Is There a Flavor Hierarchy in the Deconfinement Transition of QCD?

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We present possible indications for flavor separation during the QCD crossover transition based on continuum extrapolated lattice QCD calculations of higher order susceptibilities. We base our findings on flavor-specific quantities in the light and strange quark sector. We propose a possible experimental verification of our prediction, based on the measurement of higher order moments of identified particle multiplicities. Since all our calculations are performed at zero baryochemical potential, these results are of particular relevance for the heavy-ion program at the LHC.

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It is expected that the Universe, only a few microseconds after the big bang, underwent a transition in which matter converted from a state of free quarks and gluons to a state of color-neutral particles of finite mass, namely, the hadrons, which populate the Universe today. This transition can be reproduced in the laboratory, in relativistic heavy-ion collisions currently taking place at the Large Hadron Collider (LHC) at CERN and the Relativistic Heavy Ion Collider (RHIC) facility at Brookhaven National Laboratory.

From the theoretical point of view, this transition can be studied from first principles, by simulating QCD on a discretized lattice. Results of such studies show that the finite-temperature QCD transition is merely an analytic crossover [1]. This means that during the cooling of the Universe, the system transitioned from the phase dominated by colored particles to the hadronic phase over an extended period of time without the emission of latent heat. Since no unambiguous temperature can be assigned to this transition, the question arises whether hadrons of different quark composition freeze out simultaneously or exhibit a flavor hierarchy [2]. This question is relevant since the reported strangeness enhancement at SPS (Super Proton Synchrotron), RHIC, and LHC energies (in particular, in the multistrange particle sector [3–5]) and the discovery of hypernuclei formation at RHIC [6] suggest the possibility of increased strange bound state production at the highest available collision energies. Furthermore, recent measurements in relativistic heavy-ion collisions at the LHC indicate a separation of chemical freeze-out temperatures between light and strange quark hadrons [7,8].

In the present Letter, we show a set of observables, obtained by means of continuum-extrapolated lattice QCD simulations with physical masses, which indicate a flavor separation in the transition region of QCD. Such quantities are based on flavor-specific fluctuations, as well as on correlations between different flavors or conserved charges.

In a series of papers, the Wuppertal-Budapest Collaboration used the tree-level improved Symanzik gauge action and a staggered fermionic action with two-level stout improvement (for a precise definition of the action, see Ref. [9]). One of the advantageous features of stout smearing is the improvement for the pion mass splitting, typical for staggered QCD simulations. In this Letter, we focus on observables that are not sensitive to valence pions and remove the pion mass splitting effect in the sea by making a continuum extrapolation.

To carry out the continuum extrapolation at a given temperature T, we simulate for several N_t values, or lattice spacings, $N_t = 6$, 8, 10, 12. At T_c, these temporal extensions correspond to $a = 0.22$, 0.16, 0.13, and 0.11 fm lattice spacings, respectively. In order to keep the same physical content when we decrease a, the kaon decay constant and the kaon mass were tuned to their physical values, and we also used for the strange to light quark mass ratio its physical value ($m_s/m_{ud} \approx 28$) determined in Ref. [10]. Lattice simulations are always carried out in a finite volume. In our case, the relevant observables (baryon, strange, and light quark number fluctuations) are mostly carried by the kaons and protons in the hadronic phase. Pions which are mostly sensitive to finite volume effects do not carry a net light quark number. Therefore, we expect that finite volume effects of the present analysis are small and are hidden by the statistical errors.

Fluctuations of conserved charges can be expressed in a grand canonical ensemble as the derivatives of the partition function with respect to the conserved charge chemical potential. In QCD, the net u, d, and s quark numbers are conserved: we introduce μ_u, μ_d, and μ_s as the corresponding chemical potentials. Fluctuations are then expressed in terms of derivatives of the pressure p of the equilibrated system:

$$\chi_{lmn}^{uds} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_u/T)^l \partial(\mu_d/T)^m \partial(\mu_s/T)^n}. \quad (1)$$

Odd $l+m+n$ combinations are sensitive to nonvanishing chemical potentials, but since our main interest in this Letter is the physics at LHC, we work with vanishing chemical potentials. More precisely, we concentrate on some quadratic and quartic fluctuations (thus, $l+m+n=2$ or 4) and their ratios. Since these fluctuations are directly related to conserved currents, no renormalization ambiguity should appear.

