Coherent Control of Population Transfer between Vibrational States in an Optical Lattice via Two-Path Quantum Interference

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We demonstrate coherent control of population transfer between vibrational states in an optical lattice by using interference between a one-phonon transition at 2ω and a two-phonon transition at ω. The ω and 2ω transitions are driven by phase- and amplitude-modulation of the lattice laser beams, respectively. By varying the relative phase of these two pathways, we control the branching ratio between transitions to the first excited state and those to the higher states. Our best result shows a branching ratio of 17 ± 2, which is the highest among coherent control experiments using analogous schemes. Such quantum control techniques may find broad application in suppressing leakage errors in a variety of quantum information architectures.

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Coherent control of quantum systems is an exciting paradigm that has been applied in a number of contexts ranging from chemistry to condensed-matter physics [1–6]. In this Letter, we demonstrate that a common coherent-control technique ("1 versus 2") [7] may be extended to control leakage out of a bound-state manifold, by using coupling pulses of appropriately tailored symmetries. In particular, we show that by using this technique, it is possible to enhance the coupling of vibrational states in an optical lattice while reducing leakage into "lossy" states. This provides a new tool for the control of motional states in optical lattices, and is promising for applications in areas such as quantum information more broadly. Quantum information processing relies on the coherent manipulation of quantum two-level systems (qubits). In reality, any qubit is a subspace of a larger Hilbert space, meaning that one possible error is leakage to the states outside the computational subspace. Leakage error has been a major concern in many quantum information processing devices [8], including neutral atoms in optical lattices [9,10], superconducting qubits [11,12], trapped ions [13], and cavity QED [14]. In the present experiment, we study a qubit composed of quasibound vibrational states in a tilted-washboard potential possessing unbound states as well, and demonstrate a coherent control technique that greatly increases the branching ratio of the desired transition into an excited quasibound state to the "leakage" transitions into unbound states.

The coherent control scheme we use is analogous to the one-photon versus two-photon interference scheme (1 versus 2) [7], which we show here can be adapted to control vibrational excitations by careful control of the symmetry of the excitation pathways. In the 1 versus 2 scheme, control is achieved by coupling the initial state to the desired final state through two simultaneous, phase-coherent pathways—in one path, the transition is accomplished via absorption of one photon at frequency 2ω; in the other, via absorption of two photons each at frequency ω. The total transition amplitude is the coherent sum of the amplitudes for these two processes, allowing the final-state probability to be controlled by varying the relative phase of the ω and 2ω transitions. This concept has been applied to coherently control the photoionization of atoms [2], photocurrents in semiconductors [3] and graphene [4], and photodissociation of molecules [5], to name a few proof-of-principle examples. There have also been proposals for using this technique to study the quantum-to-classical transition [15], to control photocurrents in carbon nanotubes [16] and molecular wires [17], as well as to control the populations of different electronic states in semiconductor quantum wells [18] and molecular wires [19].

In our Letter, we extend the application of this technique into the domain of quantum information by demonstrating an analogous scheme based on one-phonon versus two-phonon interference in a two-vibrational-state system. Using this technique, we succeed in suppressing leakage during coherent coupling of the two lowest vibrational states. Our method is applicable to situations where leakage is a major source of error, such as the exchange gate experiments with neutral atoms in optical lattices [9]. The Hamiltonian of the system we study also has the same form as the one for superconducting qubits [11], suggesting that this method should be useful for suppressing leakage errors in such systems as well.

The two-level system we use is made up of the lowest two vibrational states of an atom trapped in each potential well of an optical lattice. Because our one-dimensional optical lattice is in the vertical direction, the atoms are actually trapped in a tilted-washboard potential, which is the sum of the periodic lattice potential and the linear potential due to gravity. An atom in the tilted-washboard potential possesses quasibound vibrational states, known as Wannier-Stark states [20,21]. We use a shallow tilted-washboard potential that only has two long-lived Wannier-Stark states centered on each potential well. Treating these two states as a qubit, we consider any coupling into the higher excited vibrational states as leakage. To rotate this qubit, we coherently transfer population from the ground state to the excited state by a one-phonon excitation. The one-phonon excitation is experimentally realized by phase modulation (PM) of one lattice laser beam at ω with an acousto-optic modulator (AOM) [22], where ω is the resonance frequency between the ground and first excited states and is measured to be $2\pi \times (4.99 \pm 0.01)$ kHz for this experiment [23]. The PM creates a series of sidebands at $\omega_L \pm q\omega$, where ω_L is the laser frequency and q is an integer. A Raman transition involving one photon at ω_L and another photon at $\omega_L \pm \omega$ can couple two vibrational states with energy separation of $\hbar\omega$. The effective interaction has (in the reference frame that follows the displacement of the potential) the same form as a dipole Hamiltonian [as will become clear in Eq. (1) below], and therefore couples vibrational states of opposite parity. We refer to this creation of a vibrational excitation via absorption of a single modulation quantum at ω as a one-phonon excitation at ω. An atom in the ground state can absorb one phonon at ω and be transferred into the excited state, but it can also absorb two phonons at ω and leak out of the qubit space, as shown in case (i) of Fig. 1(a). To mitigate this leakage, we introduce a second pathway of excitation, a

