Edge Reconstruction in the $\nu = 2/3$ Fractional Quantum Hall State

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The edge structure of the $\nu = 2/3$ fractional quantum Hall state has been studied for several decades, but recent experiments, exhibiting upstream neutral mode(s), a plateau at a Hall conductance of $\frac{1}{3}(e^2/h)$ through a quantum point contact, and a crossover of the effective charge, from $e/3$ at high temperature to $2e/3$ at low temperature, could not be explained by a single theory. Here we develop such a theory, based on edge reconstruction due to a confining potential with finite slope, that admits an additional $\nu = 1/3$ incompressible strip near the edge. Renormalization group analysis of the effective edge theory due to disorder and interactions explains the experimental observations.

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Ever since it has been realized that the low energy dynamics of quantum Hall (and particularly fractional quantum Hall) systems is related to the edge, and that the latter can be described by chiral Luttinger liquids [1], the interest in edge state physics has surged. While the structure of the edge, for simple fractional filling factors, is believed to be well understood, there is recently growing experimental evidence that the situation is much more intriguing and exciting than had been initially believed, when more complex fractions are involved. In this Letter we focus on the $\nu = 2/3$ edge. Ostensibly simple, the physics of this edge involves some of the intricacies applicable to other fractions, i.e., edge reconstruction and the emergence of novel elementary excitations (neutral modes in the present case). The major experimental observations addressed by us are as follows: (a) The conductance through a quantum point contact (QPC) exhibits a plateau (versus split-gate voltage) at $G = \frac{1}{3}(e^2/h)$ [2,3]. (b) An upstream heating of the QPC has been observed [4]. (c) The effective charge, detected through shot noise measurements [3], crosses over from $e^* = 1/3$ at higher temperature to $e^* = 2/3$ at lower ones.

Theoretical works have attempted to account for these observations. Observation (a) has been explained [2,5] by positing that the current at the edge of the $\nu = 2/3$ state is carried by two $\delta\nu = 1/3$ edge states, each having the characteristics of a $\nu = 1/3$ edge state. While one of these edges is reflected at the QPC, the other is adiabatically transmitted. Observation (b) is associated with a counterpropagating neutral mode, consistent with the Kane-Fisher-Polchinski (KFP) theory [6]. This theory is based on the renormalization of the original counterpropagating $\delta\nu = 1$ and $\delta\nu = -1/3$ edge channels (ECs) [7,8] due to interactions and disorder. Observation (c) has been attributed [9] to the competition between relevant operators within the context of the KFP theory. Clearly, there is no single theory currently that can account for all these experimental observations.

Our theory is based on an edge structure first proposed in Ref. [10]. It accounts for all these observations, and provides a general scheme to deal with complex edge structures. According to Ref. [10], the finite slope of the confining potential may lead to a reconstruction of the edge (see also Ref. [11]), resulting in four parallel edge channels. The latter correspond to filling factor discontinuities $\delta\nu = -1/3$, $+1$, $-1/3$, $+1/3$ (moving from the inner edge channel to the outer one; see Fig. 1). Due to the interplay between the interaction energy and the confining potential, the distance between the inner three modes and the outermost edge could be finite and large, an observation supported by the numerical calculation of Ref. [10]. Hence we expect the set of inner ECs to first mix and renormalize among themselves (due to disorder and interaction). In this regime, the emergence of the 1/3 plateau [observation (a)]

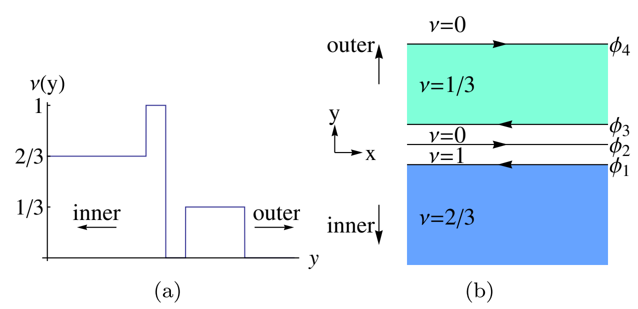

FIG. 1 (color online). The reconstructed edge. (a) The filling factor of the particle-hole conjugate wave function [7,8] grows from $\nu = 2/3$ in the bulk to $\nu = 1$ towards the edge, before falling to zero. In addition, for a smooth potential, the competition between Coulomb and potential energy admits an additional incompressible strip of $\nu = 1/3$. (b) The four associated edge modes are depicted, with arrowheads indicating the direction of each mode [downstream (right movers) or upstream (left movers)].

