Universality of Abrupt Holographic Quenches

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We make an analytic investigation of rapid quenches of relevant operators in d-dimensional holographic conformal field theories, which admit a dual gravity description. We uncover a universal scaling behavior in the response of the system, which depends only on the conformal dimension of the quenched operator in the vicinity of the ultraviolet fixed point of the theory. Unless the amplitude of the quench is scaled appropriately, the work done on a system during the quench diverges in the limit of abrupt quenches for operators with dimension $(d/2) \le \Delta < d$.

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Quantum quenches have recently become accessible in laboratory experiments [1], which has initiated much activity by theoretical physicists to understand such systems. Up until now, most analytic work on the topic of relativistic quantum quenches has assumed that the field theory is at weak coupling [2–6].

The study of quantum quenches at strong coupling is accessible through the gauge-gravity duality [7]. Much related work studying thermalization in the boundary theory was done by studying the gravity dual under the assumption that the nonequilibrium evolution can be approximated by a uniformly evolving spacetime, e.g., Refs. [8–15]. Other approaches study the evolution of a probe on the static spacetime [16]. The approach of numerically evolving the dual gravity theory was initiated in Ref. [17]. Further numerical studies of quenches in a variety of holographic systems were presented in Refs. [18–21].

In Refs. [19,21], holography was applied to study quenches of the coupling to a relevant scalar operator in the boundary theory. A numerical approach was taken to study the evolution of the dual scalar field in the bulk spacetime. For fast quenches, evidence was found for a universal scaling of the expectation value of the boundary operator. Similar scaling was observed for the change in energy density, pressure, and entropy density. However, no analytic understanding of this behavior was available.

In this Letter, we investigate these holographic quenches analytically, focusing on the work done by the quench. Unlike Refs. [19,21], in which the coupling was an analytic function of time, we abruptly (but with some degree of smoothness) switch on this source at $t = 0$. The coupling is then varied over a finite interval δt and is held constant afterwards. We find that for fast quenches, the essential physics can be extracted by solving the linearized scalar field equation in the asymptotic anti–de Sitter (AdS) geometry. Note that our analysis is naturally driven to this regime by the limit $\delta t \to 0$. In contrast to Refs. [19,21], we are not *a priori* limiting our study to a perturbative expansion in the amplitude of the bulk scalar. Our analytic results also cover any spacetime dimension d for the boundary theory, whereas Refs. [19,21] were limited to $d = 4$.

Let us describe the quenches in more detail: The coupling in the boundary theory is determined by the leading non-normalizable mode of the bulk scalar [7]. We set this mode to zero before $t = 0$, vary it in the interval $0 < t < \delta t$, and hold it fixed afterwards. Because the energy density can only change while the coupling is changing, we are only interested in the response of the scalar field during the time span $0 < t < \delta t$. Further, since the response propagates in from the boundary of the spacetime, the field will only be nonzero within the light cone $t = \rho$. Hence, to determine the work done, we need only solve for the bulk evolution in the triangular region bounded by this light cone, the surface $t = \delta t$, and the AdS boundary, as shown in Fig. 1. As is also illustrated, as $\delta t \to 0$, this triangle shrinks to a small region in the asymptotic spacetime. The normalizable component of the scalar field, which determines the expectation value of the boundary operator, can be solved analytically in this situation, and its scaling with δt can readily be seen from this solution. From this, we also obtain the scaling of the energy density in the boundary.

