Anomaly Nucleation Constrains $SU(2)$ Gauge Theories

James Halverson*
Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA
(Received 25 October 2013; published 27 December 2013)

We argue for the existence of additional constraints on $SU(2)$ gauge theories in four dimensions when realized in ultraviolet completions admitting an analog of D-brane nucleation. In type II string compactifications these constraints are necessary and sufficient for the absence of cubic non-Abelian anomalies in certain nucleated $SU(N > 2)$ theories. It is argued that they appear quite broadly in the string landscape. Implications for particle physics are discussed; most realizations of the standard model in this context are inconsistent, unless extra electroweak fermions are added.

DOI: 10.1103/PhysRevLett.111.261601 PACS numbers: 11.15.-q, 11.25.Wx, 12.60.Cn

Introduction.—Despite its many successes, the standard model of particle physics is an incomplete description of nature. Some of its shortcomings, such as the absence of a cold dark matter candidate, can be amended in quantum field theory; others, such as the absence of quantum gravity, require a more robust framework for ultraviolet completion. An important question is whether constraints on gauge theories in that framework differ from those of generic quantum field theories, and whether these differences have implications for low energy physics.

In this Letter we focus our attention on $SU(2)$ gauge theories, whose relevance for nature cannot be overstated; they govern the weak interactions [1], the recently discovered Higgs boson [2], and perhaps dark matter. We demonstrate that there are additional constraints on these theories when realized in ultraviolet completions admitting certain dynamical processes. These include broad regions of the string landscape. Most realizations of the standard model in this context are inconsistent, unless extra electroweak fermions are added.

The physical argument relies critically on a beautiful property of string theory. There, gauge theories are often carried by charged objects, such as D-branes [3]. If these objects can pair produce and embed the $SU(2)$ theory into an $SU(N)$ theory, then the $SU(2)$ theory must satisfy additional constraints necessary for consistency of the $SU(N)$ theory, but not its own. D-brane nucleation is an example of such a process, and we will study the constraints in this light. A natural objection is that such a system is unstable. Indeed this is true, but the gauge theories arising in it must, nevertheless, be anomaly free.

String consistency conditions are stronger than those of quantum field theory. For example, it has been argued [4] that there are effective theories which do not admit string embeddings, and also that matter representations are constrained (see [5] for a recent discussion). Here, we obtain a different type of constraint, placed on one theory to ensure the consistency of another related by a dynamical process. Perhaps this basic mechanism could be applied to other transitions in the string landscape.

This Letter is organized as follows. In the second section we present the physical argument for new constraints. In the third section we derive this idea in the string landscape, and in the fourth section we discuss implications for particle physics.

The physical argument.—It is already evident in quantum field theory that four-dimensional $SU(2)$ gauge theories are special within the broader class of $SU(N)$ theories. The latter receive $SU(N)^3$ anomaly contributions from Weyl fermions in complex representations of the gauge group, and consistency constrains the allowed representations. However, there is no such constraint on $SU(2)$ theories, since $SU(2)$ does not have complex representations. Furthermore, $SU(2)$ theories with an odd number of Weyl fermion doublets are inconsistent [6]. This arises from the fact the $\pi_4(SU(2)) = \mathbb{Z}_2$, but since $\pi_4(SU(N > 2)) = 0$, there is no corresponding constraint on those theories. The former constraints are stronger than the latter.

Our main point can be made in a simple example before turning to string theoretic realizations. Consider an $SU(2)$ gauge theory in four dimensions with an even number of left-handed Weyl fermion doublets. This is a consistent quantum field theory.

Now suppose that this theory is UV completed into a framework where gauge theories are carried by charged objects which have dynamics and can pair produce. If as a result of this process the $SU(2)$ theory has embedded into a nucleated $SU(N)$ theory, ensuring the absence of $SU(N)^3$ anomalies can place constraints on the chiral spectrum of the $SU(2)$ theory. For example, suppose the latter is embedded via setting the vacuum expectation value of an adjoint scalar to zero, such that doublets embed either into the $\square$ or $\overline{\square}$ of $SU(N)$, henceforth $\square_N$ or $\overline{\square}_N$. The embedding *defines* a way to distinguish two types of doublets; for notational convenience, denote those of the first and second type as $\square_2$ and $\overline{\square}_2$, respectively. Then $SU(N)^3$

anomaly cancellation requires that $\chi(\Box_N) \equiv \#\Box_N - \#\overline{\Box}_N$ satisfies

$$0 = \chi(\Box_N) = \chi(\Box_2), \tag{1}$$

where the $SU(2)$ constraint $\chi(\Box_2) = 0$ exists due to the process and ensures the absence of nucleated anomalies.

