Drift-Induced Excitable Localized States

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Excitable localized states, spatial structures which possess both the features of temporal excitable pulses and of transverse cavity solitons, have been theoretically predicted in model systems as single pulses of light localized in space with a finite and deterministic duration. We study experimentally the nucleation of laser localized structures on a device defect and its motion along a spatial gradient. We demonstrate that in the reference frame of the drifting localized structure, the resulting dynamics presents the typical features of excitable systems. In particular, for specific parameter values, we observe that the nucleation of laser localized structures is triggered by noise, while the drift of the localized structure up to a spatial region where it vanishes provides the deterministic orbit which brings the system back to its initial rest state. The control of such structures may open the way to novel applications of localized structures beyond that of simple stationary bits.

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Localized states are ubiquitous in dissipative nonlinear systems. In optics, they have been observed in many different configurations and materials including atomic vapors, liquid crystal light valves, and semiconductor microcavities [1,2]. In all these cases, they have been shown to be stable for an arbitrarily long time even after the perturbation creating them has ceased to exist; i.e., they are studied in a context of bistability. In contrast to all of these studies (which focus on stable states), excitability mediated by localized structures has been recently studied theoretically in a paradigmatic model for a nonlinear optical cavity [3]. Excitability is a property of many nonlinear dynamical systems defined by the response of the system to perturbations: Perturbations below a certain threshold decay to the inital state, while perturbations overcoming a certain threshold result in the system running through a large and well-defined excursion in phase space before returning to its original state. While the most prominent example is, of course, the neuron [4], many other systems can present this well-calibrated thresholdlike response, which can result from one of three well-known phase space configurations. Examples in optics include active photonic crystals [5] and optical amplifiers [6] (weakly saturated Hopf bifurcation), lasers with optical feedback [7] or with saturable absorber [8,9] (saddle-loop bifurcation), and lasers with optical injection [10] (saddle node on invariant circle bifurcation). Beyond the interesting parallel with neural systems, excitability could provide innovative functionalities such as optical event detection [11].

At the intersection between the phenomena of nonlinear light localization and excitability, excitable localized structures offer both the parallel mode of operation typical of localized states and the thresholdlike response of excitable systems. Thus, they appear as well-calibrated pulses of light which are localized in space and after which the system returns to its initial quiescent state. In the case studied in Ref. [3], excitability arises from the collision of a stable limit cycle (oscillating localized state) with the unstable localized state branch, leading to a saddle-loop bifurcation close to which the system shows excitable localized structures. Even in variational systems in which localized states do not have the possibility to oscillate "in place," excitable localized structures have recently been theoretically analyzed in the presence of gradients and pinning [12]. In this case, excitability is mediated by the drift of the localized states. Besides the intrinsic interest of the underlying phenomena, the distinctive features of excitable localized structures also opens novel application perspectives for information processing [13].

In this Letter, we observe experimentally that a drift instability can lead to excitable laser localized structures in a very similar way to the theoretical analysis reported in Ref. [12]. We show that the nucleation of a localized structure followed by a drift towards a spatial region where the localized structure vanishes occurs via a finite amplitude and infinite period bifurcation, which is a sufficient condition for excitable behavior [4]. Close to this bifurcation, we observe noise-induced nucleation of localized states, while the following localized structure drift and annihilation are deterministic since they are imposed by a spatial gradient. This deterministic evolution of stochastically triggered events provides further evidence of the excitable character of the drifting laser localized structures. Beyond excitable localized structures, we also observe an alternating motion of a localized structure oscillating back and forth when interacting with both a gradient and a defect.

The experimental system [14,15] consists of two coupled broad-area (200 μm) vertical cavity surface emitting lasers. One laser (L_1, amplifier) is pumped above transparency and the other one (L_2, saturable absorber) below. The devices are optically coupled in a self-imaging condition. The system parameters are the temperatures and the pump currents of the two devices. An output coupler gives two detection branches, each of them comprising a CCD camera, an iris, and a fast detector (DC-8 GHz). We monitor the time-averaged near field of the devices with the CCD, and use the iris to select the region of the space to be monitored. The recorded area has a diameter of about 20 μm, which is slightly larger than the typical dimension of the localized structures.

We measure the time traces in a spatial region where a localized structure is known to be stable (pinning on a defect) under optimal alignment conditions. Under the alignment conditions considered in the following, the parameters are slightly outside the range of stationary localized state bistability [15,16].

For specific alignment conditions (and, therefore, specific spatial gradients) and parameters, the system is in a stable trivial state (no coherent emission). Upon bias current increase, the first observation is the sporadic emission of bursts [Fig. 1(a)], all rather similar to each other, whose duration is of the order of 12 ns. For a given current of the absorber (I_2), as the current of the amplifier (I_1) is increased, the trivial state becomes unstable as the time between two successive pulses (interspike time) decreases and a clear periodicity appears [Figs. 1(b) and 1(c)]. For these parameters, the time-averaged near field image on the CCD consists of a bright 40 μm long and 15 μm wide region surrounded by homogeneous dark background. The detector is placed on the brightest part of this region (see inset in Fig. 1).