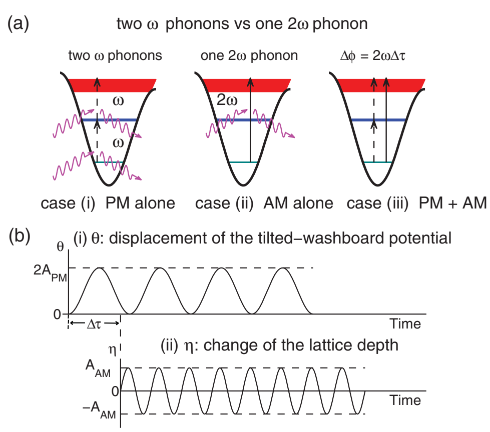

FIG. 1 (color online). (a) An atom in the ground state can leak out of the qubit space by absorbing either [case (i)] two phonons at ω (dashed arrows) or [case (ii)] one phonon at 2ω (solid arrow). When the two transitions occur together [case (iii)], the probability of leakage depends on the relative phase between these two transitions. (b) Experimentally, the ω-phonon transition is realized by phase modulation (PM) of one lattice beam at ω (i). The 2ω-phonon transition is realized by amplitude modulation (AM) of the other lattice beam at 2ω (ii). The relative phase $\Delta\phi$ between the two transitions is controlled by the difference $\Delta\tau$ between the initial times when PM and AM are applied, through a relationship of $\Delta\phi = 2\omega\Delta\tau$.

one-phonon excitation at 2ω, as shown in case (ii) of Fig. 1(a). Experimentally, we perform amplitude modulation (AM) on the other lattice laser beam at 2ω with an AOM, which creates two sidebands of $\omega_L \pm 2\omega$. A Raman transition involving one photon at ω_L and another photon at $\omega_L \pm 2\omega$ can couple two vibrational states with energy separation of $2\hbar\omega$. The even symmetry of the amplitude modulation allows this process to directly couple states of like parity. We refer to this creation of a vibrational excitation via absorption of a single modulation quantum at 2ω as a one-phonon excitation at 2ω. When the two-phonon transition at ω and the one-phonon transition at 2ω are both driven, as shown in case (iii) of Fig. 1(a), the two pathways interfere, such that the leakage probability depends on their relative phase. By adjusting the pathways to have equal but opposite amplitudes, one could in principle suppress the leakage completely.

We use a sample of ^{85}Rb atoms laser cooled to approximately 10 μK, with a sufficiently low density (roughly 10^9 atoms/cm^3) that we can neglect interactions between atoms. Our optical lattice is formed by two 15 mW laser beams, red detuned by 30 GHz from the D_2 line, which intersect at an angle of 49°, resulting in a lattice spacing of $a = 0.930$ μm, which is much larger than the 60 nm thermal de Broglie wavelength of the atoms. There is therefore vanishing coherence between neighboring wells of the lattice. This lattice has a typical depth of $19\hbar\omega_r$, where $\omega_r = 2\pi \times h/(8ma^2) = 2\pi \times 685$ Hz is the effective recoil frequency and m is the mass of one ^{85}Rb atom. Because of the 1.5-mm rms width of the Gaussian lattice beams, the lattice depth is inhomogeneously broadened; the distribution of lattice depths shown in the inset of Fig. 3(b) below is measured using the technique in Ref. [24]. The linear potential of gravity has a value of $2.86\hbar\omega_r$ per lattice spacing, and for these parameters there are only two long-lived Wannier-Stark states centered on each lattice well [20]. By adiabatically lowering the depth of the optical lattice until only one Wannier-Stark state is supported, and then adiabatically increasing it again, we prepare the atoms in the lowest Wannier-Stark state. This same filtering technique [25] is used to measure the populations of the different vibrational states after excitation.