is straightforward, as the inner three modes are reflected, while the outer $\delta\nu = +1/3$ mode is transmitted. However, in order to understand the upstream neutral mode [observation (b)] in this regime, one has to carry out a renormalization group (RG) flow calculation within the subspace of the inner three modes. We find (see below) a wide range of parameter space [the green (light gray) strips and the blue (dark gray) hexagram in Fig. 2(d)] where there are intermediate RG fixed points supporting such an upstream neutral mode (or modes) [see Fig. 2(b)]. At sufficiently low temperatures the relevant coupling to the outer edge begins to play a role, resulting in further renormalizations of the ECs. The emergent picture is that of a $\delta\nu = +2/3$ (downstream) EC, an upstream neutral EC (similar to the

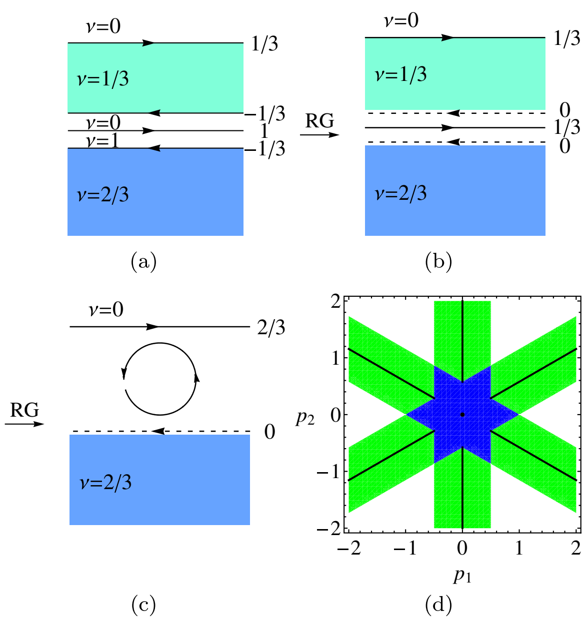

FIG. 2 (color online). The renormalization group (RG) flow. (a) The bare edge channels. (b) Under RG flow, the three inner channels are renormalized to two upstream neutral modes (as denoted by the dashed lines) and one downstream 1/3-like charged modes. (The ordering of the three modes is arbitrary.) (c) When temperature is further lowered, the relevant coupling to the outermost mode leads to localization of two modes, as symbolized by the circle, an upstream neutral mode (dashed line) and a downstream 2/3-like charged mode. (d) Basins of attraction of the various fixed points for the intermediate regime, where the outermost mode is assumed to be decoupled from the other three modes. p_1 and p_2 parametrize the interaction matrix V [17]. The green (light gray) strips mark the regions where only one of the three potentially relevant scattering operators [Eq. (3)] is relevant. The blue (dark gray) hexagram is where at least two scattering operators are relevant. The thick black lines in the middle of the green (light gray) strips are fixed lines where there is one upstream neutral mode (in addition to a downstream charged mode and an upstream charged mode) at the interface. The black dot at the origin (the center of the hexagram) is a fixed point where there are two upstream neutral modes [as depicted in (b)].

KFP theory), and two localized charged modes. The crossover from the intermediate to the low temperature regime explains observation (c).

The model.—A general clean edge is described by the action [1]

$$S_0 = \frac{1}{4\pi}\int_{x,\tau}[\partial_x\phi_i K_{ij} i\partial_\tau\phi_j + \partial_x\phi_i V_{ij}\partial_x\phi_j] \quad (1)$$

