Generalized Bose-Einstein Condensation into Multiple States in Driven-Dissipative Systems

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Bose-Einstein condensation, the macroscopic occupation of a single quantum state, appears in equilibrium quantum statistical mechanics and persists also in the hydrodynamic regime close to equilibrium. Here we show that even when a degenerate Bose gas is driven into a steady state far from equilibrium, where the notion of a single-particle ground state becomes meaningless, Bose-Einstein condensation survives in a generalized form: the unambiguous selection of an odd number of states acquiring large occupations. Within mean-field theory we derive a criterion for when a single state and when multiple states are Bose selected in a noninteracting gas. We study the effect in several driven-dissipative model systems, and propose a quantum switch for heat conductivity based on shifting between one and three selected states.

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In quantum many-body physics there is currently a huge interest in nonequilibrium phenomena beyond the hydrodynamic description of systems retaining approximate local equilibrium. Recently, intriguing results have been obtained for paradigmatic scenarios: the dynamics away from equilibrium in response to a parameter variation [1–3], the possible relaxation towards equilibrium [2,3] versus many-body localization [4,5], and the control of many-body physics by means of strong periodic forcing [6–8]. Another fundamental scenario of quantum many-body dynamics is nonequilibrium steady states of driven-dissipative systems, with transport of, e.g., mass or energy through the system [9–11]. In this context one might ask simple questions: What are the properties of an ideal Bose gas driven to a steady state far from equilibrium? In particular, what happens in the quantum degenerate regime, where in equilibrium Bose-Einstein condensation occurs?

In this Letter we investigate the quantum degenerate regime of driven-dissipative ideal Bose gases of N particles in steady states far from equilibrium, assuming weak coupling to the environment. Examples of such systems comprise bosons coupled to two heat baths of different temperature and time-periodically forced bosons in contact with a single heat bath. For large densities these systems are found to exhibit an intriguing generic behavior. Namely the single-particle states unambiguously separate into two groups: one, that we call Bose selected, whose occupations increase linearly when the total particle number is increased at fixed system size, and another one whose occupations saturate. Remarkably, this generalized form of Bose condensation is a very consequence of bosonic indistinguishability, not relying on thermodynamic equilibrium. We show examples both with the number of selected states being extensive and of order one, the latter corresponding to a fragmented condensate [12] with a macroscopic occupation of each selected state (not relying on ground-state degeneracy). We propose to switch the heat conductivity of a system by shifting between one selected state (corresponding to standard Bose condensation) and three selected states.

Our findings are relevant for artificial many-body quantum systems such as superconducting and optical circuits [13–16], exciton-polariton fluids [16,17], or photons in a dye-filled cavity [18], that are intrinsically driven dissipative. Tailored dissipation has also been used or proposed as a powerful tool for quantum engineering in ultracold atomic quantum gases [19–22] and trapped ions [21,23,24]. Our results, moreover, provide a connection between Bose condensation in quantum systems and the phenomenon of real-space condensation in classical nonequilibrium models [25–29], where also condensation into multiple states has been found [30–33].

Consider an open quantum system of a single particle weakly coupled to an environment, with reduced density operator ρ. The time evolution shall be given by a Markovian master equation $\dot{\rho}(t) = \mathcal{L}(\rho(t))$ with linear Liouvillian $\mathcal{L}$, guiding the system into a steady state ρ_∞ that is diagonal with respect to the (quasi)energy eigenstates $i = 1, 2, \ldots, M$ [34]. The dynamics of the diagonal elements $p_i \equiv \langle i|\rho|i\rangle$ is given by

$$\dot{p}_i = \sum_{j=1}^{M} (R_{ij} p_j - R_{ji} p_i) \tag{1}$$

with rates R_{ij} for a quantum jump from j to i that, for simplicity, we assume to be strictly positive, $R_{ij} > 0$.

Now we generalize the single-particle problem (1) to a gas of N noninteracting bosons. The many-body steady state $\hat{\rho}_\infty$ will be diagonal in the Fock basis $|\boldsymbol{n}\rangle$ labeled by the occupation numbers $\boldsymbol{n} = (n_1, n_2, \ldots n_M)^T$ of the single-particle states i, obeying $\sum_i n_i = N$: $\langle \boldsymbol{n}'|\hat{\rho}_\infty|\boldsymbol{n}\rangle = \delta_{\boldsymbol{n}'\boldsymbol{n}} p_{\boldsymbol{n}}$. The N-boson rate equation reads

