ called the "mutual bunching coefficient" (MBC), defined as

$$\overline{b_{12}} = \langle e^{-i(\zeta_m^0 - \zeta_k^0)} e^{i\nu_{01}\tau'} \rangle \tag{8}$$

and introduced during the average in an heuristic way. The MBC term describes several mechanisms, namely, the phase locking through an external laser or the effect induced by plasma wave oscillations associated with the current density modulation due to the FEL bunching. Without any specific assumption on the phasing mechanism, we assume $\overline{b_{12}}$ as a weight parameter quantifying the mutual interaction. Equation (7) reduces to two third-order differential equations:

$$\frac{d^3 a_1}{d\tau^3} + 2i\nu_{01} \frac{d^2 a_1}{d\tau^2} - \nu_{01}^2 \frac{da_1}{d\tau} = \frac{iC_1}{2} j_1 [a_1 + \overline{b_{12}} e^{-i\delta\nu_{12}\tau} a_2] \tag{9}$$

(and similar for a_2) and, in turn, to a system of six ordinary first-order differential equations:

$$\frac{d}{d\tau} \underset{\sim}{\Phi}(\tau) = \hat{A}(\tau) \underset{\sim}{\Phi}(\tau), \tag{10}$$

with

$$\hat{A}(\tau) = \begin{bmatrix} -2i\nu_{01} & \nu_{01}^2 & \frac{iC_1 j_1}{2} & 0 & 0 & \frac{i\bar{b}_{12} C_1 j_1}{2} e^{i\delta\nu_{12}\tau} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i\bar{b}_{12} C_2 j_2}{2} e^{-i\delta\nu_{12}\tau} & -2i\nu_{02} & \nu_{02}^2 & \frac{iC_2 j_2}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{11}$$

The solution of Eqs. (10) and (11) has been evaluated by using either a full numerical procedure or by the method of the Magnus time ordering [15] and is reported in Fig. 3 for various values of the MBC. The overall qualitative agreement between the PROMETEO results presented in Fig. 2 and the solution of Eqs. (10) and (11) is satisfactory before the onset of saturation ($\tau < 0.6$ corresponding to $z \sim 7$ m), particularly as regards the dependence of the oscillations on $\delta\nu_{12}$. The details of the curves are, however, strongly dependent on the choice of the complex factor $\overline{b_{12}}$ and the choice of the other parameters. In Fig. 3(c), a case where one of the two peaks is initially seedless is presented.

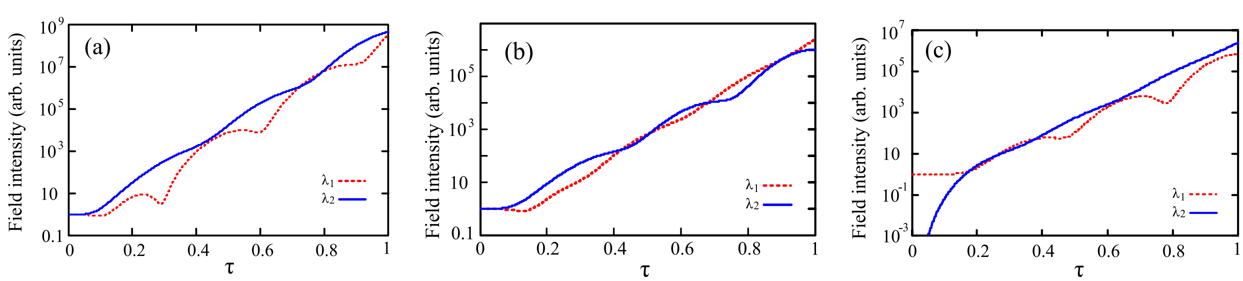

FIG. 3 (color online). Field intensity evolution for the two peaks with $\lambda_1 < \lambda_2$ ($E_1 > E_2$). Different cases are shown with different values of the mutual bunching coefficient: (a) $b_{1,2}^- = 6$, (b) $b_{1,2}^- = 2.5$. In (c), again $b_{1,2}^- = 6$, but with the lower frequency field (blue continuous line) having a vanishing input seed. Plots are calculated with $\rho \approx 10^{-3}$ and with a number of undulator periods $N = 400$.

