Stability, Higgs Boson Mass, and New Physics

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Assuming that the particle with mass ~126 GeV discovered at LHC is the standard model Higgs boson, we find that the stability of the electroweak (EW) vacuum strongly depends on new physics interaction at the Planck scale M_P, despite of the fact that they are higher-dimensional interactions, apparently suppressed by inverse powers of M_P. In particular, for the present experimental values of the top and Higgs boson masses, if τ is the lifetime of the EW vacuum, new physics can turn τ from $\tau \gg T_U$ to $\tau \ll T_U$, where T_U is the age of the Universe, thus, weakening the conclusions of the so called metastability scenario.

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Introduction.—When the particle with mass ~126 GeV, discovered at LHC [1,2], is identified with the standard model (SM) Higgs boson, serious and challenging questions arise, among them, the vacuum stability issue. The Higgs effective potential $V_{\rm eff}(\phi)$ bends down for values of ϕ larger than the electroweak (EW) minimum, an instability due to top loop corrections. By requiring stability, lower bounds on the Higgs boson mass M_H were found [3–9].

A variation on this picture is the so-called metastability scenario [4,10–12]. For ϕ much larger than v (location of the EW minimum), $V_{\rm eff}(\phi)$ develops a new minimum at $\phi_{\rm min}^{(2)}$. When M_H and M_t are such that $V_{\rm eff}(v) < V_{\rm eff}(\phi_{\rm min}^{(2)})$, the EW minimum is stable; otherwise it is a false vacuum that should decay into the true vacuum (at $\phi_{\rm min}^{(2)}$) in a finite amount of time. Depending on the values of M_H and M_t, the lifetime τ of the EW vacuum can be larger or smaller than the age of the Universe T_U. For $\tau > T_U$, we may well live in the metastable EW minimum. This is the metastability scenario.

The aim of this Letter is to study the influence of new physics interactions (at the Planck scale) on τ. Tree level and quantum fluctuation contributions are taken into account. In this Letter, however, we limit ourselves to considering the quantum corrections from the Higgs sector only. This is sufficient to illustrate our main point. The complete analysis is left for a forthcoming paper.

Let us begin with Fig. 1, where we repeat the usual analysis [10–12] and draw the phase diagram in the $M_H - M_t$ plane. The latter is divided into three different sectors: an absolute stability region [$V_{\rm eff}(v) < V_{\rm eff}(\phi_{\rm min}^{(2)})$], a metastability region ($\tau > T_U$), and an instability region ($\tau < T_U$). The dashed line separates the stability and the metastability sectors and is obtained for M_H and M_t such that $V_{\rm eff}(v) = V_{\rm eff}(\phi_{\rm min}^{(2)})$. The dashed-dotted line separates the metastability and the instability regions and is obtained for M_H and M_t such that $\tau = T_U$. For $M_t \sim 173.1$ GeV and $M_H \sim 126$ GeV, the SM lies within the metastability region. It is then concluded that the present experimental values of M_H and M_t allow for a standard model valid all the way up to the Planck scale.

Let $V_{\rm eff}(\phi)$ be normalized so as to vanish at $\phi = v$. At a much larger value $\phi = \phi_{\rm inst}$, $V_{\rm eff}(\phi_{\rm inst})$ vanishes again (for $M_H \sim 126$ GeV, $M_t \sim 173.1$ GeV, this happens for $\phi_{\rm inst} \sim 10^{10}$ GeV). For $\phi > \phi_{\rm inst}$, the potential becomes negative, later developing a new minimum.

It is assumed that the actual behavior of $V_{\rm eff}(\phi)$ for ϕ beyond $\phi_{\rm inst}$ has no impact on τ. More precisely, it is stated that even if $V_{\rm eff}(\phi)$ at $\phi = M_P$ is still negative (and the new minimum forms at a scale much larger than M_P), new physics interactions around the Planck scale must stabilize the potential (eventually bringing the new minimum around M_P), but τ does not depend on the detailed form of $V_{\rm eff}(\phi)$ beyond $\phi_{\rm inst}$ [10].