$$\dot{p}_{\boldsymbol{n}} = \sum_{i,j=1}^{M} (R_{ij} p_{\boldsymbol{n}_{ji}} - R_{ji} p_{\boldsymbol{n}}) n_i (n_j + 1), \tag{2}$$

where $\boldsymbol{n}_{ji}$ denotes the occupation numbers obtained from $\boldsymbol{n}$ by transferring one particle from i to j [35]. The steady state with $\dot{p}_{\boldsymbol{n}} = 0$ is unique, if every state can be reached from every other one via a sequence of finite-rate quantum jumps [36,37]. This is true for every N, if it is true for the single-particle problem (1). Equation (2) is classical in the sense that it involves the diagonal elements of the density matrix only. However, the bosonic quantum statistics is reflected in the fact that the rate for a jump from i to j depends both on n_i and n_j. This rate reads $R_{ji} n_i (1 + \sigma n_j)$ with $\sigma = -1, 0, +1$ for fermions, distinguishable particles, and bosons, respectively.

As a transparent model system, we will first consider rate matrices R_{ij} given by exponentially distributed, independent random numbers [38]. This choice is motivated by the distribution of rates obtained for fully chaotic systems (see the Supplemental Material [39]). In this model the number of states M corresponds to the system size and, thus, the filling factor $n \equiv N/M$ to the density. In Fig. 1 we plot the mean steady-state occupations $\bar{n}_i$ versus n, for two realizations of R_{ij} with $M = 10$ and $M = 200$; here $\bar{n}_i = \langle \hat{n}_i \rangle$ with number operator $\hat{n}_i$ and $\langle \cdot \rangle = \mathrm{tr}\{\hat{\rho}_\infty \cdot\}$. In the nondegenerate regime of low filling $n \ll 1$ the relative occupations $\bar{n}_i/N$ approach the n-independent single-particle probabilities p_i. Quantum-statistical corrections make themselves felt when entering the degenerate regime at $n \sim 1$. For even larger densities n, around a crossover value n^*, we observe that for a group of M_S single-particle states the occupation grows linearly with N, while the occupation of the remaining states saturates. This is the aforementioned effect of Bose selection. Asymptotically, in the ultradegenerate regime $n \gg n^*$, the relative occupations of the selected states $\bar{n}_i/N$ as well as the absolute occupations $\bar{n}_i$ of the nonselected states become independent of n.

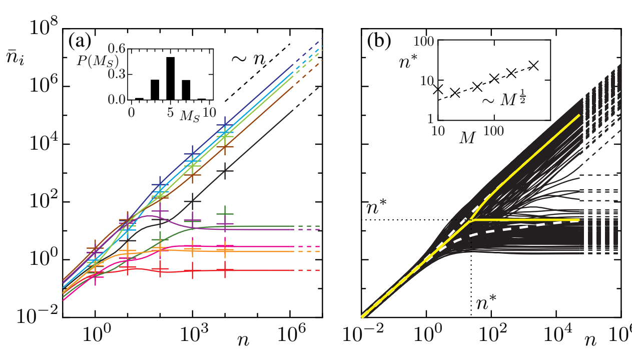

FIG. 1 (color online). (a) Mean occupations $\bar{n}_i$ versus density $n = N/M$ for one realization of the random-rate matrix R_{ij} with $M = 10$. Crosses are from quasiexact Monte Carlo theory (see the Supplemental Material [39,54]), and solid (dashed) lines are from mean-field (asymptotic) theory described below. For large n the occupations of $M_S = 5$ Bose selected states do not saturate. Inset: distribution of M_S for the ensemble of rate matrices. (b) Like (a), but with $M = 200$ and $M_S \approx M/2$. Thick lines: average occupation of a selected and a nonselected state, exactly (dashed) and assuming equal occupation n for $n < n^*$ followed by saturation of the nonselected occupations (solid). Inset: crossover density n^* versus system size M.

Within the ensemble of rate matrices the number of selected states M_S is found to be always odd [e.g.. Fig. 1(a), inset] and on average $M_S = M/2$ with fluctuations $\sim M^{1/2}$ (a system with nonextensive $M_S \sim 1$ is presented below). The crossover to Bose selection occurs around $n = n^*$, when the density reaches the saturation value of the average occupation of a nonselected state [Fig. 1(b), thick lines]. In the random-rate model n^* increases like $\sim M^{1/2}$ with M [Fig. 1(b), inset] [40]. Therefore, in this model Bose selection does not occur in the thermodynamic limit, $M \to \infty$ keeping n constant, but in finite systems (similar to finite-temperature equilibrium Bose condensation in one dimension).