(summations over repeated indices are implied). V represents inter- and intrachannel interactions. The bosonic fields ϕ_i ($i = 1, \dots, N$) are quantized with the usual commutation relations $[\phi_i(x), \phi_j(y)] = i\pi K^{-1}_{ij}\mathrm{sgn}(x-y)$. The symmetric $N \times N$ integer matrix K characterizes the internal structures (the topological orders) in the quantum Hall fluid [12,13]. In addition, the charge vector $\boldsymbol{t}$ describes the coupling between the charge density in each channel and an external electric field. The total electric charge density is thus $\rho_{\mathrm{el}} = (1/2\pi)t_i\partial_x\phi_i$, where the t_i's are the components of $\boldsymbol{t}$. The different channels may, in addition, be coupled by impurity scattering [6,11], with an additional term

$$S_{\mathrm{imp}} = \int_{x,\tau}\sum_{\boldsymbol{n}}[\xi_{\boldsymbol{n}}(x)e^{i\boldsymbol{n}\cdot\boldsymbol{\phi}(x,\tau)} + \mathrm{H.c.}], \quad (2)$$

where $\boldsymbol{\phi}$ is the vector whose entries are ϕ_i, $\boldsymbol{n}$ are constant vectors specifying the scattering processes, $\xi_{\boldsymbol{n}}(x)$ are the random scattering amplitudes. We consider here white noise correlations $\langle\langle\xi_{\boldsymbol{n}}(x)\xi_{\boldsymbol{n}'}(x')\rangle\rangle = D_{\boldsymbol{n}}\delta_{\boldsymbol{n}\boldsymbol{n}'}\delta(x-x')$ [6,14,15].

The proposed $\nu = 2/3$ ground state wave function [7,8], supports two edge modes. This wave function assumes particle-hole symmetry, and is valid for an infinitely sharp confining potential. For a confining potential of a finite slope, it has been demonstrated [10,11] that the competition between the Coulomb energy and the potential energy can lead (and will indeed do so for typical experimental parameters) to the formation of another incompressible strip, of filling factor $\nu = 1/3$ in this case. The smoother the potential, the wider the strip, and, as a consequence (see Fig. 1), the larger the distance between the outer edge mode and the other modes. Thus, in the present case, $N = 4$, and the K matrix may be written [16] in a diagonal form, with the elements $\{-3, 1, -3, 3\}$ on the diagonal, and $\boldsymbol{t} = (1, 1, 1, 1)^T$.

The intermediate temperature regime.—As mentioned above, we assume that the outermost channel is sufficiently far from the inner three channels, so we can first neglect the interaction and scattering between the former and the latter. The inner three channels are then described by three bosonic fields (cf. Fig. 1). The K matrix will consist of the upper left 3×3 block of the full K matrix, and $\boldsymbol{t} = (1, 1, 1)^T$. The allowed scattering operators that are relevant in parts of the parameter space in the RG sense [17] are $O_{\boldsymbol{n}} = e^{i\boldsymbol{n}\cdot\boldsymbol{\phi}}$ with

$$\boldsymbol{n}_1 = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \qquad \boldsymbol{n}_2 = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}, \qquad \boldsymbol{n}_3 = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}. \tag{3}$$

The RG analysis of such a three-channel system has been looked at before [14,15], though not as an intermediate state in a larger space. Since K is a real symmetric matrix and V a real symmetric positive definite matrix, they can be diagonalized simultaneously

$$\Lambda^T K \Lambda = I_{n^+,n^-}, \qquad \Lambda^T V \Lambda = V_{\rm D}, \tag{4}$$

where $V_{\rm D}$ is a diagonal matrix, and I_{n^+,n^-} is the pseudoidentity matrix with n^+ 1's and n^- -1's in the diagonal; for the current problem, $n^+ = 1, n^- = 2$. (The structure of the transformation matrix Λ is further discussed in Ref. [17].) Equation (4) amounts to transforming to the basis of the eigenmodes $\tilde{\boldsymbol{\phi}} = \Lambda^{-1}\boldsymbol{\phi}$ (we use the words "mode" and "channel" interchangeably). In addition to the amplitudes $D_{\boldsymbol{n}}$, the model has six parameters $V_{ij}(i \leq j = 1, 2, 3)$, or in the eigenmode basis, $v_i(i = 1, 2, 3)$, θ, p_1, and p_2. The first three are the diagonal elements of $V_{\rm D}$, while the latter three characterize the transformation matrix Λ [17]. In terms of the transformed fields, the scaling dimensions [17] can be calculated very easily. The scaling dimension of $e^{i\boldsymbol{c}\cdot\tilde{\boldsymbol{\phi}}}$ is simply $\Delta = \frac{1}{2}\boldsymbol{c}^2$. The relation between $\boldsymbol{c}$ and $\boldsymbol{n}$ is $\boldsymbol{c} = \Lambda^T\boldsymbol{n}$. Therefore, we find that the scaling dimensions of the three scattering operators specified by Eq. (3) are

