Yukawa Unification Predictions with Effective ''Mirage'' Mediation

Archana Anandakrishnan and Stuart Raby

The Ohio State University, 191 West Woodruff Avenue, Columbus, Ohio 43210, USA (Received 22 March 2013; revised manuscript received 23 October 2013; published 18 November 2013)

In this Letter we analyze the consequences, for the LHC, of gauge and third family Yukawa coupling unification with a particular set of boundary conditions defined at the grand unified theory (GUT) scale, which we characterize as effective "mirage" mediation. We perform a global χ^2 analysis including the observables M_W , M_Z , G_F , α_{em}^{-1} , $\alpha_s(M_Z)$, M_t , $m_b(m_b)$, M_τ , $BR(B \to X_s \gamma)$, $BR(B_s \to \mu^+ \mu^-)$, and M_h . The fit is performed in the minimal supersymmetric standard model in terms of 10 GUT scale parameters fit is performed in the minimal supersymmetric standard model in terms of 10 GUT scale parameters, while tan β and μ are fixed at the weak scale. We find good fits to the low energy data and a supersymmetry spectrum which is dramatically different than previously studied in the context of Yukawa unification.

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Gauge coupling unification in supersymmetric grand unified theories (SUSY GUTs) $[1-6]$ $[1-6]$ $[1-6]$ provides an experimental hint for low energy SUSY. However, it does not significantly constrain the spectrum of supersymmetric particles. On the other hand, it has been observed that Yukawa coupling unification for the third generation of quarks and leptons in models, such as $SO(10)$ or $SU(4)_c \times SU(2)_R$ - can place significant constraints on the $SU(2)_L \times SU(2)_R$, can place significant constraints on the SUSY spectrum in order to fit the top bottom and tau SUSY spectrum in order to fit the top, bottom, and tau masses [\[7](#page-3-2)[–11\]](#page-3-3). These constraints depend on the particular boundary conditions for sparticle masses chosen at the GUT scale (see, for example, Refs. [[9](#page-3-4),[12](#page-4-0)[–14\]](#page-4-1), which consider different GUT scale boundary conditions). In this Letter we consider effective "mirage" mediation boundary conditions and show that they are consistent with gauge and Yukawa coupling unification with a dramatically different low energy SUSY spectrum. The GUT scale boundary conditions are given by an effective mirage pattern with gaugino masses defined in terms of two parameters, $M_{1/2}$ an overall mass scale and α the ratio of the anomaly mediation to gravity mediation contribution [[15](#page-4-2)[–18\]](#page-4-3). Scalar masses are given in terms of m_{16} (for squarks and sleptons) and m_{10} (for Higgs doublets). In addition, the H_u and H_d masses are split, either with "just-so" splitting or with a $U(1)$ D term which affects all scalar masses. Note, as in Ref. [[15](#page-4-2)], we allow for several origins of SUSY breaking. For example, the dilaton and conformal compensator fields break SUSY at a scale of order $M_{1/2}$, while the dominant contribution to SUSY breaking is at a scale of order $m_{3/2} \ge m_{16} \approx m_{10}$. We fit the low energy observables, M_W , M_Z , G_F , α_{em}^{-1} , $\alpha_s(M_Z)$, M_t , $m_b(m_b)$, M_{τ} ,
RR $(B \rightarrow X \gamma)$, RR $(B \rightarrow u^+ u^-)$ and M, in terms of 12 $BR(B \to X_s\gamma)$, $BR(B_s \to \mu^+\mu^-)$ and M_h in terms of 12 arbitrary parameters. The low energy sparticle spectrum is imminently amenable to testing at the LHC. Two benchmark points are contained in Table [III.](#page-2-0)

Fermion masses and quark mixing angles are manifestly hierarchical. The simplest way to describe this hierarchy is with Yukawa matrices which are also hierarchical.