Consider a generic deformation of a conformal field theory in d spacetime dimensions by the time-dependent coupling $\lambda = \lambda(t)$ of a relevant operator $\mathcal{O}_\Delta$ of dimension Δ: $\mathcal{L}_0 \to \mathcal{L} = \mathcal{L}_0 + \lambda \mathcal{O}_\Delta$. The quenches are then characterized by two distinct scales: the mass scale set by the change $\delta\lambda$ and that associated with the rate of change, i.e., $1/\delta t$. As described above, we will be particularly interested in rapid quenches where the second scale is much larger than the first, that is, quenches where $\delta\lambda(\delta t)^{d-\Delta} \ll 1$. The gravity dual describing this system is given by

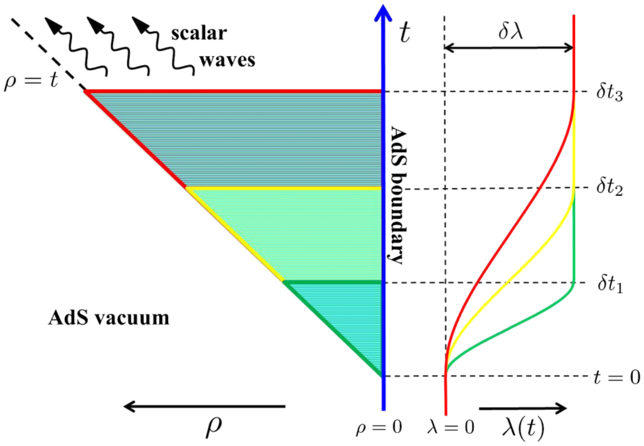

FIG. 1 (color online). The shaded triangle is the region close to the boundary of the AdS spacetime where we must solve for the scalar field. We show several cases with $\delta t_1 < \delta t_2 < \delta t_3$. The profile $\lambda(t/\delta t)$ is held fixed in each case. In particular, the amplitude $\delta\lambda$ of the quench remains constant as δt becomes smaller. As the quench becomes more rapid, the bulk region shrinks closer to the asymptotic boundary.

$$I_{d+1} = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-g}\bigg(R + d(d-1) - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - u(\phi)\bigg), \quad (1)$$

where we have chosen an AdS radius of 1. The bulk scalar ϕ is dual to $\mathcal{O}_\Delta$ with $m^2 = \Delta(\Delta - d)$. The potential $u(\phi)$ contains terms of order ϕ^3 or higher. To simplify our discussion, we will consider quenches where the conformal dimension of the operator is a noninteger (for even d and not a half-integer for odd d—see comments below). Further, we initially consider dimensions in the range $(d/2) \le \Delta < d$.

Since we are interested in quenches that are homogeneous and isotropic in the spatial boundary directions, we assume that both the background metric and the scalar field depend only on a radial coordinate ρ and a time t. We will work in a spacetime asymptotic to the AdS Poincaré patch as $\rho \to 0$. Hence, the bulk metric is

$$ds^2 = -A(t,\rho)dt^2 + \Sigma(t,\rho)^2 d\vec{x}^2 + \rho^{-4}A(t,\rho)^{-1}d\rho^2. \quad (2)$$

The (nonlinear) Einstein equations and the scalar field equation then take the form

$$0 = -\frac{2(d-3)}{(d-1)A}u(\phi) + \frac{2d(d-3)}{A} + \rho^4(\phi')^2 - \frac{d-3}{(d-1)A}m^2\phi^2 - \left(\frac{\dot\phi}{A}\right)^2 + 2(d-2)(d-1) \times\left[\left(\frac{\dot\Sigma}{A\Sigma}\right)^2 - \left(\frac{\rho^2\Sigma'}{\Sigma}\right)^2\right] + \frac{2\rho^2(\rho^2A')'}{A} - 4\left(\frac{\dot A}{A^2}\right)^2 + 2\frac{\ddot A}{A^3}, \quad (3)$$

$$0 = d - \frac{u(\phi)}{(d-1)} - \frac{m^2\phi^2}{2(d-1)} + \frac{\rho^4 A}{2(d-1)}(\phi')^2 + \frac{\dot\phi^2}{A} - \rho^4\frac{A'\Sigma'}{\Sigma} - (d-2)\rho^4 A\frac{(\Sigma')^2}{\Sigma^2} + \frac{2\ddot\Sigma}{A\Sigma} - \frac{\dot A\,\dot\Sigma}{A^2\Sigma} + (d-2)\frac{\dot\Sigma^2}{A\Sigma^2}, \quad (4)$$