Each of the bursts is composed of much faster pulses separated by 1.8 ns, which is the round-trip time of the cavity composed by the two devices. This fine structure is, therefore, associated to the multiple longitudinal modes which are involved in the dynamics of the system. Even if there is certainly interest in this very fast time scale, we chose to focus in this Letter on another relevant dynamical variable of the system, which is the slow envelope of the dynamics [17]. Thus, we Fourier filter the time series allowing only components below 1.3 GHz (thick line on Fig. 1). Although the choice of this value is arbitrary, we have checked that the observations reported below do not depend on the details of the filter, provided that it is sufficient to smooth out the details of the longitudinal mode dynamics.

Although the dynamics looks very reminiscent of the well-known Q-switching instability in lasers with saturable absorbers (a situation which leads to excitability in systems with no spatial degrees of freedom [9]), the typical duration and period of pulses shown in Fig. 3 are rather far from the typical time scales of the semiconductor gain medium or absorber.

In the following, we analyze in more detail the spatial dynamics of the system around this bifurcation. To this end, we acquire time series simultaneously from two different detectors in the spatial region of interest. The detectors are aligned along the elongated direction of the bright area shown on the insets of Fig. 1. Each detector monitors a 20 μm diameter area, whose centers are separated by 23 μm. Typical results are shown on Fig. 2. In the strongly periodic regime, the occurrence of a pulse in one region of space is followed by a pulse in the neighboring region with a delay, and the clear correlation between the two traces demonstrates a propagation in the transverse dimension. The fact that the intensity emitted in the area monitored by $D2$ increases only upon decrease of the intensity emitted in the area monitored by $D1$, calls for an interpretation of the dynamics in terms of a localized state nucleated in front of $D1$, which then drifts across the spatial region of interest. If detector $D2$ is moved farther away, then no intensity is detected, which indicates that the structure has reached a spatial region where it cannot exist due to spatial inhomogeneities [18]. This deterministic trajectory (in space) leads the system back to the initial state, where no structure exists. We note that even in an ideal (perfectly homogeneous) but finite system, the localized structure would cease to exist when reaching the system's edge, again leading to a deterministic lifetime for the localized

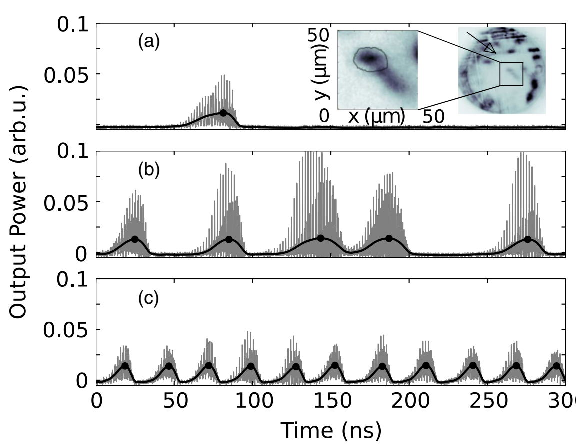

FIG. 1 (color online). Single point dynamics for three different values of the current in the amplifier: (a) $I_1 = 308.4$ mA, (b) $I_1 = 310.9$ mA, (c) $I_1 = 315.4$ mA, $I_2 = 5.5$ mA. Grey lines are the measured time series, black ones are the low-frequency part of the dynamics (1.3 GHz cutoff). Top right inset: Near field of the amplifier device. The arrow indicates a material defect line (see text). The data are acquired in the squared area of which a zoom is shown on the left inset.

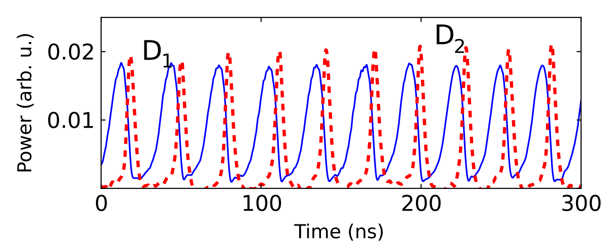

FIG. 2 (color online). Spatially resolved dynamics showing periodic nucleation and drift ($I_2 = 10.5$ mA, $I_1 = 302.9$ mA). Detector 1 is the continuous line, detector 2 is the dashed one.

structure. The speed of the localized structure can be estimated from the time separation between the falling edges in $D1$ and $D2$, i.e., when the structure leaves the area monitored by each detector or from the duration of the pulse measured in $D2$, and the estimated values are coherent (5.2 μm/ns vs 4.8 μm/ns). The motion does not take place along material defect lines, which can be seen on the top and left parts of the inset in Fig. 1.