Moreover, the most natural way to obtain the hierarchy is in terms of effective higher-dimension operators of the form

$$
W \supset \lambda 16_3 10 16_3 + 16_3 10 \frac{45}{M} 16_2 + \cdots. \qquad (1)
$$

This version of $SO(10)$ models has the nice feature that it only requires small representations of $SO(10)$, has many predictions, and can, in principle, find an UV completion in string theory. The only renormalizable term in W is λ $16₃$ 10 $16₃$ which gives Yukawa coupling unification

$$
\lambda = \lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \tag{2}
$$

at M_{GUT} . Note, one *cannot* predict the top mass due to large SUSY threshold corrections to the bottom and tau masses, as shown in Refs. [\[19–](#page-4-4)[21](#page-4-5)]. These corrections are of the form

$$
\delta m_b / m_b \propto \frac{\alpha_3 \mu M_{\tilde{g}} \tan \beta}{m_{\tilde{b}}^2} + \frac{\lambda_t^2 \mu A_t \tan \beta}{m_{\tilde{t}}^2} + \log \text{ corrections.}
$$
\n(3)

So instead we use Yukawa unification to predict the soft SUSY breaking masses. In order to fit the data, we need

$$
\delta m_b / m_b \sim -2\%.\tag{4}
$$

We take $\mu < 0$, $M_{\tilde{g}} > 0$. For a short list of references on this subject, see $[7-11, 22-27]$ $[7-11, 22-27]$ $[7-11, 22-27]$ $[7-11, 22-27]$ $[7-11, 22-27]$ $[7-11, 22-27]$ $[7-11, 22-27]$.

We assume the following GUT scale boundary conditions, namely, a universal squark and slepton mass parameter m_{16} , universal cubic scalar parameter A_0 , mirage mediation gaugino masses,

$$
M_{i} = \left[1 + \frac{g_{G}^{2}b_{i}\alpha}{16\pi^{2}}\log\left(\frac{M_{Pl}}{m_{16}}\right)\right]M_{1/2}
$$
 (5)

(where $M_{1/2}$ and α are free parameters and $b_i = (33/5, 1, -3)$ for $i = 1, 2, 3$). Note this expression is $(33/5, 1, -3)$ for $i = 1, 2, 3$. Note, this expression is equivalent to the gaugino masses defined in Ref. [[28\]](#page-4-8).

 α in the above expression is related to the ρ in Ref. [[15](#page-4-2)] as $(1/\rho) = (\alpha/16\pi^2) \ln(M_{PL}/m_{16})$. We consider two differ-
ent cases for nonuniversal Higgs masses (NHHM) with ent cases for nonuniversal Higgs masses (NUHM) with just-so Higgs splitting

$$
m_{H_{u(d)}}^2 = m_{10}^2 - (+)2D \tag{6}
$$

or, D-term Higgs splitting, where, in addition, squark and slepton masses are given by

$$
m_a^2 = m_{16}^2 + Q_a D, \{Q_a = +1, \{Q, \bar{u}, \bar{e}\}; -3, \{L, \bar{d}\}\}, (7)
$$

with the U(1) D term D, and SU(5) invariant charges Q_a . Note, we take μ , $M_{1/2} < 0$. Thus, for $\alpha \ge 4$ we have $M_{1} > 0$, $M_{1} M_{2} < 0$. Note the case of *D*-term splitting $M_3 > 0$, M_1 , $M_2 < 0$. (Note, the case of D-term splitting is similar to the analysis of Ref. [[13\]](#page-4-9). However, our low energy SUSY spectrum is much different.) In the set of boundary conditions above, the scalar masses and trilinear couplings are large (of order $m_{3/2}$), while the magnitude of the gaugino masses is given by $M_{1/2} \ll m_{3/2}$. Note, this does not agree with the examples of mirage mediation in the literature. For example, in the context of type IIB strings, Refs. [\[16–](#page-4-10)[18](#page-4-3)], the scalar, gaugino, and trilinear couplings are all of order $m_{3/2}$, while in the heterotic version of mirage mediation, Ref. [\[15\]](#page-4-2), the soft terms for scalar masses are of order $m_{3/2}$, while the gaugino masses and trilinear couplings are given by $M_{1/2} \ll m_{3/2}$. Finding a SUSY breaking mechanism with the set of boundary conditions presented here is still an open challenge. Nevertheless, we are using the SO(10) symmetry to justify Yukawa unification for the third family and then finding the minimal set of SUSY breaking parameters at the GUT scale consistent with the low energy data. This forces A_0 to be large and we have ignored the small corrections to both A_0 and the quadratic scalar mass terms due to anomaly mediation.