In this respect, it is worth it to note that for $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV, not only the effective potential at the Planck scale is negative, but it also continues to go down beyond M_P. The new minimum is formed at $\phi_{\rm min}^{(2)} \sim 10^{31}$ GeV (see Fig. 3).

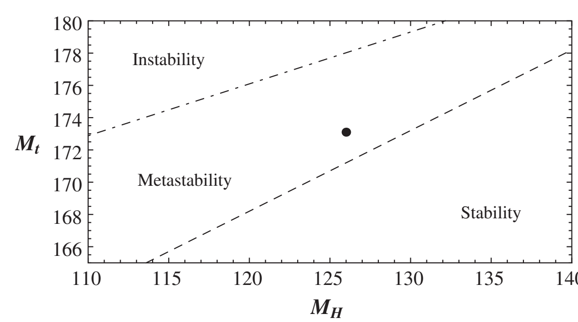

FIG. 1. In this picture, we repeat the analysis of [10–12], which is done in the absence of new interactions at the Planck scale. The $M_H - M_t$ plane is divided in three sectors: absolute stability, metastability, and instability regions. The dot indicates $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV.

Note also that the instability of the effective potential occurs for very large values of ϕ, ($\phi_{inst} \sim 10^{10}$ GeV). In this range, $V_{eff}(\phi)$ is well approximated by keeping only the quartic term [6]. Accordingly, following [13,14], the tunneling time τ is computed by considering the bounce solutions to the Euclidean equation of motion for the potential $V(\phi) = (\lambda/4)\phi^4$ with negative λ, a good approximation in this range.

Lifetime of the EW vacuum.—In order to study the impact of new physics interactions at the Planck scale, we add two higher dimension operators ϕ^6 and ϕ^8 to the SM Higgs potential

$$V(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}. \qquad (1)$$

Naturally, we can also consider higher dimensional operators. However, the examples we are going to study (with different choices of λ_6 and λ_8) are sufficient for illustrating some interesting cases we can face when new physics interactions at the Planck scale are considered.

The influence of λ_6 and λ_8 on the RG flow of the quartic coupling $\lambda(\mu)$, for values of μ below M_P, is negligible (see Fig. 2). The RG functions for the SM parameters at the two-loop level (with the corresponding boundary conditions) can be found, for instance, in [11,15]. Further (slight) improvement is obtained by considering three-loop contributions [11,12,16].

Let us now consider two different representative cases. For $\lambda_6(M_P) = -2$ and $\lambda_8(M_P) = 2.1$, the potential is given by the dashed line of Fig. 3. Because of the large range of scales involved, the plot is done in a double logarithmic scale. As λ_6 is negative, when ϕ approaches M_P, $V_{eff}^{new}(\phi)$, which is the renormalization group improved effective potential in the presence of λ_6 and λ_8, bends down much more steeply than $V_{eff}(\phi)$ and forms a new minimum at about $\phi_{min}^{(2)} \sim M_P$. This is clear from the zoom around the Planck scale in panel (b) of Fig. 3.

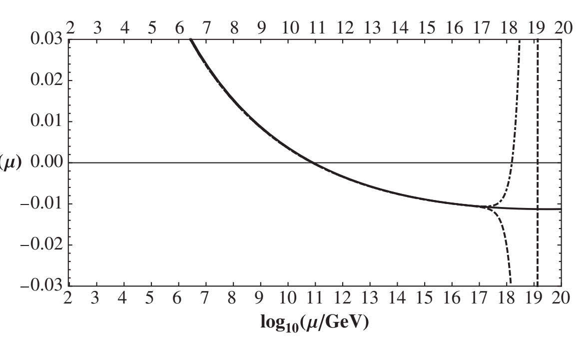

FIG. 2. For $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV, the running of $\lambda(\mu)$ as determined by SM interactions only (solid line) and in the presence of λ_6 and λ_8. Dashed-dotted line: $\lambda_6(M_P) = 1$ and $\lambda_8(M_P) = 0.5$. Dashed line: $\lambda_6(M_P) = -2$ and $\lambda_8(M_P) = 2.1$. Clearly, the tree lines coincide for values of μ below the Planck scale.