Our focus is on μ_L and μ_S, which couple to the net light flavor density and the strangeness density, respectively. To study correlations with the flavor-mixed baryon number, we also introduce the respective chemical potential μ_B:

$$\frac{\partial}{\partial\mu_L} = \frac{1}{2}\frac{\partial}{\partial\mu_u} + \frac{1}{2}\frac{\partial}{\partial\mu_d}; \qquad \frac{\partial}{\partial\mu_S} = -\frac{\partial}{\partial\mu_s},$$
$$\frac{\partial}{\partial\mu_B} = \frac{1}{3}\frac{\partial}{\partial\mu_u} + \frac{1}{3}\frac{\partial}{\partial\mu_d} + \frac{1}{3}\frac{\partial}{\partial\mu_s}. \quad (2)$$

The method for extracting the diagonal and off-diagonal quark number susceptibilities as well as the second and fourth order derivatives has been worked out in detail in Refs. [11–14]. Besides the exact numerical approach used in this Letter, other analytical approximations are emerging such as improvement perturbation theory [15,16] or holographic methods [17].

Already, in 2006, the Wuppertal-Budapest Collaboration showed that the characteristic temperature of the transition in the strange sector (based on χ_2^s) is about 20 MeV higher than that for the light quark sector (based on the chiral susceptibility [18] or χ_2^u in Ref. [19]). In order to understand these differences, we determined χ_2^u and χ_2^s with unprecedented accuracy [11]. Figure 1 shows the continuum extrapolated T dependence of the light quark $\chi_2^L(T)$ and strange quark $\chi_2^s(T)$ susceptibilities normalized to the Stefan Boltzmann limit (SB). One striking observation is an approximate scaling relation between the T dependencies of the light and strange quark susceptibilities, respectively. A rescaling in T for one of these observables closely reproduces the other one: $\chi_2^L(Tx) = \chi_2^s(T)$. The figure also shows this rescaling relation, for which the rescaling factor $x = 1.11$ is preferred. The most important message of this relationship can be summarized as follows: independently of a chosen characteristic point in the crossover region (e.g., inflection point, halving point, or any other) and its relation to some physically motivated definition of the transition temperature, the similarity transformation between the two curves leads to a quite precise prediction for the difference between these transition temperatures of light and strange quarks. We find that the characteristic temperature defined, e.g., by the inflection point is $(x-1)T_c \simeq 15$ MeV higher for the strange quark than for the light quarks. (Here, T_c is the characteristic temperature for the transition in the light quark sector.)

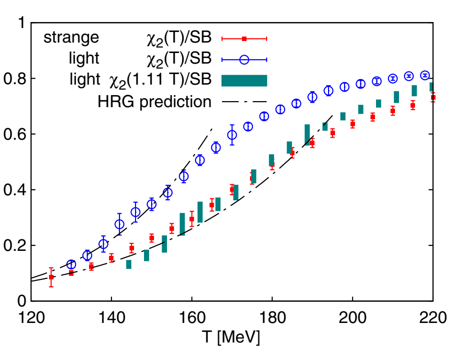

FIG. 1 (color online). Light and strange quark susceptibilities in the continuum limit (plotted as blue circles and red squares, respectively). The transition temperatures defined by the inflection points for χ_2^L (150 MeV) and for χ_2^s (165 MeV) differ by ≈ 15 MeV. A rescaling transformation is shown with bars.

A model-dependent but enlightening approach to locate T_c is to compare lattice data to the hadron resonance gas (HRG) model prediction. For the data in Fig. 1, the highest temperature of agreement with the HRG result is ambiguous and may depend on the number of resonances included in the HRG partition sum. Therefore, we continue our discussion with higher order fluctuations where the HRG prediction is more robust, and the point where lattice and HRG results start to deviate is also a characteristic point of the data set.