This is the phenomenon we wish to investigate broadly in the landscape. In certain corners such constraints have already been derived and been noted to be stronger than anomaly cancellation; it is in these corners that we derive the relationship to anomalies in nucleated theories. In other corners, we utilize nucleation processes and dualities to argue for their existence.

Traversing the landscape.—We will begin in type IIa, since there the relationship between the chiral spectrum of four-dimensional theories and topological consistency conditions is simple.

(i) Large volume type IIa compactifications: Consider a large volume type IIa compactification (see [7] for reviews) on a compact Calabi-Yau threefold X with an anti-holomorphic orientifold involution with fixed point locus a three-cycle $\pi_{O6} \in H_3(X, \mathbb{Z})$ wrapped by a spacetime filling O6-plane. Stacks of N_a spacetime-filling $D6$-branes which wrap a generic three-cycle π_a and its orientifold image π_a' give rise to $U(N_a)$ gauge theories in four dimensions; the $U(1) \subset U(N_a)$ is often massive, giving $SU(N_a)$ in the infrared. Chiral matter is localized at points of $D6$-brane intersection in X; the possible representations are bifundamentals of two unitary groups or two-index tensor representations of one.

We would like to study the chiral spectrum of a distinguished $D6$-brane stack on π_N and its image with $U(N)$ gauge symmetry. Since the $D6$-branes and O6-plane carry Ramond-Ramond charge, Gauss's law requires

$$N(\pi_N + \pi_N') + \sum_{a\neq N} N_a(\pi_a + \pi_a') - 4\pi_{O6} = 0. \tag{2}$$

This is the $D6$-brane tadpole cancellation condition [8]. The topological intersection numbers of the branes compute the chiral spectrum as

$$\chi(\Box_a, \overline{\Box}_b) \equiv \pi_a \cdot \pi_b \qquad \chi\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a\right) \equiv \frac{1}{2}(\pi_a \cdot \pi_a' + \pi_a \cdot \pi_{O6})$$

$$\chi(\Box_a, \Box_b) \equiv \pi_a \cdot \pi_b' \qquad \chi(\Box\!\Box_a) \equiv \frac{1}{2}(\pi_a \cdot \pi_a' - \pi_a \cdot \pi_{O6}),$$

Using these and intersecting (2) with π_N gives

$$T_N \equiv \chi(\Box_N) + (N-4)\,\chi\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_N\right) + (N+4)\,\chi(\Box\!\Box_N) = 0, \tag{3}$$

a constraint on the chiral spectrum necessary for $D6$-brane tadpole cancellation and thus global consistency.

This interplay between $D6$-brane tadpole cancellation and constraints on the chiral spectrum has been discussed extensively in the type IIa literature; see [8] for critical early works. In particular, $T_N = 0$ for $N > 2$ is the $SU(N)^3$ anomaly cancellation condition; such anomalies do not exist for $N = 1, 2$. In addition, certain $U(1)$ anomalies are cancelled by a combination of the condition $T_N = 0$ and axionic couplings via the Green-Schwarz mechanism; these include, for example, $U(1)^3$ anomalies for the particular $U(1) \subset U(N)$. This gives a low-energy interpretation of the constraints $T_2 = 0$ and $T_1 = 0$; they play a partial role in $U(1)$ anomaly cancellation.

We would like to present a different physical understanding of the T_2 and T_1 constraints. Though $D6$-brane charge cancellation in X is required for consistency, stability is not. To the system we have discussed, add a single $D6$ on π_N and a $\overline{D6}$ on a distant but homologous cycle $\bar{\pi}_N$ (as well as their orientifold images). Such a configuration can be reached (with energy cost) by nucleating a $D6$-$\overline{D6}$ pair on $\bar{\pi}_N$ and its image, and then setting the vacuum expectation value of the adjoint scalar of the combined $D6$-brane system to zero.