To investigate the interference between the two different transition pathways, we vary the relative phase $\Delta\phi$ between the two-phonon transition at ω and the one-phonon transition at 2ω, while keeping the probability of each transition constant. The probability of each transition depends on the modulation amplitudes A_{PM}, A_{AM}, and modulation duration t_m. We always perform the same modulation duration $t_m = 2n\pi/\omega$ for both PM and AM, where n is an integer. Figure 1(b) shows examples of PM and AM with $n = 4$. The modulation duration we use in this experiment is always much smaller than the measured photon scattering time of about 50 ms. The relative phase $\Delta\phi$ between the two transitions depends on the difference

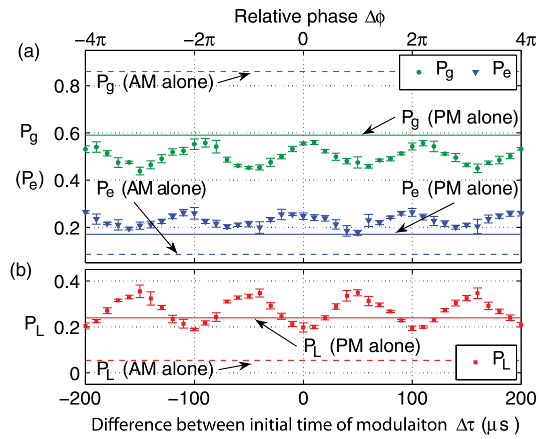

FIG. 2 (color online). The oscillation curves of P_g (green dots), P_e (blue triangles), and P_L (red squares) vs $\Delta\tau$ when PM and AM are applied together. The control parameters of PM and AM used for these interference fringes are $n = 4$, $A_{\rm PM} = 8°$, $A_{\rm AM} = 10\%$.

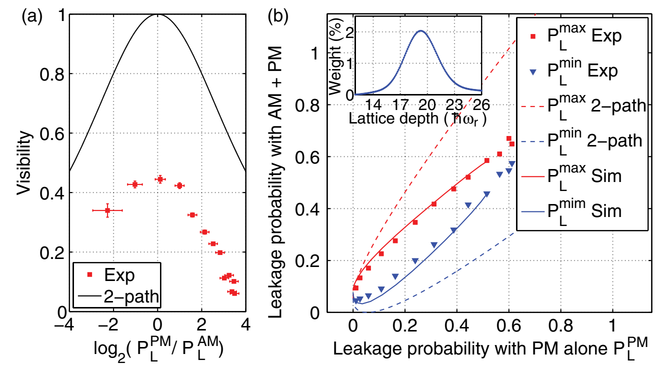

FIG. 3 (color online). (a) Visibility $(P_L^{\max} - P_L^{\min})/(P_L^{\max} + P_L^{\min})$ vs $\log_2(P_L^{\rm PM}/P_L^{\rm AM})$; experiment: red dots; idealized two-path model: solid curve. (b) $P_L^{\max}$ and $P_L^{\min}$ vs $P_L^{\rm PM}$ for $P_L^{\rm AM} = 0.06 \pm 0.01$. Inset: lattice depth distribution.

$\Delta\tau$ between the initial time of PM and AM, through the relationship $\Delta\phi = 2\omega\Delta\tau$. In the first part of the experiment, we measure the probabilities of leaking out of the qubit space P_L, of being transferred into the excited state P_e, and of being left in the ground state P_g, when $\Delta\tau$ is varied and $A_{\rm PM}$, $A_{\rm AM}$, and n are kept constant. Typical oscillation curves of P_g, P_e, and P_L vs $\Delta\tau$ measured in the experiment are shown in Fig. 2, clearly demonstrating interference between the two transition pathways. The constructive and destructive interference conditions for P_L are found to be $\Delta\phi = (2l + 1)\pi$ and $\Delta\phi = 2l\pi$, respectively, where l is an integer. Figure 2 shows that compared to the leakage $P_L^{\rm PM}$ when PM alone is applied, P_L is suppressed at the points where destructive interference occurs. The probability of transition into the excited state is increased at the same points. This demonstrates that the leakage due to the two-phonon transition at ω is suppressed by simultaneously driving the one-phonon transition at 2ω with the appropriate phase.