A recent work [20] suggested two interesting susceptibility combinations v_1 and v_2, which vanish in the hadronic phase and become nonzero as soon as s quark degrees of freedom start to be liberated in the system. We generalize these expressions to any flavor f:

$$v_1^f = \chi_{11}^{Bf} - \chi_{31}^{Bf};$$
$$v_2^f = \frac{1}{3}(\chi_2^f - \chi_4^f) + 2\chi_{13}^{Bf} - 4\chi_{22}^{Bf} + 2\chi_{31}^{Bf}.$$

Based on these combinations, it was shown that strange quarks start to be deconfined at $T \gtrsim 157$ MeV. Here, we compare, for the first time, such parameters for strange and light quarks. We supplement this information with a third combination:

$$w^f = \chi_{13}^{Bf} - \chi_{11}^{Bf}. \quad (3)$$

This is more sensitive to the flavor content based on the higher order of quark derivatives with respect to the baryon derivatives: in particular, in the hadronic phase, it only receives contributions from hadrons containing more than one quark of flavor f. In the following, we consider $f = u$, s, or L.

Figure 2 shows our continuum extrapolated results in comparison to HRG calculations. Out of the three shown observables, w shows the strongest flavor separation. In all

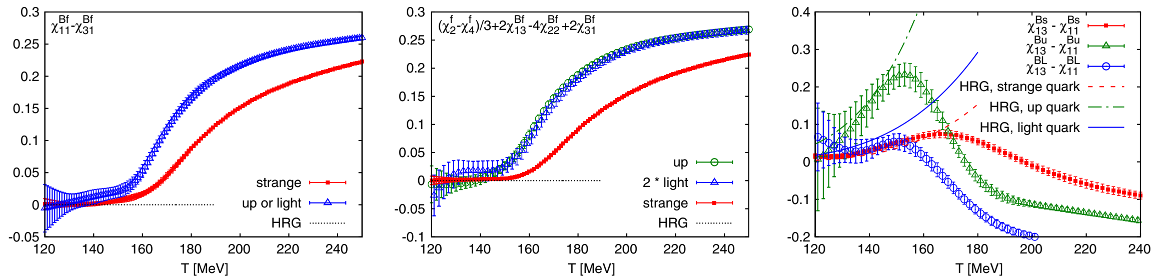

FIG. 2 (color online). The continuum extrapolated temperature dependence of the v_1, v_2, and w susceptibility combinations for light and strange quarks in comparison to HRG calculations.

cases, a deviation from the HRG at a certain minimum temperature can be considered as the onset of the liberation of quarks of a given flavor. The advantage of the HRG being strictly zero for the first two derivatives is balanced by rather large error bars in the lattice results, whereas in the case of w, the lattice calculation shows a clearly identifiable characteristic peak at the temperature where it starts to deviate from the HRG result. Notice that the temperatures at which v_1^f and v_2^f deviate from the HRG model are lower than the inflection points extracted from χ_2^f: this reflects the fact that the inflection point of χ_2^f defines the "steepest" point of the transition, while v_1^f and v_2^f deviate from the HRG model as soon as the first deconfined quark of flavor f appears. For observables that are most sensitive to multistrange content, we see a pronouncedly higher temperature of deviation from HRG than for the analogous quantity in the light sector. The contribution of multistrange hadrons is enhanced in a combination with higher $(\partial/\partial\mu_s)$ derivatives, like χ_2^s or w, whereas v_1 and v_2 signal the liberated strangeness from all strange hadrons equally.

We move now to more basic susceptibility combinations which can, in principle, be measured in experiments. The most attractive quantity to the purpose is χ_4^f/χ_2^f, since the ratio does not depend on the volume. Similar ratios have been proposed to determine the chemical freeze-out temperature independent of any statistical model assumptions [21–23]. Its nonmonotonic behavior as a function of the temperature has also been suggested as an indicator for the deconfinement transition [24]. Figure 3 shows the T dependence of χ_4/χ_2 for light and strange quarks. By choosing a susceptibility ratio measurement, the leading finite volume effects will cancel out. Furthermore, the crossover region requires an extended time for the system to reach the purely hadronic phase, which is larger than the typical characteristic time scale for strong interactions. Therefore, we expect finite time effects to have little influence on the proposed fluctuation measures. For the light quark susceptibilities (and thus for observables related to net up + down quark numbers), the pion contribution, which is notoriously difficult to calculate on the lattice, is absent by definition. The figure shows two characteristic features: (a) each lattice calculation exhibits a kink (or peak) at a particular temperature, and (b) this kink coincides with the temperature at which the lattice curve starts to deviate from the HRG result. Interestingly, the separation between the kinks of the two flavors corresponds to the previously mentioned ≈ 15 MeV. In a scenario in which the highest temperature where the HRG and lattice QCD agree is indicating a "deconfinement" or "liberation" temperature for a particular flavor, Fig. 3 further supports the flavor separation of the characteristic temperatures.