$$\Delta_{\boldsymbol{n}_1} = 1 + 2p_1^2, \tag{5}$$

$$\Delta_{\boldsymbol{n}_2} = \frac{1}{2}(2 + p_1^2 - 2\sqrt{3}p_1p_2 + 3p_2^2), \tag{6}$$

$$\Delta_{\boldsymbol{n}_3} = \frac{1}{2}(2 + p_1^2 + 2\sqrt{3}p_1p_2 + 3p_2^2). \tag{7}$$

Note that the scaling dimensions only depend on two parameters p_1 and p_2. For an operator to be relevant, its scaling dimension has to be smaller than $3/2$ [18]. The regions $\Delta_{\boldsymbol{n}_i} < 3/2$ are the three green (light gray) stripes in Fig. 2(d). The blue (dark gray) hexagram is where at least two of the three operators are relevant.

In regions where none of the three operators is relevant (uncolored regions), the amplitudes of impurity scattering will be renormalized to zero. The other parameters will be somewhat renormalized as well. The result of the RG flow is a clean but nonuniversal interface between the bulk incompressible region and the additional incompressible strip.

In regions of the phase diagram [see Fig. 2(d)] where one or more impurity operators are relevant, the theory will be renormalized. This means that $\{V_{ij}\}$ will be modified in a nontrivial manner. Alternatively, from the point of view of the eigenmodes, the charge vector $\tilde{\boldsymbol{t}}$, and the vectors $\boldsymbol{c}$ in the bosonized form of the electron and quasiparticle tunneling operators will all be renormalized. The coefficients $\boldsymbol{n}$ are, evidently, unchanged.

Within any of the green (light gray) strips, the theory flows to the center line (the thick black lines). On such a fixed line, there is one upstream-moving neutral mode, one upstream-moving charged mode, and one downstream-moving charged mode [not counting the outermost mode, i.e., the topmost one in Fig. 1(b)]. The neutral sector realizes the $\widehat{\mathfrak{su}}(2)_1$ current algebra, so the strong impurity fixed point is exactly solvable, and by simple power counting one can show that it is a stable fixed point. The charge vector in the eigenmode basis depends on the position on the fixed line. For example, on the $p_1 = 0$ line, we have (assuming, without loss of generality, $v_2 > v_3$)

$$\tilde{\boldsymbol{t}} = \left(\frac{\sqrt{1 + p_2^2}}{\sqrt{3}}, \frac{p_2}{\sqrt{3}}, 0\right)^T. \tag{8}$$

Note that the discontinuity in the filling factor (between the bulk value $\frac{2}{3}$ and the value before the outermost channel $\frac{1}{3}$) $\delta\nu = \tilde{t}_1^2 - \tilde{t}_2^2 - \tilde{t}_3^2 = 1/3$ is independent of p_2 as it should be [19,20].

Turning our attention to the hexagram, within this region the RG flows are towards the center (the origin of the $p_1 - p_2$ plane). The (0, 0) strong impurity fixed point of the three inner modes corresponds to a downstream-moving charge mode (with $\tilde{t}_1 = 1/\sqrt{3}$), and two upstream neutral modes. The neutral sector realizes the $\widehat{\mathfrak{su}}(3)_1$ current algebra (assuming the neutral mode velocities v_2 and v_3 are renormalized to the same value in higher order RG analysis, i.e., beyond first order in the impurity strength $D_{\boldsymbol{n}}$'s). We note the similarity between Fig. 2(d) and the basins of attraction for the principal hierarchy states $\nu = 3/5$, $3/11$, etc. [14].