$$0 = \frac{(\phi')^2}{2(d-1)} + \frac{1}{2(d-1)}\left(\frac{\dot\phi}{\rho^2 A}\right)^2 + \frac{\Sigma''}{\Sigma} + \frac{2\Sigma'}{\rho\Sigma} + \frac{\ddot\Sigma}{\rho^4A^2\Sigma}, \quad (5)$$

$$0 = \frac{\phi'\dot\phi}{d-1} + \frac{\dot A\Sigma'}{A\Sigma} - \frac{A'\dot\Sigma}{A\Sigma} + 2\frac{\dot\Sigma'}{\Sigma}, \quad (6)$$

$$0 = -\frac{\delta u(\phi)}{\delta\phi} - m^2\phi + \rho^4A\phi'' + 2\rho^3A\phi' + \rho^4A'\phi' + \frac{(d-1)\rho^4A\Sigma'\phi'}{\Sigma} + \frac{\dot A\,\dot\phi}{A^2} - \frac{(d-1)\dot\Sigma\,\dot\phi}{A\Sigma} - \frac{\ddot\phi}{A}, \quad (7)$$

where dots and primes denote derivatives with respect to t and ρ, respectively. The scalar field will have an asymptotic expansion of the form

$$\phi(t,\rho) \sim \rho^{d-\Delta}[p_0(t) + o(\rho^2)] + \rho^\Delta[p_{2\Delta-d}(t) + o(\rho^2)], \quad (8)$$

where the non-normalizable coefficient p_0 is proportional to λ, while the normalizable coefficient $p_{2\Delta-d}$ is proportional to $\langle\mathcal{O}_\Delta\rangle$. Similarly,

$$A \sim \rho^{-2}[1 + a_{d-2}(t)\rho^d + o(\rho^{d+4-2\Delta})]. \quad (9)$$

Here, the coefficient a_{d-2} controls the energy density (and pressure) of the dual field theory, as shown in Ref. [19]. Equation (6) is a constraint, which in the limit $\rho \to 0$, determines $\partial_t a_{d-2}$. Integrating over t, we then find

$$a_{d-2}(t) = \mathcal{C} - \frac{(2\Delta - d + 1)(d-\Delta)}{(d-1)^2}p_0(t)p_{2\Delta-d}(t) + \frac{2\Delta - d}{d-1}\int_0^t d\tilde t\, p_{2\Delta-d}(\tilde t)\frac{d}{d\tilde t}p_0(\tilde t). \quad (10)$$

Here, $\mathcal{C} = a_{d-2}(-\infty)$ is an integration constant. With $d = 4$, this expression matches that found in Ref. [21], using Eddington-Finkelstein coordinates.

In our quenches, the coupling to $\mathcal{O}_\Delta$ is made time dependent with a characteristic time δt as

$$\lambda = \lambda(t/\delta t). \quad (11)$$

For general δt, the response $p_{2\Delta-d}$ in Eq. (8) cannot be solved analytically. However, as described in Refs. [19,21], for large δt (adiabatic quenches), we can find a series solution for ϕ in inverse powers of δt and, in principle, we can solve for $p_{2\Delta-d}$ analytically.

We now present a new analytic approach for the opposite limit of fast quenches, that is, for quenches where δt is much smaller than any other scale. As described above, to answer the question of how much work is done by the quench, we need only consider the interval $0 \leq t \leq \delta t$. Intuitively, we may expect that when δt is very short, there is no time for nonlinearities in the bulk equations to become important, i.e., for the metric to backreact on the scalar.

To make this intuition manifest, we rescale the coordinates and fields by the parameter δt considering their (leading) dimension in units of the AdS radius: $\rho = \delta t \hat{\rho}$, $t = \delta t \hat{t}$, $A = \hat{A}/\delta t^2$, $\Sigma = \hat{\Sigma}/\delta t$, and $\phi = \delta t^{d-\Delta}\hat{\phi}$. With this rescaling, the limit $\delta t \to 0$ then removes the scalar from the Einstein equations (3)–(6), while leaving the form of the Klein-Gordon equation (7) unchanged.