The second case we consider is when λ_6 and λ_8 are both positive. For $\lambda_6(M_P) = 1$ and $\lambda_8(M_P) = 0.5$, the potential is given by the dotted-dashed line of Fig. 3. As λ_6 is positive, when ϕ approaches M_P the potential $V_{eff}^{new}(\phi)$ lies above (rather than below) $V_{eff}(\phi)$.

In both cases, the potential is stabilized at the Planck scale by new physics terms. However, it is commonly believed that, although such a stabilization has to take place, the presence of new physics interactions has no impact on the EW vacuum lifetime [10]. We shall see that this is not generically true. When $V_{eff}^{new}(\phi)$ lies above $V_{eff}(\phi)$, which in our example is realized with $\lambda_6(M_P) > 0$ and $\lambda_8(M_P) > 0$, τ is almost insensitive to the presence of these new terms. On the contrary, when $V_{eff}^{new}(\phi)$ lies below $V_{eff}(\phi)$, which in our example is realized with $\lambda_6(M_P) < 0$ and $\lambda_8(M_P) > 0$, τ strongly depends on new physics.

The tunneling time τ is given by [10,13,14],

$$\frac{1}{\tau} = T_U^3 \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det'[-\partial^2 + V''(\phi_b)]}{\det[-\partial^2 + V''(v)]} \right|^{-1/2} e^{-S[\phi_b]}, \qquad (2)$$

where $\phi_b(r)$ is the $O(4)$ bounce solution to the Euclidean equation of motion ($r^2 = x_\mu x_\mu$), $S[\phi_b]$ the action for the bounce, $[-\partial^2 + V''(\phi_b)]$ the fluctuation operator around the bounce (V'' is the second derivative of V with respect to ϕ). The prime in the det$'$ means that, in the computation of the determinant, the zero modes are excluded and $S[\phi_b]^2/4\pi^2$ comes from the translational zero modes.

Let us compute τ for the potential of Eq. (1) with $\lambda_6(M_P) = -2$ and $\lambda_8(M_P) = 2.1$.

When $V_{eff}(\phi)$ (the usual SM Higgs potential without new interaction terms) is computed within the $\overline{MS}$ scheme and the renormalization scale μ is chosen to coincide with the inverse of the bounce size R_{max} that maximizes the tunneling probability [10], we have $\mu = 2R_{max}^{-1}e^{-\gamma_E} = 1.32 \times 10^{17}$ GeV (γ_E is the Euler gamma) and the coupling constant $\lambda(\mu)$ is $\lambda(\mu) \simeq -0.014$.

For our potential, we find that, up to the scale $\eta \simeq 0.780 M_P$, it is very well approximated by an upside down quartic parabola, $V_{eff}^{new}(\phi) = (\lambda_{eff}/4)\phi^4$, with $\lambda_{eff} = \lambda + \frac{2}{3}\lambda_6(\eta^2/M_P^2) + \frac{1}{2}\lambda_8(\eta^4/M_P^4) \simeq -0.437$. For $\phi > \eta$, $V_{eff}^{new}(\phi)$ bends down very steeply [see Fig. 3(b)], eventually creating a new minimum very close to M_P : $\phi_{min}^{(2)} = 0.979 M_P$. Therefore, for values of ϕ larger than (but close to) η, $\phi \gtrsim \eta$, it can be linearized and we get $V(\phi) = [(\lambda_{eff}/4)\eta^4 - (\lambda_{eff}\eta^3/\gamma)(\phi - \eta)]$, where

$$\gamma = -\lambda_{eff}\eta^3\left(\lambda\eta^3 + \lambda_6\frac{\eta^5}{M_P^2} + \lambda_8\frac{\eta^7}{M_P^4}\right)^{-1}. \qquad (3)$$

Interestingly, in order to compute τ, this is all that we need to know [17]. Moreover, the parameter γ plays an essential role in determining when new physics interactions influence τ.