We further study the dependence of this interference on the leakage probability $P_L^{\rm PM}$ by measuring oscillation curves of P_L vs $\Delta\tau$ for different values of the control parameter $A_{\rm PM}$, while $A_{\rm AM}$ and n are kept constant. As all the measured interference fringes show the same constructive and destructive interference conditions, we focus our study on the dependence of the visibility on $P_L^{\rm PM}$. We denote the leakage probability when constructive (destructive) interference conditions are met as $P_L^{\max}$ ($P_L^{\min}$), which means visibility can be expressed as $(P_L^{\max} - P_L^{\min})/(P_L^{\max} + P_L^{\min})$. To find $P_L^{\max}$ and $P_L^{\min}$, we perform a sinusoidal fit for each interference fringe with the oscillation frequency held constant at 10 kHz. Figure 3(a) shows a plot of visibility versus $\log_2(P_L^{\rm PM}/P_L^{\rm AM})$, where $P_L^{\rm AM}$ is the leakage probability when AM alone is applied. We find experimentally (red dots) that the maximum visibility occurs when $P_L^{\rm PM} = P_L^{\rm AM}$, as would be expected from an idealized two-path interference (two-path) model (solid black line). Such a model assumes the total probability amplitude for leakage is the sum of the amplitudes for the two individual transition pathways, with perfect phase coherence. We would then expect $P_L^{\max,\min} = P_L^{\rm PM} + P_L^{\rm AM} \pm 2\sqrt{P_L^{\rm PM}P_L^{\rm AM}}$. Although the experimental result agrees with the two-path model as to when the maximum visibility should occur, it shows lower visibility.

We find, with a simulation of our experiment, that the discrepancy in visibility between the experimental results and the idealized two-path model is due to the coupling into neighboring potential wells. To perform the simulation, we consider the Hamiltonian for an atom in the tilted-washboard potential $H_0 = p^2/(2m) + U_0\sin^2(\pi x/a) + mgx$, where U_0 is the optical lattice depth and g is the acceleration due to gravity. By introducing dimensionless parameters $\tilde{x} = \pi x/a$, $\tilde{p} = ap/(\pi\hbar)$, $r = U_0/(\hbar\omega_r)$, and $s = mga/(\hbar\omega_r)$, we can write the dimensionless time-dependent Hamiltonian in the reference frame that follows the displacement of the potential [20] in the form

$$\tilde{\mathbf{H}}_{\mathbf{U}}(t) = \tilde{\mathbf{p}}^2 + r\sin^2\tilde{\mathbf{x}} + \frac{s}{\pi}\tilde{\mathbf{x}} - \frac{\ddot{\theta}(t)}{2}\tilde{\mathbf{x}} + r\eta(t)\sin^2\tilde{\mathbf{x}}, \qquad (1)$$

where $\theta(t) = A_{\rm PM}(1 - \cos\omega t)$ is the displacement of the tilted-washboard potential and $\eta(t) = A_{\rm AM}\sin[2\omega(t - \Delta\tau)]$ is the fractional potential depth modulation, as shown in Fig. 1(b). Using the Hamiltonian [Eq. (1)], we employ a split-operator method to numerically solve the time-dependent Schrödinger equation with the ground state as the initial state. In this simulation, we find that the total leakage is made up of both leakage to other states in the same well and into other wells. Although we see interference for both kind of leakage in our simulation, the destructive interference happens at different times for the two channels. We believe this is the major cause of the reduced visibility, as shown in Fig. 3(a). Further study of the simulation results shows that the total leakage $P_L^{\rm PM}$ has a monotonically increasing but nonlinear dependence on $A_{\rm PM}$ when t_m is fixed. It also shows that, for $A_{\rm PM}$ corresponding to the total leakage above our experimental uncertainty, the ratio of the same-well leakage to the interwell leakage decreases as $A_{\rm PM}$ increases when t_m is fixed. This means, in our experiment, the ratio of the same-well leakage to the interwell leakage decreases as $P_L^{\rm PM}$ increases. Hence our coherent control scheme works better for small $P_L^{\rm PM}$, where the interwell leakage is much smaller than the same-well leakage and can be safely ignored. To compare our simulation to experimental results, we average the simulation results for each lattice depth r over the distribution of lattice depths the atoms actually experience, as shown in the inset of Fig. 3(b). In Fig. 3(b) we plot $P_L^{\rm max}$ ($P_L^{\rm min}$) vs $P_L^{\rm PM}$ from these averaged simulation results to compare with the experimentally measured ones. The simulation results (red and blue solid lines) agree much better with the experimental data (red squares and blue triangles) than the idealized two-path model (red and blue dashed lines).