Localized states in models of bistable lasers are known to be able to move at either constant or oscillating velocity in the absence of any parameter gradient [19,20]. In the present case, however, motion always takes place along a fixed direction which critically depends on alignment. This suggests that perturbations external to the localized structure are responsible for the drift. Therefore, we interpret the periodic pulsing described above in terms of periodic nucleation and motion of a localized structure in the presence of a gradient and a local inhomogeneity.

We argue that the limit cycle constituted by this periodic nucleation and motion until the structure vanishes arises via a finite amplitude and infinite period bifurcation and can, thus, lead to excitable localized structures. Accordingly, we show that noise can trigger excitable events.

We show on the top panel of Fig. 3 how the average time between pulses depends on the amplifier bias for four different values of the absorber bias. The error bars indicate the standard deviation of the time between pulses. In all cases, we observe a clear dependence of the average period on the parameter value with a strong increase of the average period when decreasing the parameter. We interpret this as an indication of the divergence of the period, which is compatible with the period of a limit cycle arising via saddle node on a circle or saddle-loop bifurcations (both of them leading to excitable dynamics). When the average period is largest, the dynamics consists essentially in a disordered emission of pulses as indicated by the very large standard deviation.

Upon amplifier bias increase, this regime is followed by a decrease of the average value of the interspike time. The dispersion of the interspike time also strongly diminishes upon parameter increase, indicating that the dynamics is more and more periodic. The ratio between the standard deviation and the average value of the time between pulses can reach a value of 0.9 (a value of 1 being a distinctive feature of a Poisson process). On the other hand, the amplitude of the pulses is very well defined in any case, as shown on the bottom panel of Fig. 3. This (together with the very well-defined duration of the pulses within a few sampling points) indicates that the trajectory followed by the system in phase space is regular even when the temporal distribution is disordered. All the data shown in Fig. 3 have been obtained in the exact same configuration (alignment and spatial region) for consistency. Even if the evolution of the time interval average and standard deviation as a function of the pump values do not depend on the specific realization, the absolute values of the parameters does depend on the spatial region and alignment configuration in which the measurements are performed.

Histograms of the time between pulses for regimes are shown in Fig. 4 (left). While the periodicity of the time series is very clear in case 4(c), the exponential decay at long times in the other two histograms indicates that the pulse emission process, when not periodic, can be described as a noise-induced barrier crossing (see, e.g., Ref. [21]), as expected for noise-induced excitable pulses. In this framework, the cutoff at short times (15 ns) visible in all these histograms is caused by the refractory time of excitable pulses.

At variance with the observations reported in Ref. [18], which were realized in a coherently driven system, the distribution of time intervals between events in the present

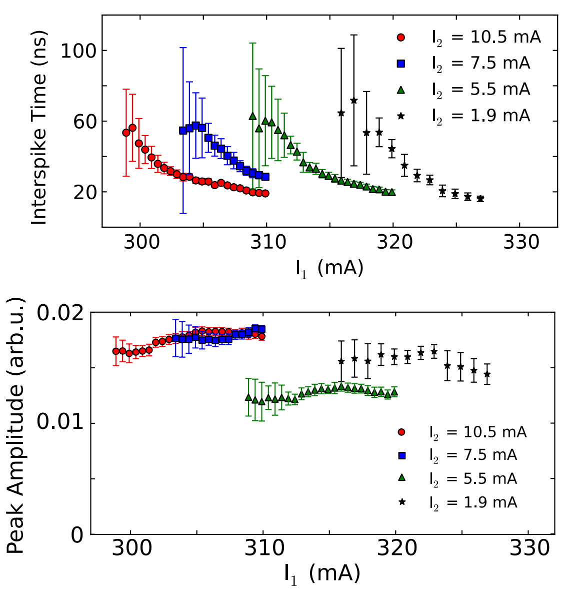

FIG. 3 (color online). Top: Mean value of interspike time as a function of amplifier pump I_1 for different values of the absorber current. Bottom: Amplitude of the pulses as a function of the amplifier current for different values of the absorber current. $I_2 = 10.5$ mA (filled circle), $I_2 = 7.5$ mA (open square), $I_2 = 5.5$ mA (open upward triangle), $I_2 = 1.9$ mA (filled star). The error bars indicate the standard deviation.