The presented lattice results show that these quantities, if measured with high accuracy, are good thermometers. The challenge is to unambiguously demonstrate such a flavor hierarchy experimentally. The effect could manifest itself to first order in the multiplicity distribution of identified particles. In case strange hadrons form at a higher temperature than their light quark counterparts, their abundance will be enhanced relative to a common low temperature freeze-out scenario. Therefore, the comparison of measured yields to a statistical hadronization model enables one to determine chemical freeze-out temperatures (T_{ch}) in a flavor-separated way. The first results from ALICE indicate that the T_{ch} of strange hadrons is about 16 MeV higher than that of light hadrons (164 vs 148 MeV) [7,8]. As in the case of the lattice parameters, this sensitivity to the freeze-out temperature, extracted from a statistical hadronization fit, is most pronounced for the

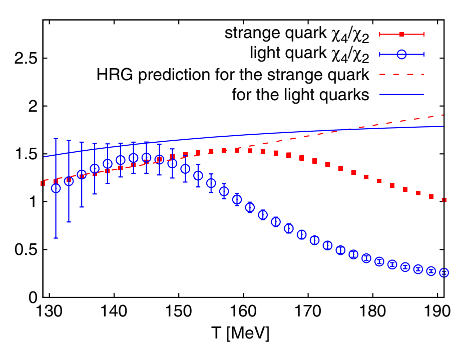

FIG. 3 (color online). The T dependence of the χ_4/χ_2 ratio for light and strange quarks in the continuum limit. The lattice data are compared to HRG calculations.

multistrange baryons. These temperature fits are model dependent, though, and a direct comparison to temperatures extracted from susceptibilities in lattice QCD requires correction; e.g., it was suggested that final state interactions between hadrons might modify the baryon yields [25,26].

A more precise verification, less prone to alternate explanations, can be obtained by using a higher order moment analysis of particle identified yields, since those moments can be directly related to the higher order susceptibilities on the lattice [21]. In particular, the product of kurtosis and variance of the net multiplicity distributions corresponds to the susceptibility ratio in Fig. 3. Even for this analysis, though, the caveat is the exact relation between the quark-based observable and the hadron-based measurement. Specifically, one needs to determine how many hadron species have to be measured in order to fully capture the transition behavior of the respective quark flavor. Preliminary studies show that taking into account only the dominant mesonic states is not sufficient [27]. Baryonic states require significant acceptance and efficiency corrections, though, even under the assumption of statistical behavior of the higher moments of reconstructed particles in the detector acceptance [28], but recent RHIC results have shown that these corrections do not have a large impact on the final results [29]. Therefore, we expect that the suggested flavor separation on the lattice can be experimentally verified for the freeze-out temperatures, too. Once the experimental results are approaching a certain value with small errors, one can extend the studies in Fig. 3 in this particular temperature region with more high precision lattice data.

In conclusion, we have presented, for the first time, high precision continuum extrapolated lattice calculations of flavor-specific higher order susceptibility combinations at zero baryochemical potential and high temperatures. We have shown that flavor-dependent patterns emerge in the crossover region of the QCD transition. The T dependence of the examined observables hints at a flavor separation, especially for quantities which are most sensitive to multistrange states. This flavor separation is an obvious consequence of the mass difference between the light and strange quarks. These subtle differences of about 15 MeV in the characteristic temperatures are only visible thanks to the large accuracy of the latest lattice calculations. We have proposed an experimental program that might allow an observation of a possible similar separation in the freeze-out temperatures. One can look for these effects at LHC, and potentially RHIC energies if the net multiplicity distributions of identified particles in the relevant quark sectors can be measured and efficiency corrected to the same high accuracy as the lattice QCD calculations.

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