We perform a global χ^2 analysis varying the parameters in Table [I](#page-1-0) used to calculate the total χ^2 function in terms of all the observables given in Table II defined at the electro-weak scale as discussed in Ref. [\[14\]](#page-4-1). We minimize the χ^2 function using the MINUIT package maintained by CERN [\[33\]](#page-4-11). Note that MINUIT is not guaranteed to find the *global* minimum, but will in most cases converge on a local one. For that reason, we iterate $\mathcal{O}(100)$ times the minimization procedure for each set of input parameters, and in each step we take a different initial guess for the minimum (required by MINUIT) so that we have a fair chance of finding the true minimum. We realize that the system is underconstrained and thus we obtain values of $\chi^2 \ll 1$. For this reason, it is not possible to define a goodness of fit or χ^2 /d.o.f. However, in Fig. [1](#page-2-1), we fix certain parameters such that we have 2 d.o.f., and plot contours of χ^2 /d.o.f. = 1, 2.3, 3 corresponding to a 95%, 90%, and 68% C.L., respectively. One could also add more observables to the fit and this is possible when one considers a three family model, which is the subject of an ongoing study. The additional parameters determining fermion masses, mixing angles, and flavor

TABLE I. The model is defined by three gauge parameters α_G , M_G [where $\alpha_1(M_G) = \alpha_2(M_G) \equiv \alpha_G$], and $\epsilon_3 = (\alpha_3 - \alpha_G/\alpha_G)$,
one large Yukawa coupling λ six SUSY parameters defined at one large Yukawa coupling, λ , six SUSY parameters defined at the GUT scale, m_{16} (universal scalar mass for squarks and sleptons), $M_{1/2}$ (universal gaugino mass), α (the ratio of anomaly mediation to gravity mediation contribution to gaugino masses), m_{10} (universal Higgs boson mass), A_0 (universal trilinear scalar coupling), and D which fixes the magnitude of Higgs splitting in the case of just-so Higgs splitting or the magnitude of all scalar splitting in the case of D-term splitting. The parameters μ , tan β are obtained at the weak scale by consistent electroweak symmetry breaking.

observables for the first two families introduce more degrees of freedom (as discussed previously in Ref. [\[14\]](#page-4-1) with different GUT scale boundary conditions), but they do not significantly affect the SUSY spectrum.

Consider first the SUSY spectrum in our analysis. Two benchmark points are given in Table [III](#page-2-0) with fixed $m_{16} =$ 5 TeV. The first and second family squarks and sleptons have mass of order m_{16} , while top squarks, bottom squarks, and tau squarks are all a factor of about 2 lighter. In addition, gluinos are always lighter than the third family squarks and sleptons, and the lightest charginos and neutralinos are even lighter. Figure [1](#page-2-1) shows that the gluino mass increases as α increases and we are able to find good fits for gluino masses up to at least 3 TeV. In models with universal gaugino masses, however, it was found that for fixed values of m_{16} , there is an upper bound on the gluino

TABLE II. The 11 observables that we fit and their experimental values. Capital letters denote pole masses. We take LHCb results into account, but use the average by Ref. [[29](#page-4-12)]. All experimental errors are 1σ unless otherwise indicated. Finally, the Z mass is fit precisely via a separate χ^2 function solely imposing electroweak symmetry breaking.

Observable	Exp. Value	Ref.
$\alpha_3(M_Z)$	0.1184 ± 0.0007	$\lceil 30 \rceil$
$\alpha_{\rm em}$	1/137.035 999 074(44)	$\lceil 30 \rceil$
G_μ	$1.16637876(7) \times 10^{-5}$ GeV ⁻²	[30]
M_W	80.385 ± 0.015 GeV	$\lceil 30 \rceil$
M_{Z}	91.1876 ± 0.0021	$\lceil 30 \rceil$
M_{t}	173.5 ± 1.0 GeV	$\lceil 30 \rceil$
$m_h(m_h)$	4.18 ± 0.03 GeV	[30]
M_{τ}	1776.82 ± 0.16 MeV	$\lceil 30 \rceil$
M_h	$125.3 \pm 0.4 \pm 0.5$ GeV	$\lceil 31 \rceil$
$BR(b \rightarrow s\gamma)$	$(343 \pm 21 \pm 7) \times 10^{-6}$	$\lceil 29 \rceil$
$BR(B_s \to \mu^+ \mu^-)$	$(3.2 \pm 1.5) \times 10^{-9}$	$\left[32\right]$