It is interesting to compare the emerging picture to Beenakker's model [5]. In our picture, the interface between $\nu = 1/3$ and $\nu = 2/3$ corresponds to three renormalized modes, $\tilde{\phi}_1$ (downstream $\frac{1}{3}$ charged mode) and two upstream modes. It is $\tilde{\phi}_1$ [assuming we are at the $\widehat{\mathfrak{su}}(3)_1$ point] that conducts electric current (along with the outermost channel). In Beenakker's picture this interface between $\nu = 1/3$ and $\nu = 2/3$ corresponds to an electric current-conducting compressible strip with no inner structure. When there is a QPC with appropriate split-gate voltage, it is possible to form a region of $\nu = 1/3$ under the constriction, with the three inner modes fully reflected [similar to Fig. 2(b) of Ref. [5]], but with the outermost mode fully transmitted. This would lead to a plateau in two-terminal conductance, $G = \frac{1}{3}(e^2/h)$, in agreement with experiment [3]. On top of Beenakker's picture of two downstream charged modes, our picture includes the upstream neutral modes [two if the theory flows to the $\widehat{\mathfrak{su}}(3)_1$ point, one if to the $\widehat{\mathfrak{su}}(2)_1$ lines], which have also been observed [4].

We can also consider the leading backscattering processes at a QPC (assuming the transmission is close to 1). With the assumption that the outermost mode is weakly coupled to the inner three, only the latter participate in the backscattering from the top edge of the sample to the bottom edge. At the $\widehat{\mathfrak{su}}(3)_1$ point, the scaling dimension of the backscattering processes is simply $2 \times \frac{1}{2} \boldsymbol{n}^T M_1 M_1^T \boldsymbol{n}$, where M_1 is as given in the Supplemental Material [17], $\boldsymbol{n}$ is an integer vector specifying the quasiparticle that is backscattered. The factor 2 is due to the fact that now we are considering two edges. The minimum of this expression is 1, corresponding to the most relevant tunneling operators with $\boldsymbol{n} = (0, -1, 0)^T$, $(1, 2, 0)^T$, and $(2, 2, 3)^T$. All three operators backscatter $e/3$ charge from one edge to the other, so if one measures the shot noise at the weak backscattering limit, the effective charge inferred should be $e/3$.

The low temperature regime.—Since the coupling between the outermost channel and the inner channels (through interaction and impurity scattering) is relevant in the RG sense, then even if its bare value is small, which allowed us to neglect it in the intermediate temperature regime above, it becomes significant at lower temperatures. There is a qualitative difference between the ensuing four-channel problem and the three-channel problem considered above. We now discuss the RG flow of the theory, and the ensuing picture approaching the stable low temperature fixed point. If the impurity amplitudes are small we can still use the same RG formalism as for the three-channel problem. We find that under renormalization, the theory will flow to regions where only one of the (infinitely many) zero-conformal-spin operators is still relevant (actually its scaling dimension flows to zero). This operator involves two of the eigenmodes, say $\tilde{\phi}_1$ and $\tilde{\phi}_4$, whose charge vector components will be the same, $\tilde{t}_1 = \tilde{t}_4$. These two modes are unstable against localization. This is consistent with Haldane's null vector criterion about edge stability [19]. If the initial point of the RG trajectory has a projection in the $p_1 - p_2$ plane close to the origin, the amplitude of yet another operator will also grow and eventually have the form $e^{i\sqrt{2}\tilde{\phi}_3}$. As an impurity operator, it cannot create electric charge. This implies that $\tilde{\phi}_3$ has to be neutral, which is indeed what we see in the result of the numerical integration of the RG equations. In other words, $\tilde{\phi}_3$ is an (upstream-moving) neutral mode, while $\tilde{\phi}_2$ is a (downstream-moving) charged mode ($\tilde{t}_2$ is renormalized to $\sqrt{2/3}$), as in the KFP fixed point [6]. The neutral sector realizes the $\widehat{\mathfrak{su}}(2)_1$ current algebra. At the low temperature fixed point, the quasiparticles involved in weak backscattering at a QPC are made of the remaining nonlocalized modes only, whose action is

$$
\begin{aligned}
S = {} & \frac{1}{4\pi} \int_{x,\tau} \partial_x \tilde{\phi}_2 (i\partial_\tau + v_2 \partial_x) \tilde{\phi}_2 \\
& + \frac{1}{4\pi} \int_{x,\tau} \partial_x \tilde{\phi}_3 (-i\partial_\tau + v_3 \partial_x) \tilde{\phi}_3 \\
& + \int_{x,\tau} [\xi(x) e^{i\sqrt{2}\tilde{\phi}_3} + \mathrm{H.c}]; \qquad (9)
\end{aligned}
$$