In order to test the effectiveness of this coherent control technique, we carry out a search for the best branching ratio in the parameter space of $A_{\rm PM}$, $A_{\rm AM}$, and n, while keeping $\Delta\tau = 0$ (where the destructive interference occurs). We define the branching ratio as the probability of transition into the excited state divided by the probability of leakage, $B = P_e/P_L$. If we could completely suppress the leakage, the branching ratio B would go to infinity. It would not be useful to increase the branching ratio by reducing both transition probabilities simultaneously, so we study both figures of merit B and P_e. Figure 4 shows B vs P_e on a log-log graph for a typical set of our experimental data, where $n = 2$. For each curve in the graph, we hold $P_L^{\rm PM}$ constant and plot the branching

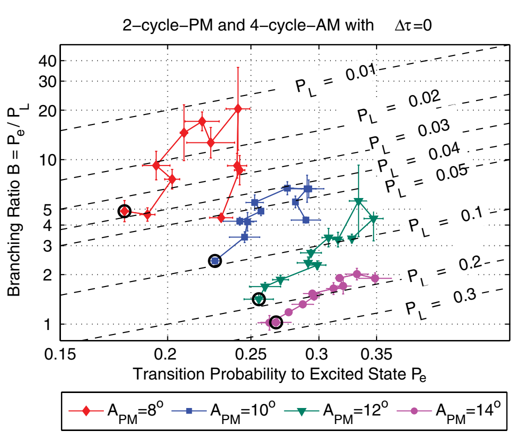

FIG. 4 (color online). Branching ratio $B = P_e/P_L$ vs P_e on a log-log plot for $n = 2$. For each curve, $P_L^{\rm PM}$ is kept constant by holding $A_{\rm PM}$ constant. The black circle on each curve marks the data when PM alone is applied. The points on the same curve are connected in order of increasing $A_{\rm AM}$ by 2% increments with the black circle as the starting point. The dashed lines in the plots are equiloss lines.

ratio in the absence of AM as a black circle for reference. All the other points on the same curve correspond to different values of $P_L^{\rm AM}$ (experimentally, different values of $A_{\rm AM}$). As we have learned that the two-path model does not work perfectly for large values of $P_L^{\rm PM}$ and $P_L^{\rm AM}$, we expect our coherent control technique to work better for small n, $A_{\rm PM}$, and $A_{\rm AM}$. This is confirmed by our experimental results. For $n > 5$, we never observe any suppression of the leakage. For $n \leq 5$, the branching ratio is always higher for smaller $A_{\rm PM}$ with a given value of P_e; one can see this in Fig. 4. For a given value of $A_{\rm PM}$, increasing the amplitude of AM at first decreases the leakage while simultaneously increasing the excitation probability (and hence the branching ratio). This process reaches an optimum, after which the leakage begins to grow again and the branching ratio declines. The largest enhancement in branching ratio was seen for the smallest value of $A_{\rm PM}$ tested; for the largest values of $A_{\rm PM}$, very little improvement was observed. This agrees with the simulation: when $P_L^{\rm AM}$ is too large, we expect the interwell leakage to become significant, and the degree of leakage suppression is reduced. The largest increase we observed in branching ratio was by a factor of 3.5 ± 0.7, achieved for $n = 2$ and $A_{\rm PM} = 8°$ when $A_{\rm AM}$ was set to 10%: the resulting branching ratio was 17 ± 2. Experimental uncertainties prevent us from measuring smaller values of P_L, but our observations are consistent with the expectation that the enhancement continues to improve for lower drive amplitudes. In summary, we have succeeded in reducing the leakage down to a level limited only by our measurement accuracy.

To conclude, we have experimentally demonstrated a novel coherent control technique for effectively suppressing the leakage error for a two-level system. It is the first coherent control experiment on vibrational excitations in an optical lattice. Our experiment shows that leakage can be suppressed by interference between a two-phonon transition at ω and a one-phonon transition at 2ω during the coherent population transfer between the ground and excited states of an atom in a tilted-washboard potential. Using this technique, we are able to simultaneously suppress the leakage and increase the transition probability into the excited state. Although our data and analysis show that there are other leakage transitions when the excitation becomes large, it may well be possible to engineer the interference conditions for multiple interference pathways through pulse engineering techniques such as GRAPE [26]. The best achieved branching ratio $B = P_e/P_L$ in our system was 3.5 ± 0.7 times the branching ratio in the absence of coherent control. We believe that similar techniques will prove useful for minimizing leakage in a variety of quantum information architectures, and particularly in those which rely on periodic or washboard potentials, including both optical lattices and superconducting qubits.

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