FIG. 1 (color online). The figure shows total χ^2 in the α - $M_{1/2}$ plane. The different shades of blue regions have $\chi^2/\text{d.o.f.} = 1, 2.3, 3,$ and greater (from light to dark), and the solid olive lines show contours of constant gluino mass.

mass [[14](#page-4-1)], which is not the case here. Note, CMS and ATLAS have used simplified models to place lower bounds on the gluino mass. However, the allowed decay modes for our model, as presented below, do not in any way resemble any simplified model. Preliminary analysis, Ref. [\[34\]](#page-4-16),

TABLE III. Benchmark points and SUSY spectrum. For each case we have $\chi^2 \ll 1$. The chargino and neutralino masses are tree level and the one loop correction to the mass difference is given by ΔM . All masses are in GeV.

NUHM	Just-so	D term
$\ensuremath{m_{16}}$	5000	5000
\sqrt{D}	1877	1242
m_{10}	6097	5261
A_0	8074	5593
μ	-615	-1294
$M_{1/2}$	-105	-100
α	11.59	12.00
$M_{\rm GUT} \times 10^{-16}$	4.50	2.38
$1/\alpha_{\text{GUT}}$	25.11	25.64
ϵ_3	-0.039	-0.007
λ	0.59	0.56
$tan \beta$	49.43	48.73
M_A	1558	1237
$m_{\tilde{t}_1}$	1975	2921
$m_{\tilde{b}_1}$	2049	2159
$m_{\tilde{\tau}_1}$	2473	3601
$m_{\tilde{u}}$	4905	5081
$m_{\tilde{d}}$	4944	4467
$m_{\tilde{e}}$	4947	4477
$m_{\tilde{\chi}^0_1}$	231.98	219.11
$m_{\tilde{\chi}_1^+}$	232.05	219.11
$\Delta M \equiv M_{\tilde{\chi}^+} - M_{\tilde{\chi}^0}$	0.519	0.438
$M_{\tilde{g}}$	882	874

shows that with such decay branching fractions the bounds coming from published LHC data are at least 20% lower than obtained using any simplified model. The states $\tilde{\chi}_1^{\pm}$ and ${\tilde{\chi}}_1^0$ are approximately degenerate. In Table [III](#page-2-0) we include the running masses for the chargino and neutralino and the dominant one-loop contribution to the mass splitting, ΔM [[35](#page-4-17)]. Thus the chargino signature at the LHC is dominated by the decay $\tilde{\chi}^+ \to \tilde{\chi}^0 + \pi^+$ [\[36\]](#page-4-18). This typically results in a disappearing charged track since the pion cally results in a disappearing charged track since the pion would carry too little energy. The present limits from ATLAS are not very constraining [[37](#page-4-19)]. Our lightest supersymmetric particle (LSP) is a winolike neutralino. As a result, the thermal abundance of the LSP (obtained using micrOMEGA 2.4 [\[38\]](#page-4-20)) is of order 10^{-5} due to the large annihilation cross section to W^+W^- , i.e., too small for dark matter. However, nonthermal production of wino dark matter can give the correct abundance [\[39–](#page-4-21)[42](#page-4-22)]. Finally, for the two benchmark points, the dominant decay modes for the gluino are (calculated using Sdecay [\[43\]](#page-4-23)) as follows: (i) for just-so Higgs splitting—(63% $\rightarrow \tilde{\chi}^0 g$; 28% \rightarrow $\tilde{\chi}^+ b \bar{t} + \tilde{\chi}^- t \bar{b}$ and $8\% \rightarrow \tilde{\chi}^0 t \bar{t}$ and (ii) for D-term split-
ting (76% $\rightarrow \tilde{\chi}^+ b \bar{t} + \tilde{\chi}^- t \bar{b}$; 14% $\rightarrow \tilde{\chi}^0 t \bar{t}$; 3.5% \rightarrow $\lim_{\tilde{\chi}^0 b \bar{b}}$ $-(76\% \rightarrow \tilde{\chi}^+ b \bar{t} + \tilde{\chi}^- t \bar{b};$ $14\% \rightarrow \tilde{\chi}^0 t \bar{t}$
 $\tilde{\chi}^0 b \bar{b}$ and the rest to light quarks or gluons) $3.5\% \rightarrow$ $\tilde{\chi}^0 b \bar{b}$, and the rest to light quarks or gluons).