The Euclidean equation of motion admits the following bounce solution:

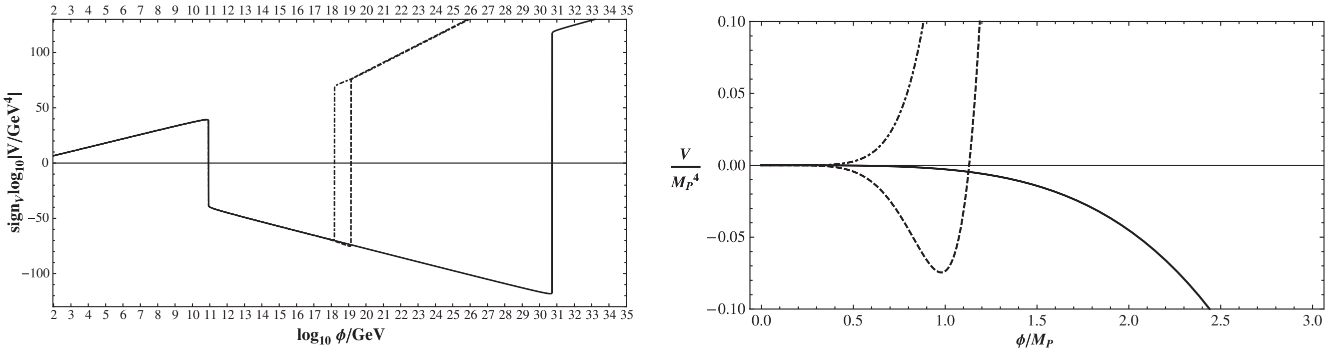

FIG. 3. (a) The effective potential $V_{\rm eff}(\phi)$ (solid line) for $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV. Note that the new minimum forms at $\phi^{(2)}_{\rm min} \sim 10^{31}$ GeV. For the same values of M_H and M_t, $V^{\rm new}_{\rm eff}(\phi)$ for $\lambda_6 = -2$ and $\lambda_8 = 2.1$ (dashed line); $V^{\rm new}_{\rm eff}(\phi)$ for $\lambda_6 = 1$ and $\lambda_8 = 0.5$ (dashed-dotted line). In order to include such a vast range of scales, a log-log plot has been considered. (b) Zoom of the panel (a) figure near the Planck scale. $V_{\rm eff}(\phi)$, $V^{\rm new}_{\rm eff}(\phi)$, and ϕ are normalized to Planck units (in this range no log-log plot is needed). $V^{\rm new}_{\rm eff}(\phi)$ for $\lambda_6 < 0$ bends down steeply and forms a new minimum at $\phi^{(2)}_{\rm min} = 0.979 M_P$. With $\lambda_6 > 0$, $V^{\rm new}_{\rm eff}(\phi)$ falls down less steeply than $V_{\rm eff}(\phi)$, and in the picture, we cannot resolve the minimum which forms at $\phi = 0.119 M_P$.

$$\phi_b(r) = \begin{cases} 2\eta - \eta^2 \sqrt{\frac{|\lambda_{\rm eff}|}{8}} \frac{r^2 + \bar{R}^2}{\bar{R}} & 0 < r < \bar{r} \\ \sqrt{\frac{8}{|\lambda_{\rm eff}|}} \frac{\bar{R}}{r^2 + \bar{R}^2} & r > \bar{r}, \end{cases} \tag{4}$$

where

$$\bar{r}^2 = \frac{8\gamma}{\lambda_{\rm eff}\eta^2}(1+\gamma), \tag{5a}$$

$$\bar{R}^2 = \frac{8}{|\lambda_{\rm eff}|}\frac{\gamma^2}{\eta^2}. \tag{5b}$$

From Eq. (5a) we see that the solution (4) exists only for $-1 < \gamma < 0$. $\bar{R}$ is the size of the bounce (4) and the action at ϕ_b is