In order to treat large systems and to understand the behavior visible in Fig. 1, we derive a mean-field (MF) theory from the equation of motion $\dot{\bar{n}}_i = \mathrm{tr}(\dot{\hat{\rho}} \hat{n}_i)$ for the $\bar{n}_i$ by approximating two-state correlations $\langle \hat{n}_i \hat{n}_j \rangle$ by the trivial ones given by Wick decomposition, $\langle \hat{n}_i \hat{n}_j \rangle \approx \bar{n}_i \bar{n}_j$ (for $i \neq j$). This gives a closed set of nonlinear equations

$$\dot{\bar{n}}_i = \sum_{j=1}^{M} [R_{ij} \bar{n}_j (\bar{n}_i + 1) - R_{ji} \bar{n}_i (\bar{n}_j + 1)], \tag{3}$$

for the $\bar{n}_i$ and is equivalent to a Gaussian ansatz $\hat{\rho} \propto \exp(-\sum_i \nu_i \hat{n}_i)$ with $\nu_i = \ln(\bar{n}_i^{-1} + 1)$. The MF theory is confirmed by the Monte Carlo data [Figs. 1(a), 2(a), and 2(d)] [41].

An asymptotic theory for the ultradegenerate regime, particle number to infinity at fixed system size, (not to be confused with the thermodynamic limit: system size to infinity at fixed density) can be derived from the MF Eq. (3) for $\dot{\bar{n}}_i = 0$. The naive approximation $(\bar{n}_k + 1) \simeq \bar{n}_k$ leads to the set of equations $0 = \bar{n}_i \sum_j (R_{ij} - R_{ji}) \bar{n}_j$ that generally does not possess a physical solution with nonnegative occupations $\bar{n}_i \geq 0$, unless several of the $\bar{n}_i$ vanish. This gives already a hint why Bose selection occurs, but it does not tell us which states become selected, since, e.g., $\bar{n}_i = N \delta_{ik}$ would be a solution for any state k. A systematic theory is obtained by assuming that there is some (yet to be determined) set S of Bose selected single-particle states with occupation numbers $\sim n$ that are large compared to one as well as to the occupations of the nonselected states $\sim n^0$. This allows us to expand the $\bar{n}_i$ in powers of n^{-1}. In leading order we obtain the closed set of linear equations for the Bose selected states

$$0 = \sum_{j \in S} (R_{ij} - R_{ji}) \bar{n}_j, \quad i \in S. \tag{4}$$

The fact that $(R_{ij} - R_{ji})$ is a skew-symmetric matrix guarantees a zero determinant and a solution of Eq. (4) provided the set S contains an odd number M_S of states (for even M_S

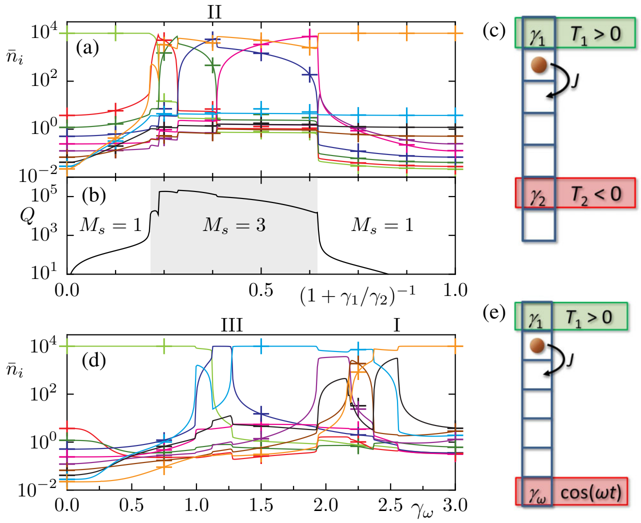

FIG. 2 (color online). (a) Occupations $\bar{n}_i$ from mean-field (lines) and Monte Carlo (crosses) calculations for $N = 10^4$ bosons on a tight-binding lattice of $M = 10$ sites, weakly coupled with strengths $\gamma_{1,2}$ to two baths of temperature $T_1 = -T_2 = J$, as depicted in (c). (b) Heat flow Q from bath 2 to bath 1 (arbitrary units, $\gamma_1 + \gamma_2$ kept constant); the shaded (unshaded) area corresponds to $M_S = 3$ ($M_S = 1$). (d) Occupations of single-particle Floquet states for the tight-binding chain with the coupling to bath 2 replaced by a driving term of strength γ_ω as depicted in (e).