The coefficient a_{d-2} controls the next-to-leading-order term in A at small ρ. As we will show, this coefficient scales as $\delta t^{d-2\Delta}$. Further, in Eq. (9), this coefficient is accompanied by a factor of ρ^d and hence this term has an overall scaling of $\delta t^{2(d-\Delta)}$. Hence, as long as we are considering a relevant operator, this term vanishes in the limit $\delta t \to 0$. The same is true of the subleading contributions in the expression of Σ. Hence, for fast quenches with small δt, we can approximate the metric coefficients as simply

$$\hat{\Sigma} = \hat{\rho}^{-1}, \qquad \hat{A} = \hat{\rho}^{-2}. \tag{12}$$

The equation for $\hat{\phi}$ becomes the Klein-Gordon equation in the AdS vacuum spacetime, i.e.,

$$\hat{\rho}^2 \partial_{\hat{\rho}}^2 \hat{\phi} - (d-1)\hat{\rho}\partial_{\hat{\rho}}\hat{\phi} - \hat{\rho}^2\partial_{\hat{t}}^2\hat{\phi} + \Delta(d-\Delta)\hat{\phi} = 0. \tag{13}$$

That is, in the limit of small δt, the work done in the full nonlinear quench can be determined by simply solving the linear scalar field equation (13) in empty AdS space.

Now, we consider sources that vanish for $t \leq 0$ and are constant for $t \geq \delta t$. In $0 < t < \delta t$, we vary the source as

$$p_0(t) = \delta p (t/\delta t)^{\kappa}, \tag{14}$$

where κ is a positive exponent. Note that here $p_0(t \geq \delta t) = \delta p$. Since $\phi = 0$ before we switch on the source at $t = 0$, it remains zero throughout the bulk up to the null ray $t = \rho$. Therefore, we impose

$$\phi(t = \rho, \rho) = 0. \tag{15}$$

Evaluating the scalar field equation (13) subject to the boundary conditions (14) and (15), we find [22]

$$p_{2\Delta-d}(t) = b_{\kappa}\delta t^{d-2\Delta}\delta p (t/\delta t)^{d-2\Delta+\kappa}, \tag{16}$$

with

$$b_{\kappa} = -\frac{2^{d-2\Delta}\Gamma(\kappa+1)\Gamma(\frac{d+2}{2}-\Delta)}{\Gamma(d+1+\kappa-2\Delta)\Gamma(\Delta-\frac{d-2}{2})}. \tag{17}$$

Of course, if we construct more complicated sources with a series expansion of monomials as in Eq. (14), then since Eq. (13) is linear, the response is simply given by the sum of corresponding terms as in Eq. (16).

The response coefficient (16) exhibits two noteworthy features: First, we see that the overall scaling of the response is $\delta t^{d-2\Delta}$. This is precisely the behavior found in the numerical studies of Ref. [21] in the case $d = 4$. Second of all, $p_{2\Delta-d}$ varies in time as $t^{d+\kappa-2\Delta}$. Therefore, if $\kappa < 2\Delta - d$, the response (i.e., the operator expectation value $\langle \mathcal{O}_\Delta \rangle$ in the boundary theory) diverges at $t = 0$. For a source constructed as a series, both of these features in the response are controlled by the smallest exponent.