Equation (9) predicts indeed that the mutual bunching acts as the source for the associated intensity, similar to what occurs in the dynamics of prebunched beams.

Equation (7) describes the case of an initially non bunched e-beam. The generalization to the case of prebunched beams, with an initial nonuniform phase distribution, is

$$\frac{da_m}{d\tau} = -j_m e^{-i\nu_{0m}\tau}\left\{\langle e^{-i\zeta_m^0}\rangle - i\frac{C_m}{2}\sum_n\left(1+\frac{\delta_{mn}}{2}\right)\right.$$
$$\left.\times[\langle e^{-i(\zeta_m^0-\zeta_n^0)}\rangle I_n(\tau) + \langle e^{-i(\zeta_m^0+\zeta_n^0)}\rangle I_m^*(\tau)]\right\}, \qquad (12)$$

with $I_n(\tau) = \int_0^\tau d\tau'(\tau-\tau')a_n(\tau')e^{i\nu_{0n}\tau'}$. The first term is responsible for the coherent spontaneous emission from prebunched e-beams, while the new terms lead to the trembling behavior even in the absence of both initial seeds. A discussion of Eq. (12) in the context of FEL processes with coherent spontaneous emission for one single bunch is found in Ref. [16]. From the experimental point of view, the problem is how to resolve the field oscillations with available detectors, typically Joule meters, for wavelength differences larger than 100 nm. The oscillations may be weak and hardly observable, and the measure of the spectral distribution along the undulator might be a more reliable tool. To study the spectrum it is necessary to modify the integral equation with the inclusion of short pulses effects [16]:

$$\frac{da_1}{d\tau} = \frac{iC_1 j_1(\zeta+\Delta\tau\ \ )}{2}\int_0^\tau(\tau-\tau')[e^{-i\nu_{01}(\tau-\tau')}a_1(\zeta+\Delta\tau',\tau')$$
$$\times d\tau' + \overline{b_{1,2}}e^{-i(\nu_{01}\tau-\delta\nu_{12}\tau')}a_2(\zeta+\Delta\tau',\tau')d\tau'], \qquad (13)$$

with Δ the slippage length. The field intensity distributions along the bunch coordinate are reported in Fig. 4, for different beam overlapping and energies. Fringes in the time domain with distance depending on the energy gap between the two beamlets are observed. In the first case, the fringes,

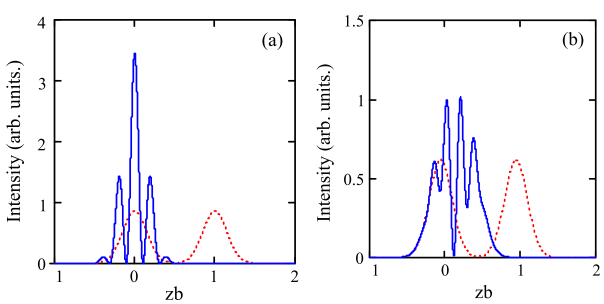

FIG. 4 (color online). Dimensionless intensity versus zb, packet coordinate normalized to the bunch length. The red dotted curve refers to the radiation produced by two identical bunches separated by a rms bunch length. The blue curve shows (a) completely overlapping bunches and (b) bunches with distance $d_b \approx 0.4\sigma_z$.

due to the mutual interference, are evident; in the second, they are less pronounced and disappear when the bunches are separated. An analogous behavior has been observed in the two color experiment at SPARC [8].

The two color operation in FEL amplifiers has been analyzed in Ref. [17]. The authors employed a 3D nonlinear polychromatic simulation to study the case of a FEL seeded with two narrow band fields at closely spaced wavelengths within the gain band and predicted the growth of a discrete spectrum of beat waves outside the gain band. The beat waves grew parasitically due to electron bunching in the seeded waves with rates higher than the seeded waves themselves.