the existence of a solution requires fine tuned rates R_{ij}). Thus generically one expects an odd number of selected states, in accordance with the numerically obtained distribution [Fig. 1(a), inset]. The next order describes the occupations of the nonselected states

$$\bar{n}_i = \frac{1}{g_i - 1} \quad \text{with} \quad g_i = \frac{\sum_{j\in S} R_{ji}\bar{n}_j}{\sum_{j\in S} R_{ij}\bar{n}_j}, \qquad i \notin S, \quad (5)$$

and gives also corrections to the occupations of the selected states that we omit here [even higher orders can become relevant when allowing some rates R_{ij} to be zero (see the Supplemental Material [39])]. Equation (4) determines the relative occupations among the selected states. These are independent of the total particle number N and, in turn, dictate the absolute occupations of the nonselected states via Eq. (5). The latter, thus, do not depend on N, corresponding to the saturation behavior visible in Fig. 1. The total number of particles occupying the selected states, including corrections to the leading order (4), is given by $N - \sum_{i\notin S}\bar{n}_i$ and increases linearly with N (since the "depletion" $\sum_{i\in S}\bar{n}_i$ is independent of N). This behavior is generic for the ultradegenerate regime and generalizes Bose condensation, where the occupation of a single state k increases with N. Remarkably, Bose selection is a very consequence of the bosonic quantum statistics, not relying on equilibrium statistical mechanics.

The set S of selected states has to be determined by the physical requirement that the occupations $\bar{n}_i$ of both the selected and the nonselected states are nonnegative. It can be shown that a unique physical solution with positive occupations exists [42], as expected from the fact that a unique steady state of Eq. (2) exists (see the Supplemental Material [39]). We are not aware of an easy strategy (beyond trial and error) that generally allows us to determine which and how many states are selected. However, if there is a ground-state-like single-particle state k, characterized by $R_{ki} > R_{ik}$ for all $i \neq k$, then only this state k will be selected and $M_S = 1$, corresponding to Bose-Einstein condensation. Namely, since Eq. (4) is fulfilled trivially and Eq. (5) gives positive occupations $\bar{n}_{i\neq k} = [R_{ki}/R_{ik} - 1]^{-1} > 0$ for the nonselected states, this is the (unique) physical solution. In contrast, as soon as there is no such ground-state-like state k anymore, then more than a single state must be selected.

An important special case is rate matrices for a system with single-particle energies $E_1 < E_2 \leq E_3 \ldots$ in weak contact with a thermal bath of inverse temperature β for which the rate matrices obey $R_{ji}/R_{ij} = \exp[\beta(E_i - E_j)]$. Such rates guarantee detailed balance, i.e., the existence of an equilibrium steady state for which each summand on the right-hand side of Eqs. (1) and (2) vanishes independently. In the ultradegenerate regime, one then recovers from Eq. (5) the familiar expression $\bar{n}_i = \{\exp[\beta(E_i - E_1)] - 1\}^{-1}$ for $i > 1$ while $\bar{n}_1 = N - \sum_{i>1}\bar{n}_i$. Therefore (excluding ground-state degeneracy $E_1 = E_2$) a nonequilibrium steady state breaking detailed balance, as it is found in driven-dissipative systems, is a necessary condition for observing Bose selection of more than a single state. However, breaking detailed balance is not sufficient, as can be inferred from the example of a system driven between two baths of different positive temperature. In this case the rates sum up $R_{ij} = R_{ij}^{(1)} + R_{ij}^{(2)}$ and, despite the fact that the combined rates do not lead to detailed balance anymore, they still obey $R_{1i} > R_{i1}$ for all $i \neq 1$ such that only the ground state will be selected. Below we will discuss concrete systems of two classes for which $M_S > 1$ is found naturally: (i) systems in weak contact with two baths, one with positive temperature and another, energy-inverted one with negative temperature, and (ii) time-periodically driven systems in weak contact with a thermal bath.