Though there is a force between the brane-antibrane pair and they will annihilate via open string tachyon condensation [9], the worldvolume gauge theories of the D-branes must nevertheless be anomaly free prior to annihilation; in particular, the $U(N+1)$ theory on π_N must not have $SU(N+1)^3$ anomalies. Similar ideas regarding anomaly cancellation after brane nucleation have been studied in ten- and six-dimensional theories [10].

This relationship between the T_2 (or T_1) constraint and nucleated anomalies can be derived. After adding this pair, there is a $U(N+1)$ theory on π_N and its image, a $U(1)$ theory on the $\overline{D6}$ on $\bar{\pi}_N$ and its image, and the $U(N_a)$ theories are left untouched. Quantitatively, we have added zero (in homology) to (2), which now reads

$$\begin{aligned}(N+1)(\pi_N + \pi_N') + \sum_{a\neq N} N_a(\pi_a + \pi_a') - 4\pi_{O6} \\ - \bar{\pi}_N - \bar{\pi}_N' = 0.\end{aligned} \tag{4}$$

Intersecting π_N with this equation, the terms in the first line give the contribution to $SU(N+1)^3$ anomalies from chiral fermions localized at $D6$-$D6$ intersections, whereas $-\pi_N \cdot \bar{\pi}_N = 0$, but $-\pi_N \cdot \bar{\pi}_N'$ can be nonzero. The latter counts chiral fermions localized at these $D6$-$\overline{D6}$ intersections, and the relative sign is important since the GSO projection in this sector projects out the opposite chirality fermion. In all, this calculation gives $T_{N+1} = 0$, with the subtlety that $\chi(\Box_{N+1})$ receives contributions from both $D6$-$D6$ and $D6$-$\overline{D6}$ intersections.

In summary, the computation before and after nucleation gives $T_N = 0$ and $T_{N+1} = 0$, respectively. This can be iterated M times, giving the relationship between the constraint in the setups without and with M $\overline{D6}$-branes,

$$T_N = 0 \leftrightarrow T_{N+M} = 0. \qquad (5)$$

This immediately gives a simple understanding of the T_2 and T_1 conditions: they are necessary and sufficient for the absence of $SU(N+M)^3$ anomalies in this nucleated $U(N+M)$ theory with $N+M>2$. This should be contrasted with their role in $U(1)$ anomaly cancellation; since there axionic terms are also required, they are necessary but not sufficient for $U(1)$ anomaly cancellation.

(ii) More derivations in the landscape: We would like to discuss two more derivations of the constraints in the landscape: one in type IIb, and the other in type I and their heterotic $SO(32)$ duals.

Consider large volume type IIb flux compactifications with intersecting stacks of $D7$-branes carrying Abelian worldvolume fluxes F_a. The brane stacks wrap divisors D_a of a Calabi-Yau threefold X and carry $U(N_a)$ gauge symmetry. For brevity, consider the case without orientifolds and a distinguished $U(N)$ theory, this time with a $D7$-brane stack on D_N with flux F_N. The $D7$ and $D5$ tadpole cancellation conditions read

$$ND_N + \sum_{a\neq N} N_a D_a = 0$$
$$ND_N \wedge F_N + \sum_{a\neq N} N_a D_a \wedge F_a = 0, \qquad (6)$$

respectively, with Poincaré duality implied. Wedging the $D7$ tadpole with $F_N \wedge D_N$ and the $D5$ tadpole with D_N gives $\sum N_a D_a \wedge D_N \wedge (F_N - F_A) = 0$. Rewriting in terms of the spectrum, this gives $\sum_a N_a \chi(\square_N, \bar{\square}_a) = \chi(\square_N) = 0$, a constraint necessary for $D7$ and $D5$ tadpole cancellation. Now consider a brane nucleated system with a $D7$ on D_N and a $\overline{D7}$ on a distant but homologous divisor $\bar{D}_N$ with worldvolume fluxes F_N and $\bar{F}_N$ in the same cohomology class. This adds zero in cohomology to the $D7$ and $D5$ tadpole condition but gives the constraint $\chi(\square_{N+1}) = 0$. Taking $N=2$, the constraint on the $U(2)$ theory is necessary and sufficient for the absence of cubic non-Abelian anomalies in the nucleated theory.