Another mechanism, namely, the FEL nonlinear wave mixing, occurs at saturation when the primary two colors are nonlinearly mixed, and also if the two bandwidths are initially very narrow and separated, and can be particularly intense in seeded cases. The energy distribution around the two peaks ε_1 and ε_2 as a function of the energy ε is modulated by the FEL interaction according to

$$f(\varepsilon,\tau) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{[\varepsilon-\varepsilon_1-\tau|a_1|\sin(\zeta_1+\varphi_1)]^2}{2\sigma_1^2}\right)$$
$$\times\exp\left(-\frac{[\varepsilon-\varepsilon_2-\tau|a_2|\sin(\zeta_2+\varphi_2)]^2}{2\sigma_2^2}\right), \qquad (14)$$

with ($\sigma^{-2} = \sigma_1^{-2} + \sigma_2^{-2}$) and a_1, a_2, ϕ_1, ϕ_2 the wave intensities and phases. By using the Anger-Jacobi expansion, Eq. (14), leads to

$$f(\varepsilon,\tau) \approx \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{i(m\zeta_1+n\zeta_2)}J_m\left(\frac{i\tau|a_1|}{\sigma_1}\right)J_n\left(\frac{i\tau|a_2|}{\sigma_2}\right), \qquad (15)$$

where J_m are Bessel functions. The trend of the higher order bunching coefficients associated with the beatings is then

$$b_{mn} = \frac{1}{4\pi^2}\int_0^{2\pi}d\zeta_1\int_0^{2\pi}d\zeta_2 f(\varepsilon,\tau)$$
$$\sim J_m\left(\frac{i\tau|a_1|}{\sigma_1}\right)J_n\left(\frac{i\tau|a_2|}{\sigma_2}\right). \qquad (16)$$

The beating coefficients b_{mn} lead to fields oscillating at frequency $\omega_{mn} = m\omega_1 + n\omega_2$, where $\omega_1$and ω_2 are the frequencies corresponding to the two e-beam energy peaks ε_1, ε_2. The gain length of the beating wave is $L_g^{mn} = (L_g^1 L_g^2/mL_g^1 + nL_g^2)$ and reduces to $L_g^{mn} = [L_g/(m+n)]$ for $L_g^1 = L_g^2 = L_g$. In the case of first-order beating, $m = n = 1$, a third peak appears at $\omega_1 + \omega_2$, between the second harmonics $2\omega_1$ and $2\omega_2$. The growth of FEL nonlinear wave mixing modes is, therefore, similar to that of nonlinear coherent harmonics and is characterized by a fast blowup with saturation power $P_s^{mn} \propto \rho_{mn} P_E$, where $\rho_{mn} \approxeq (f_{Bm} f_{Bn} / f_{B1})^{2/3} \rho$, with f_{Bk} the Bessel factor associated with the gain of the kth harmonics, and having assumed that $\rho_1 = \rho_2 = \rho$. The electron bunch behaves as a nonlinear medium, and the radiative mechanism is analogous to what occurs in crystals.

In this Letter we have touched on different points regarding the physics of FEL devices operating with two colors generated by two almost independent electron bunches. We have developed a theoretical description of the underlying physical mechanism and have pointed out the possibility of various interference mechanisms. In particular, we have considered the mutual bunching effect which might be responsible for the mode coupling phenomenology discussed here. Finally, it has been pointed out that the e-beam behaves as a nonlinear medium allowing for the generation of nonlinear (harmonic) beating waves. The observation of this further effect is within the possibilities offered by the SPARC test facility, when the two color experiment is operated at the onset of the saturation. The presence of this additional contribution should manifest through a further peak in the spectrum, located in between the two resonance frequency peaks.

The formalism we have provided in this Letter, for the theoretical treatment of two color FEL operation induced by two independent electron bunches, can be extended to the case of a two frequency undulator, as will be shown elsewhere.

The authors express their sincere appreciation to the members of the SPARC team for sharing with them their own view on the intriguing two color FEL physics. It is also a pleasure to recognize the help of Dr. Amalia Torre for providing her support in the analysis of Eq. (1).

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