Note, in the case of just-so Higgs splitting, top squarks are the lightest fermion squarks, while in the case of D-term splitting, bottom squarks are lighter. In addition,

TABLE IV. Generic features of the just-so Higgs splitting with the mirage pattern for gaugino masses and with different values of m_{16} and $M_{1/2}$. For each case we have $\chi^2 \ll 1$. The chargino and neutralino masses are tree level and the one loop correction to the mass difference is given by ΔM . All masses are in GeV.

m_{16}	4000	4000	10000	8000
\sqrt{D}	1725	1511	5516	3207
m_{10}	5144	5079	13036	10168
A_0	7050	7542	15789	14687
μ	-259	-391	-1364	-612
$M_{1/2}$	-100	-240	-120	-260
α	12.00	11.99	10.88	11.58
$M_{\rm GUT} \times 10^{-16}$	2.69	2.27	2.52	2.55
$1/\alpha_{\rm GUT}$	25.29	25.53	25.88	25.76
ϵ_3	-0.019	-0.017	-0.005	-0.018
λ	0.616	0.616	0.560	0.606
$tan \beta$	50.25	49.96	48.68	49.93
M_A	1658	1041	6975	2825
$m_{\tilde{t}_1}$	1308	1679	4028	2751
$m_{\tilde{b}_1}$	1279	1760	3068	2861
$m_{\tilde{\tau}_1}$	1613	1580	5021	3282
$m_{\tilde{u}}$	3929	4144	9659	7910
$m_{\tilde{d}}$	3974	4155	9876	7978
$m_{\tilde{e}}$	3952	3995	9808	7924
$m_{\tilde{\chi}_{1}^{0}}$	187	367	278	525
$m_{\tilde{\chi}_1^+}$	190	371	278	526
ΔM	3.61	4.54	0.452	1.67
$M_{\tilde{g}}$	858	1834	853	1902

TABLE V. Generic features of D-term Higgs splitting with the mirage pattern for gaugino masses and with different values of m_{16} and $M_{1/2}$. For each case we have $\chi^2 \ll 1$. The chargino and neutralino masses are tree level and the one loop correction to the mass difference is given by ΔM . All masses are in GeV.

$\ensuremath{m_{16}}$	4000	4000	8000	8000
\sqrt{D}	1037	1018	2531	1641
m_{10}	4598	4594	8094	7351
A_0	5588	5654	8325	4810
μ	-541	-591	-2945	-2636
$M_{1/2}$	-100	-280	-100	-280
α	12.00	11.91	12.00	10.39
$M_{\rm GUT} \times 10^{-16}$	2.41	1.87	1.93	2.35
$1/\alpha_{\text{GUT}}$	25.46	25.73	26.05	26.00
ϵ_3	-0.011	-0.009	0.007	-0.009
λ	0.582	0.599	0.540	0.569
$tan \beta$	49.20	49.31	48.13	48.70
M_A	969	728	3719	726
$m_{\tilde{t}_1}$	2026	2421	5178	5344
$m_{\tilde{b}_1}$	1255	1825	2634	4702
$m_{\tilde{\tau}_1}$	2622	2644	5266	6218
$m_{\tilde{u}}$	4091	4324	8233	8105
$m_{\tilde{d}}$	3553	3825	6594	7445
$m_{\tilde{\rho}}$	3546	3615	6607	7445
$m_{\tilde{\chi}^0_1}$	215	529	226	529
$m_{\tilde{\chi}_1^+}$	216	531	226	529
ΔM	0.554	2.25	0.436	0.475