We see that $SU(N>2)^3$ anomaly cancellation and the additional $T_2 = 0$ and $T_1 = 0$ constraints are a consequence of $D7$ and $D5$ tadpole cancellation. Since background three-form fluxes contribute to the $D3$ tadpole, they can be added without spoiling these chiral spectrum constraints. These are the fluxes critical in the popular moduli stabilization scenarios [11], which give most of the known landscape of string vacua. Thus, the constraints appear broadly in the known landscape.

As a final example, consider the type I or $SO(32)$ heterotic string compactified to four dimensions on a Calabi-Yau threefold X endowed with a holomorphic vector bundle $V = \bigoplus_{m=K+1}^{K+L} V_m \oplus \bigoplus_{i=1}^{K} L_i$, where the structure group of V_m and L_i are $U(N_m)$ and $U(1)$, respectively. This is more generic than the common ansatz of V with $SU(N)$ structure group. The four-dimensional gauge algebra has an $SU(N_i)$ factor, and in [12] it was shown that the $SU(N_i)^3$ anomaly is $A_{SU(N_i)^3} \sim 2\int c_1(L_i) \times \text{Tad}$, where Tad is a four-form expression which must be cohomologically trivial for consistency via $D5$ tadpole cancellation and the B-field Bianchi identity in the type I and heterotic string, respectively.

Again, $SU(N_i)^3$ anomaly cancellation is ensured by a topological consistency condition and there is also a constraint for $N_i = 2$. It is interesting that such a constraint exists in the heterotic string, since D-brane nucleation has played a major role thus far and does not exist in the heterotic string. Via nucleation of magnetized $D9$-branes in the type I case, the $SU(2)$ constraints can likely be related to nucleated $SU(N_i)^3$ anomaly cancellation. Doing so in a way consistent with $D7$-brane tadpole cancellation likely requires introducing an instability in the gauge bundle, pointing to a heterotic interpretation.

(iii) Existence arguments: Although (to our knowledge) similar constraints have not been explicitly derived in four-dimensional compactifications of M theory, F theory, and the heterotic $E_8 \times E_8$ superstring, we would like to present arguments in favor of their existence, utilizing the existence of nucleation-type processes and string dualities.

We began by discussing type IIa intersecting $D6$-brane compactifications. If supersymmetric, these lift to compactifications of M theory on seven manifolds with G_2 holonomy, in which case vector (chiral) multiplet data are captured by codimension four (seven) singularities in the geometry. An important question is whether the $D6$-$\overline{D6}$ annihilation process critical to the physics of the constraints in IIa has a known M-theory lift, even locally. Such gravitational solutions have in fact been constructed [13]; prior to annihilation the system is described by a bolt singularity, and afterwards a Taub-NUT. If such a process relates an $SU(2)$ and $SU(N)$ theory, it is plausible that $SU(N)^3$ anomaly cancellation can be used to constrain the $SU(2)$ theory obtained after annihilation.

The T-dual type IIb picture with $D7$-branes lifts to F theory. In its weak coupling limit, F theory is simply a geometrization of type IIb. $D7$-$\overline{D7}$ nucleation modifies the IIb axiodilaton profile, and to the author's knowledge the geometric F theory lift is not known; it certainly must modify the geometry significantly, as the presence of new seven-branes introduces new scalar fields. Perhaps the appropriate modification of the geometry can be extended outside of the weakly coupled regime; if so, consistency requires the absence of nucleated anomalies. As further evidence, it is known [14] that $SU(N)^3$ anomalies are automatically cancelled in $d=4F$ theory compactifications with appropriately specified "G_4 flux," and the mechanism is equivalent to $D7$ and $D5$ tadpole cancellation in the weak coupling limit, which were critical above.