$$S[\phi_b] = [1 - (\gamma + 1)^4]\frac{8\pi^2}{3|\lambda_{\rm eff}|}. \tag{6}$$

There are also other bounce solutions

$$\phi_b^{(2)}(r) = \sqrt{\frac{2}{|\lambda_{\rm eff}|}}\frac{2R}{r^2 + R^2}, \tag{7}$$

where R the size of these bounces, can take any value in the range $\sqrt{(8/|\lambda_{\rm eff}|)}(1/\eta) < R < \infty$. A numerical analysis (presented in detail in a forthcoming paper) shows that these latter solutions are related to the approximation that we are considering for $V(\phi)$. Nevertheless, for $|\phi| \ll M_P$ (that in turn means for very large values of R), configurations of the kind given in Eq. (7), with $\lambda_{\rm eff}$ replaced by λ, are good approximate solutions of the (Euclidean) equation of motion, and, in principle, should be taken into account in the computation of τ. Their action is degenerate with R and is

$$S = \frac{8\pi^2}{3|\lambda|}. \tag{8}$$

However, even if for a moment we limit ourselves to the tree level contribution only, from Eqs. (2), (6), and (8) we see that for those values of γ such that the solution (4) exists ($-1 < \gamma < 0$), the contribution to the tunneling probability coming from the bounces (7) (with $\lambda_{\rm eff}$ replaced by λ) is exponentially suppressed with respect to the contribution of (4). For λ, $\lambda_6(M_P)$, $\lambda_8(M_P)$, and η given above, we have $\gamma \simeq -0.963$.

Let us now compute the fluctuation determinant in Eq. (2) for the bounce (4) and for $\lambda_6(M_P) = -2$ and $\lambda_8(M_P) = 2.1$, which is the case of interest for us.

Because of radial symmetry, $V''(\phi_b)$ in $[-\partial^2 + V''(\phi_b)]$ only depends on r. Following [18], the logarithm of the fluctuation determinant is obtained (see below for some specifications) as

$$\log\left(\frac{\det'[-\partial^2 + V''(\phi_b)]}{\det(-\partial^2)}\right)^{1/2} = \frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2 \ln\rho_l, \tag{9}$$

$$\text{where } \rho_l = \lim_{r\to\infty}\rho_l(r), \tag{10}$$

and each $\rho_l(r)$ is a solution of the differential equation

$$\rho_l''(r) + \frac{(2l + d - 1)}{r}\rho_l'(r) - V''[\phi_b(r)]\rho_l(r) = 0, \tag{11}$$

with boundary conditions $\rho_l(0) = 1$ and $\rho_l'(0) = 0$. [$\rho_l''(r)$ is the second derivative of $\rho_l(r)$ with respect to r,...].

Equation (9) is ill defined in three respects. The eigenvalue related to $l = 0$ is a negative mode ($\rho_0 < 0$), while the $l = 1$ modes correspond to the four translational zero modes. We exclude the $l = 0$ and $l = 1$ modes from the above sum. They can be separately treated in a standard way [18,19]. Finally, the sum in Eq. (9) is divergent. This is the usual UV divergence, and we know how to take care of it through renormalization [19].

Let us now consider Eq. (11) for $l > 1$. We can easily solve this equation numerically for each value of l (for increasing l, the ρ_l's rapidly converge to one). Following [19], the $\overline{MS}$ renormalized sum in Eq. (9) is given by

$$\left[\frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2\ln\rho_l\right]_r = \frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2\ln\rho_l - \frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2 \times\left[\frac{\int_0^\infty drrV''}{2(l+1)} - \frac{\int_0^\infty drr^3(V'')^2}{8(l+1)^3}\right] - \frac{1}{8}\int_0^\infty drr^3(V'')^2\left[\ln\left(\frac{\mu r}{2}\right) + \gamma_E + 1\right], \quad (12)$$

where μ is the renormalization scale. We then get

$$\left[\frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2\ln\rho_l\right]_r = -2.49 - 5.27\ln\left(\frac{1.48\mu}{M_P}\right). \quad (13)$$

This result is obtained by truncating the sum to a value of l where it shows saturation (standard renormalization procedure). Strictly speaking, the "angular momentum" cutoff L in this sum is given by $L = \bar{R}M_P$, which, from Eq. (5b) is $L \sim 5$. However, the series in Eq. (12) converges very fast. Even truncating it to $l = 5$ we get a less than 3% difference with the result of Eq. (13). The standard renormalization procedure is then well justified.