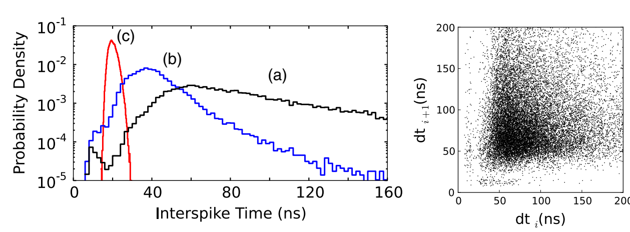

FIG. 4 (color online). Left: Histograms of interspike times in noise-dominated [(a) $I_1 = 343.9$ mA, $I_2 = 1.9$ mA], periodic [(c) $I_1 = 343.4$ mA, $I_2 = 5.9$ mA], and intermediate regimes [(b) $I_1 = 333.6$ mA, $I_2 = 5.9$ mA]. Right: Return map of time intervals corresponding to case (a).

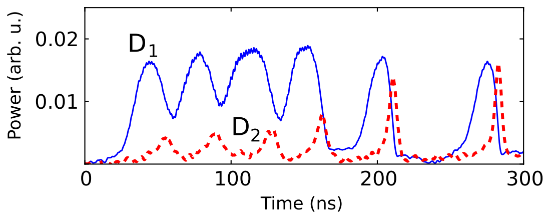

FIG. 5 (color online). Spatially resolved dynamics showing oscillatory motion ($I_2 = 10.5$ mA, $I_1 = 298.9$ mA).

case can have a very strong stochastic component. This characteristic is essentially absent in Ref. [18], where the nucleation of the localized takes place in a spatial region in which the trivial solution is unstable. In that case, the small variation of the period is then due to imperfect control of the parameters which slowly drift. Here, instead, noise appears to be sufficient to randomly trigger the nucleation and drift of localized structures, which are, therefore, well described in terms of excitable localized states. This is confirmed by the observation of the return map of time intervals shown in the right panel of Fig. 4: While fluctuations of the period of a limit cycle would lead to an accumulation of points in the $y = x$ region, the distribution of points filling the whole phase space clearly indicates the random origin of the time intervals. Since this experimental system involves lasing and not only nonlinear amplification, as occurs in driven systems, we propose that the projection of spontaneous emission noise onto the localized states is sufficient to trigger their nucleation, a situation much less likely to happen in coherently driven systems. Of course, other noise sources (current noise, for instance) could also play a role in the present dynamics.

A low-dimensional phase space (in the presence of noise) might be sufficient to describe the periodic and excitable dynamics of localized states observed above, but such description cannot be reduced to simply a translation mode. In fact, it must take into account the internal degrees of freedom of the localized states since they are not infinitely damped [3], as we illustrate in the following.

For the lowest amplifier bias current values, (i.e., when the average period is large and not well defined) the time series sometimes presents an unexpected dynamical feature shown in Fig. 5. During the interval from 30 to 150 ns, the power measured in $D1$ oscillates around some value without ever reaching the background level (the nonlasing solution), and, correspondingly, the power observed in $D2$ oscillates without ever reaching the maximum value corresponding to the presence of a localized state. Since an increase in power in $D2$ is associated to a decrease in $D1$ (and the larger the signal in $D1$, the smaller the signal in $D2$), we interpret this dynamics as back and forth motion of the localized structure close to the position monitored by $D1$. Indeed, it has recently been shown in Ref. [12] that a localized state interacting with defects and submitted to a gradient can oscillate in this way whenever the defect is strong enough to prevent instantaneous drift away of the localized state.

The amplitude of these oscillations increases until, at time $t \approx 200$ ns, the localized structure detaches and fully passes in front of $D2$, restoring the regime of mostly periodic nucleation and drift that were observed previously. The back and forth oscillation events are extremely rare as compared to regular drift events, and, therefore, do not clearly appear in the histograms [slight shoulder on the left part of the Figs. 3(b) and 3(c) histograms].

Clearly, the back and forth motion of the localized structure close to its nucleation site departs from the "particlelike" description of localized structure under external perturbations whose inertia is completely hidden by dissipation [22]. Since this inertialike behavior of the localized structure results from the external perturbation coupling to the structure's internal modes, it is understandable that the oscillatory motion is observed at the very edge of the localized structure's stability domain [23] (see also Ref. [12]), and, therefore, in the parameter range in which the periodicity breaks up and is strongly influenced by noise.

In conclusion, we have shown that the destabilization of the nonlasing solution in a system of coupled broad-area lasers in an absorber-amplifier configuration can occur via periodic or stochastic emission and deterministic drift and cancellation of localized structures. Since this limit cycle arises with a finite amplitude, the corresponding bifurcation possesses the characteristics required for the generation of excitable localized structures. Indeed, we have observed the random emission of deterministic pulses constituted by the nucleation, drift, and annihilation of single laser localized structures. In addition, to noise-triggered excitable localized structures, we have also observed oscillations of localized states around a pinning location. This dynamics, which departs from the "Aristotelian particle" description of a localized structure's motion, requires taking into account the interaction of the structure with a defect as an additional phase space dimension.

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