Finally, consider the $E_8 \times E_8$ heterotic string on a Calabi-Yau threefold. Suppose the compactification has an F theory dual; then vector bundle moduli which Higgs $E_8 \times E_8$ to the four-dimensional gauge group map to complex structure moduli in F theory, which determine the breaking of the two E_8 seven-branes through the Higgs mechanism via unfolding. The additional seven-brane moduli associated to the nucleation process suggested in the last paragraph would require passing to a different heterotic vector bundle with more moduli; since supersymmetry would be broken via the nucleation process, the new bundle must be unstable. This may provide an avenue for an explicit derivation.

(iv) Symplectic realizations of $SU(2)$: We would like to note another possibility, where an $SU(2)$ gauge theory is realized as $Sp(1)$. In known cases, there are no constraints analogous to $T_2 = 0$; e.g., in type IIa there are no constraints on their chiral spectra necessary for $D6$ tadpole cancellation. This matches nicely with the fact that such theories would nucleate $Sp(N)$ rather than $U(N)$ theories, which do not have cubic non-Abelian anomalies. See [15] for a IIa discussion.

Realizing $Sp(N)$ theories can require additional geometric constraints; e.g., in IIa, $D6$-brane three-cycles must be orientifold invariant, and in F theory codimension two singular loci must induce an automorphism of codimension one fibers. Additionally, conventional grand unification is difficult when $SU(2)_L$ is realized as $Sp(1)$.

Implications for particle physics.—Given the necessity of ultraviolet completion, it is important to study the potential implications of these constraints for physics beyond the standard model.

Model building from the bottom up in this context [16], the standard model (or MSSM) itself is incomplete: one must specify more input data, labeling $SU(2)_L$ doublets as $\square_2$ or $\overline{\square}_2$ according to their embedding into the nucleated theory. Our scheme for counting is as follows: for each set of F fermion doublets with the same standard model quantum numbers, consider all possible tuples $(\#\square_2, \#\overline{\square}_2)$ such that the sum is F. Then the three families of quark and lepton doublets split as (3,0), (2,1), (1,2), or (0,3). These contribute to T_2 as

$$T_2^l \in \{\pm 1, \pm 3\} \quad \text{and} \quad T_2^Q \in \{\pm 3, \pm 9\}, \tag{7}$$

where there is a factor of 3 for color in T_2^Q. In all, this gives 16 possibilities for $T_2^{SM} \equiv T_2^l + T_2^Q$, only two of which satisfy $T_2^{SM} = 0$. In the MSSM, the Higgsinos split as (1,0) or (0,1), contributing

$$T_2^{\tilde{h}_u} \in \{\pm 1\} \quad \text{and} \quad T_2^{\tilde{h}_d} \in \{\pm 1\}, \tag{8}$$

with only 6 of the 64 possibilities being consistent.

Most standard model and MSSM configurations do not satisfy the additional constraint on the $SU(2)$ spectrum. Without probabilistic information about the likelihood of realizing one configuration over another in the landscape, it is difficult to draw definitive conclusions.

Conservatively, though, there are only two possibilities that should be considered. (1) If $T_2^{SM} = 0$, then the quark doublet sector exhibits a $(2 + 1)$ family nonuniversality [17]. (2) If $T_2^{SM} \neq 0$, then new electroweak fermions are required for the absence of nucleated anomalies.

Identical statements hold when considering T_2^{MSSM}. The quark doublet nonuniversality could have further implications; for example, in type II realizations with $U(2)$ gauge symmetry the diagonal $U(1)$ will forbid some quark Yukawa couplings in string perturbation theory.

The second possibility is striking: it provides a new theoretical motivation for exotic electroweak fermions. In many cases (see [18] for systematic studies in type II) these new states are vectorlike with respect to the standard model but chiral under another symmetry, in which case they give smaller corrections to precision electroweak observables than do chiral exotics, but, nevertheless, have protected mass. If protected from decay, the neutral components of the exotics are excellent WIMP dark matter candidates; see [19] for a broad discussion in type II. These exotic particles could be discovered at LHC or in direct detection experiments in the near future.