i.e., the quasiparticle spectrum should be the same as in the KFP problem (for a more detailed discussion of this point see Ref. [17]). It is well known that in that case the minimum scaling dimension for the backscattering processes is $2/3$ corresponding to a process with charge $2e/3$ and two with charge $e/3$. When one lowers the temperature in an experiment, there could be a crossover from the $\widehat{\mathfrak{su}}(3)_1$ point (where the leading QPC backscattering processes have $e/3$) of the three-channel problem to the KFP point. If at the latter the amplitude of the $2e/3$ process is larger than the $e/3$ processes, one would see a crossover in effective charge from $e/3$ at high temperature to $2e/3$ at low temperature. This has actually been observed [9].

The emergence of two separate fixed points for $\nu = 2/3$ is in perfect agreement with the observed power law dependence of the transmission $\mathcal{T}$ through a QPC in this regime. Figure 4 of Ref. [3] demonstrates that the transmission starts as a power law at low impinging current (or low voltage) and then saturates at $\mathcal{T} = 1/2$ for high voltage. The low voltage data are well described by the KFP fixed point, which predicts that $\mathcal{T}$ scales at V^2 [see Fig. 1(a) in the Supplemental Material [17]]. On the other hand, the KFP theory will predict that at high voltage $\mathcal{T}$ will saturate at unity, with a correction that goes like $V^{-2/3}$, in clear contradiction to the data. Our theory, on the other hand, claims that the high voltage data is described by a different fixed point that supports an outer edge state of $\delta\nu = 1/3$. If the three inner edge states are fully reflected, and the backscattering is only due to the outer edge state, the transmission will saturate at $\mathcal{T} = 1/2$, with a correction that behaves like $V^{-4/3}$, in excellent agreement with the data [see Fig. 1(b) in the Supplemental Material [17]].

The localization transition also has an effect on the current-voltage characteristics (for $eV \gg k_B T$) of tunneling from a Fermi liquid to the edge of the fractional quantum Hall liquid (assuming edge reconstruction also happens in the kind of samples used in that kind of experiment)—when one lowers the temperature from the range where the effective charge is $e/3$ to where it is $2e/3$, the power in the $I - V$ characteristics should decrease from 3 to 2 [17,21] (or 3/2 if the neutral mode is saturated).

Summary.—We have considered a reconstructed edge at $\nu = 2/3$, consisting of four edge channels, in order to explain outstanding experimental observations. For a smooth potential, the interaction and impurity backscattering between the outermost channel and the other channels at the $\frac{1}{3}/\frac{2}{3}$ interface can be neglected at high enough temperatures. Then we have a trivial one-channel problem plus a nontrivial three-channel problem. The latter system may be renormalized by interaction and impurity scattering to high symmetry fixed points. The reflection of the inner three modes by a quantum point contact explains the observed $\frac{1}{3}(e^2/h)$ conductance plateau [2,3], while the emergence of the neutral mode at the symmetric fixed points or lines explains the observed upstream heat current [4]. At lower temperature, when the interaction and impurity scattering between the outermost mode and those at the interface cannot be neglected, the system is unstable and will flow towards a stable low temperature fixed point: one pair of counterpropagating modes will localize each other. The two remaining modes form a $\frac{2}{3}$/neutral KFP fixed point [6]. This results in a crossover in the effective charge, from $e^* = 1/3$ at high temperature to a higher effective charge (which could be as high as $e^* = 2/3$, depending in the bare amplitudes) at low temperatures, again consistent with experiment [3]. Our theory is also in excellent agreement with the the measured scaling of the transmission through a point contact with voltage [3]. Additionally we made a prediction concerning the crossover in the tunneling exponent into the edge from a Fermi liquid. Recent experiments [22–24] have demonstrated the complexity of the edge even for simple filling factors. Venkatachalam *et al.* [24], for example, have observed edge reconstruction at $\nu_{\rm bulk} = 1$, where the filling factor at the edge first goes down to $\nu = 2/3$, then to $1/3$. The edge structure from the $\nu = 2/3$ region outwards is completely consistent with our picture, giving further credence to the theory presented here.

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