For $l = 0$, ρ_0 has to be replaced with its absolute value [19]. Solving Eq. (11), we find that its contribution to the sum in Eq. (9) is $\frac{1}{2}\ln|\rho_0| = -0.806$.

Finally, the contribution of the zero modes ($l = 1$) is also obtained in a standard manner [19]. The solution of Eq. (11) for $l = 1$ vanishes in the $r \to \infty$ limit: $\rho_1 = 0$. Actually, ρ_1 has to be replaced with ρ_1', defined as

$$\rho_1' = \lim_{k\to 0}\frac{\rho_1^k}{k^2}, \quad (14)$$

where ρ_1^k is obtained by solving Eq. (11) with $V''(\phi_b)$ replaced by $V''(\phi_b) + k^2$. Note that ρ_1' has the dimension of a length square and is given in terms of $\bar{R}$, the size of the bounce (4). The zero modes contribution to the sum in Eq. (9) finally is $\frac{1}{2}4\ln\rho_1' = 2ln(0.0896\bar{R}^2)$.

For the purposes of comparing our results [from $V_{\rm eff}^{\rm new}(\phi)$] with those obtained with $V_{\rm eff}(\phi)$, it is useful to choose the same renormalization scale as before, $\mu = 1.32\times 10^{17}$ GeV. Then, collecting the different results, from Eq. (2) we find

$$\tau = 5.45\times 10^{-212}T_U, \quad (15)$$

a ridiculously small fraction of a second.

This result is at odds with what is shown in Fig. 1, where for $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV, the EW vacuum lies inside the metastability region, close to the stability line and shows that the phase diagram of Fig. 1 has to be reconsidered. Actually, when the EW vacuum lifetime for these values of M_H and M_t is computed in the absence of new physics interactions, we have

$$\tau = 1.49\times 10^{714}T_U. \quad (16)$$

Accordingly, the EW vacuum would be an extremely long-lived metastable state. This is why it is often stated that, for the present experimental values of M_H and M_t, the SM is an effective theory that is valid all the way up to the Planck scale.

Equation (15) shows that this is not generically true. As a result of the presence of new physics interactions, the EW vacuum may turn from a very long-lived metastable state to a highly unstable one. As we have already seen, in fact, when $V_{\rm eff}^{\rm new}(\phi)$ lies above $V_{\rm eff}(\phi)$, τ is not dramatically affected by new physics. On the contrary, when $V_{\rm eff}^{\rm new}(\phi)$ lies below $V_{\rm eff}(\phi)$, the UV completion of the standard model has a very strong impact on τ, turning it from $\tau \gg T_U$ to $\tau \ll T_U$.

Conclusions and outlook.—In this Letter, we show that the lifetime τ of the EW vacuum strongly depends on new physics. The metastability scenario (which is based on the assumption that τ does not depend on new physics) and the whole phase structure of Fig. 1 have to be entirely reconsidered.

Clearly, when the quantum fluctuations from other sectors of the SM are taken into account, the specific value of τ in Eq. (15) is modified. However, this does not change the core result of the present analysis, namely the huge influence of new physics on τ.

A very important outcome of our result is that it poses constraints on possible candidates to the UV completion of the SM. In this respect, we note also that a similar analysis can be done when the new physics scale lies below (even much below) the Planck scale.

Finally, we note that the considerations developed in this Letter should be relevant for related scenarios, Higgs potential with two degenerate minima [20] and Higgs driven inflation scenarios [21,22]. In all of these cases, in fact, the physical scale relevant to the involved mechanism is dangerously close to the Planck scale and we expect high sensitivity to new physics interactions.

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