Note that since the $T_3 = 0$ condition is just the $SU(3)^3$ anomaly cancellation condition, there is not a similar motivation for new colored fermions. Thus, the additional constraints motivate exotics that are not in complete GUT multiplets. One might naively think that such exotics would ruin gauge coupling unification (GCU), and in fact they will if added as chiral supermultiplets to the MSSM. However, in theories without weak scale supersymmetry grand unification could be a virtue of these exotic $SU(2)_L$ states; see [20] for a broad treatment. For example, pairs of exotic $(1, 2)_{\pm 1/2}$ fermions have the quantum numbers of Higgsinos and can improve GCU; this is the minimal extension [21] of the standard model giving dark matter and grand unification.

It is a pleasure to thank F. Chen, A. Collinucci, L. Everett, T. Hartman, G. Kane, F. Marchesano, A. Puhm, G. Shiu, P. Soler, and especially D. R. Morrison, A. Pierce, and J. Polchinski for useful conversations. I am deeply grateful for conversations and collaborations on related topics with M. Cvetič, P. Langacker, H. Piragua, and R. Richter, and for the support and encouragement of J. L. Halverson. This research was supported by the National Science Foundation under Grant No. PHY11-25915.

*jim@kitp.ucsb.edu

[1] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
[2] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.* (CMS Collaboration), *ibid.* **716**, 30 (2012).

[3] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).
[4] C. Vafa, arXiv:hep-th/0509212.
[5] A. Grassi, J. Halverson, and J. L. Shaneson, J. High Energy Phys. 10 (**2013**) 205.
[6] E. Witten, Phys. Lett. B **117**, 324 (1982).
[7] R. Blumenhagen, M. Cvetič, P. Langacker, G. Shiu, Annu. Rev. Nucl. Part. Sci. **55**, 71 (2005); R. Blumenhagen, B. Kors, D. Lüst, and S. Stieberger, Phys. Rep. **445**, 1 (2007); M. Cvetič and J. Halverson, arXiv:1101.2907.
[8] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan, and A. M. Uranga, J. Math. Phys. (N.Y.) **42**, 3103 (2001); A. M. Uranga, Nucl. Phys. **B598**, 225 (2001); L. E. Ibanez, F. Marchesano, and R. Rabadan, J. High Energy Phys. 11 (**2001**) 002.
[9] A. Sen, J. High Energy Phys. 08 (**1998**) 012.
[10] J. H. Schwarz and E. Witten, J. High Energy Phys. 03 (**2001**) 032.
[11] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003); V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, J. High Energy Phys. 03 (**2005**) 007.
[12] R. Blumenhagen, G. Honecker, and T. Weigand, J. High Energy Phys. 08 (**2005**) 009.
[13] A. Sen, J. High Energy Phys. 10 (**1997**) 002; R. Rabadan and J. Simon *ibid.* 05 (**2002**) 045.
[14] M. Cvetič, T. W. Grimm, and D. Klevers, J. High Energy Phys. 02 (**2013**) 101.
[15] D. Cremades, L. Ibanez, and F. Marchesano, J. High Energy Phys. 07 (**2003**) 038.
[16] That is, when such constraints hold; when $SU(2)$ is realized as $Sp(1)$, for example, this is not the case.
[17] For brevity, we have not considered the possibility that e_l^c or ν_l^c is realized as an antisymmetric tensor; in that analysis the family nonuniversality can be relaxed. Adjoint breaking of $SU(5)$ GUTs yields such configurations.
[18] P. Anastasopoulos, T. Dijkstra, E. Kiritsis, and A. Schellekens, Nucl. Phys. **B759**, 83 (2006); M. Cvetič, J. Halverson, and P. Langacker, J. High Energy Phys. 11 (**2011**) 058.
[19] M. Cvetič, J. Halverson, and H. Piragua, J. High Energy Phys. 02 (**2013**) 005.
[20] V. Barger, N. Deshpande, J. Jiang, P. Langacker, and T. Li, Nucl. Phys. **B793**, 307 (2008).
[21] N. Arkani-Hamed, S. Dimopoulos, and S. Kachru, arXiv: hep-th/0501082; R. Mahbubani and L. Senatore, Phys. Rev. D **73**, 043510 (2006).