at low energies, A_t , A_b are small and thus we have small left-right mixing. These affect the gluino decay branching ratios. Finally, since both μ , M_2 and M_1 are negative we obtain the correct sign for the SUSY correction to $(g-2)_{\mu}$; however, in practice, our sleptons are too heavy to give a good fit, and therefore $(g - 2)_{\mu}$ is not included in the χ^2 function. This does not agree with the results of Badziak *et al.*, Ref. [[13](#page-4-9)] who are able to fit $(g - 2)_{\mu}$, with nonuniversal gaugino masses and Yukawa unification. Unfortunately, the sparticle spectrum obtained in their Letter is now ruled out by LHC Higgs data [\[44](#page-4-24)]. In Tables [IV](#page-2-2) and [V](#page-3-5) we give different benchmark points, all with $\chi^2 \ll 1$, in order to present the variation of sparticle masses with different values of m_{16} and $M_{1/2}$.

With regards to GUT scale parameters, we find $\alpha \approx 12$
vich corresponds to approximately equal dilaton and which corresponds to approximately equal dilaton and anomaly mediated contributions to gaugino masses. We also find $|\epsilon_3| \leq 1\%$ in the case of D-term splitting or precise gauge coupling unification [[45](#page-4-25)].

In conclusion, we have performed a global χ^2 analysis of an $SO(10)$ SUSY GUT with gauge coupling unification and top, bottom, τ , ν_{τ} Yukawa unification at M_{GUT} . We have analyzed the model for the third family alone. We have shown that the SUSY spectrum is predominantly determined by fitting the third family and light Higgs masses and the branching ratio BR $(B_s \to \mu^+ \mu^-)$.

A generic prediction of third family Yukawa unification is that we have $tan \beta \approx 50$. In addition, in order to fit the branching ratio $BR(B_s \to \mu^+ \mu^-)$ we find the CP odd Higgs mass, $m_A \gg M_Z$. Hence, we are in the decoupling limit and the light Higgs boson is predicted to be standard model-like. Our model, makes several additional predictions which are unique to the effective mirage mediation boundary conditions. (i) The first and second family of squarks and sleptons obtain mass of order m_{16} , while the third family scalars are naturally about a factor of 2 lighter. Gluinos and the lightest chargino and neutralino are always lighter than the third family squarks and sleptons. We also find that there is no upper bound on the gluino mass. (ii) Our LSP is predominantly wino and thus assuming a thermal calculation of the relic abundance, we find $\Omega_{\tilde{\chi}_1^0} \sim 10^{-5}$. (iii) $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_1^0$ are approximately degenerate. Thus the chargino signature at the LHC is predominantly due to the decay $\tilde{\chi}^+ \to \tilde{\chi}^0 + \pi^+$. This typically results in a disappearing charged track since the pion would carry too disappearing charged track since the pion would carry too little energy. (iv) For the two benchmark points, Table [III](#page-2-0), the dominant decay modes for the gluino are for just-so Higgs splitting $(63\% \rightarrow \tilde{\chi}^0 g; 28\% \rightarrow \tilde{\chi}^+ b\bar{t}, \rightarrow \tilde{\chi}^- t\bar{b}$ and $8\% \rightarrow \tilde{\chi}^0 t\bar{t}$ and D -term splitting $(76\% \rightarrow \tilde{\chi}^+ b\bar{t} \rightarrow \tilde{\chi}^- t\bar{b})$ $8\% \rightarrow \tilde{\chi}^0 t\bar{t}$ and D-term splitting $(76\% \rightarrow \tilde{\chi}^+ b\bar{t}, \rightarrow \tilde{\chi}^- t\bar{b};$
 $14\% \rightarrow \tilde{\chi}^0 t\bar{t}$ 3.5% $\rightarrow \tilde{\chi}^0 b\bar{b}$ and the rest to light quarks or $14\% \rightarrow \tilde{\chi}^0 t\bar{t}$; 3.5% $\rightarrow \tilde{\chi}^0 b\bar{b}$, and the rest to light quarks or gluons) gluons).

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