Let us now investigate the effect of Bose selection in the concrete physical model system of a one-dimensional tight-binding lattice. Such a model describes, e.g., an array of Josephson junctions, ultracold atoms in optical lattices, or vibrons in an ion chain [23,43]. On the single-particle level, the lattice sites $\ell = 1, \ldots, M$ are coupled by tunneling, $\langle \ell' | H | \ell \rangle = -J\delta_{\ell',\ell\pm1}$ with $J > 0$. The eigenstates i are delocalized; thus, a highly coordinated rate matrix results from coupling a bath to a local operator like $v_\ell = |\ell\rangle\langle\ell|$. The resulting rate matrix can be derived microscopically within the Born-Markov approximation [34] (see the Supplemental Material for details of the Ohmic baths used here and a plot of the rate matrix [39]). In order to achieve Bose selection with $M_S > 1$ we consider two baths, as sketched in Fig. 2(c): one with positive temperature $T_1 = J$ couples with strength γ_1 to v_1 and another one with negative temperature $T_2 = -J$ couples with strength γ_2 to v_{M-1} [44]. Here the negative temperature models a bath with occupations that increase with energy. In Fig. 2(a) we plot the mean occupations of the eigenstates of the tight-binding chain versus the relative coupling strength $(1 + \gamma_1/\gamma_2)^{-1}$ for large filling. One can observe Bose selection as a clear separation between highly occupied states on the one hand and states with occupations $\lesssim 1$ on the other. For $\gamma_2 = 0$ the system is in equilibrium and Bose condensation, the selection of a single state, is found. When the coupling to the inverted bath is switched on, at $(1 + \gamma_1/\gamma_2)^{-1} \approx 0.2$ three states become selected [Fig. 2(a), shaded area]. While the data of Fig. 2(a) correspond to $M = 10$, for $\gamma_2/\gamma_1 = 1$ we have studied also larger systems with up to $M = 300$ sites and always found three states selected. This suggests, that the model of Fig. 2(c) is an example where, in contrast to the random-rate model, the number of selected states remains of order one (while still being larger than one). This corresponds to a fragmented condensate with a macroscopic occupation of each selected state.

As a striking signature for the selection of more than a single state, at the transition from $M_S = 1$ to $M_S = 3$ a significant steady-state heat flow Q from bath 2 to bath 1 is established abruptly [Fig. 2(b)]. The heat flow from bath $b = 1, 2$ into the system reads $Q_b = \sum_{ij} R_{ji}^{(b)} \bar{n}_i(\bar{n}_j + 1) \times (E_j - E_i)$. This explains an increase by orders of magnitude from $\sim n$ to $\sim n^2$ when the transition from one to three selected states occurs [since $\bar{n}_i \sim n$ ($\bar{n}_i \sim 1$) for selected (nonselected) states]. Thus, the mechanism of Bose selection might be used to design quantum devices working far from equilibrium that allow us to switch the heat conductivity via the number of selected states.

Let us now consider time-periodically driven quantum systems (Floquet systems) with Hamiltonian $H(t) = H(t + 2\pi/\omega)$ [45–47]. When coupled weakly to a thermal bath, these systems can be described within Floquet-Born-Markov theory [48–52]. One obtains Eqs. (1) and (2) with i labeling single-particle Floquet states. In the Supplemental Material we show that the rate differences $R_{ij} - R_{ji}$ are independent of the bath temperature [39]. According to Eqs. (4) and (5) this implies that the selected states and their relative occupations are temperature independent, whereas the occupations of the nonselected states (and thus also the crossover density n^*) are temperature dependent.

Replacing the energy-inverted bath coupled to one end of the tight-binding chain by a coherent periodic driving term $\gamma_\omega J \cos(\omega t) v_M$ with $\hbar\omega = 1.5J$ [Fig. 2(e)], we obtain the occupations of the single-particle Floquet states versus the driving strength γ_ω [Fig. 2(d)]. In this driven-dissipative system we observe again both Bose condensation into a single state—which state is controlled by the parameters—and Bose selection of $M_S = 3$ states.

Two more examples that emphasize that Bose selection is a generic and robust effect in open time-periodically driven systems are given in the Supplemental Material [39]: the N-boson generalizations of the open kicked rotor and the open driven quartic oscillator of Ref. [53].

In Figs. 2(a) and 2(d), we can study the evolution of the occupations with respect to a parameter controlling the rate matrix. Within the asymptotic theory (4) and (5) transitions of states between the groups of selected and nonselected states are triggered either by the occupation of a selected state approaching zero or by the occupation of a nonselected state diverging. Both require the fine tuning of a single parameter. While at the transition point an even number of states is selected, after the transition again the generic situation with an odd number of states has to be recovered. Thus, a second state has to make a transition at the transition point, too. When approaching the transition point from the other side, this second state plays the role of the triggering one. One finds three types of two-state processes, examples of which are labeled by I, II, and III in Figs. 2(a) and 2(d): the transition is triggered from one side by a selected and from the other one by a nonselected state (I, M_S changes by 2), or the transition is triggered on both sides either by selected (II) or nonselected (III) states (M_S does not change).

In future work, it will be interesting to study the impact of, e.g., dimensionality, particle reservoirs, disorder, and interactions on the effect of Bose selection in nonequilibrium steady states. A concrete application of Bose selection in a physical system is the quantum switch for heat conductivity proposed here.

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