For homogeneous quenches, the diffeomorphism Ward identity reduces to $\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \partial_t \lambda$ [19,21]. Hence, we can evaluate the change in the energy density as

$$\Delta\mathcal{E} = -\mathcal{A}_{\mathcal{E}} \int_{-\infty}^{+\infty} p_{2\Delta-d}\partial_t p_0 dt, \tag{18}$$

with [23]

$$\mathcal{A}_{\mathcal{E}} = \frac{2\Delta - d}{16\pi G_{d+1}} = \frac{(2\Delta-d)\pi^{d/2}\Gamma(\frac{d}{2})}{2d(d+1)\Gamma(d-1)} C_T. \tag{19}$$

Since $\partial_t p_0$ vanishes for $t < 0$ and $t > \delta t$, the above integral reduces to an integral from 0 to δt. It is for this reason that we do not need to determine the response $p_{2\Delta-d}$ after $t = \delta t$ [25]. Further, for fast quenches, the change in energy density will scale as $\delta t^{d-2\Delta}$. Note that $\partial_t p_0$ scales as δt^{-1}, but the range of the integral $0 < t < \delta t$ adds an additional scaling of δt^{+1}. Hence, the net scaling of $\Delta\mathcal{E}$ is precisely the scaling of $p_{2\Delta-d}$. Again, this precisely matches the scaling found numerically in Ref. [21] for $d = 4$. In fact, this behavior can be fixed as follows: Since Eq. (13) is linear, we must have $p_{2\Delta-d} \propto \delta p$ and hence $\Delta\mathcal{E} \propto \delta p^2$ from Eq. (18). Finally, dimensional analysis demands $\Delta\mathcal{E} \simeq \delta p^2/\delta t^{2\Delta-d}$, up to numerical factors.

However, recall the singular behavior in the response at $t = 0$ for $\kappa < 2\Delta - d$. Despite this divergence, one can easily see that in fact, the corresponding integral (18) remains finite as long as $\kappa > \Delta - (d/2)$. That is, for fixed Δ and d, we are constrained as to how quickly the source may be turned on. In fact, a more careful examination [22] of the bulk solutions indicates that our analysis is valid for $\kappa > \Delta - (d/2) + (1/2)$. For quenches not satisfying this inequality, we can no longer ignore the backreaction of the scalar on the spacetime geometry.

To summarize, we have shown that in the limit of fast quenches, the response and the energy density of a strongly coupled system which admits a dual gravitational description scales as $\delta t^{d-2\Delta}$. Here, $(d/2) \leq \Delta < d$ is the conformal dimension of the quenched operator in the vicinity of the ultraviolet fixed point. Although we considered a quench from a vacuum state at $t = 0$, our results are universal. That is, they are independent of the initial state of the system; e.g., we may start with a thermal state, as in

Refs. [19,21]. This is again a reflection of the fact that abrupt holographic quenches are completely determined by the UV dynamics of the theory—see Fig. 1. Also, if different operators are quenched simultaneously, the response is dominated by the one with the largest conformal dimension.

We emphasize that while our calculations only considered the linearized scalar equation (13), our results apply for the full nonlinear quench. In the limit $\delta t \to 0$, the relevant physics occurs in the far asymptotic geometry (see Fig. 1) where the bulk scalar and perturbations of the AdS metric are all small. This contrasts with Refs. [19,21], which only worked within a perturbative expansion in the amplitude of the scalar. Of course, the scalings determined there match those found here, but it was uncertain there if they would persist in a full nonlinear analysis.

Of course, the present analysis does not predict the dynamical evolution of the system for $t > \delta t$. However, for the fast quenches above, an arbitrarily large energy density is injected into the bulk in the limit $\delta t \to 0$, and hence we can expect quite generally that a black hole forms. Further, we can deduce the properties of the final state black hole or, alternatively, of the equilibrium thermal state on the boundary as $t \to \infty$. Indeed, since the coupling and energy density are constant for $t > \delta t$, $\lambda(+\infty) = \lambda(\delta t)$ while Eq. (18) determines the final energy density of the system, to leading order in δt. Together, these parameters completely specify the final thermal equilibrium state.

Note that our analysis strictly applies to relevant operators, for which $d - \Delta > 0$. With a marginal operator (i.e., $\Delta = d$), we can expect $\Delta\mathcal{E} \propto \delta t^{-d}$ on purely dimensional grounds [17]. While this matches the scaling found above, our numerical coefficients would no longer be valid. Marginal operators were also considered in Refs. [9,15] with a four-dimensional bulk. This case is analytically accessible because the scalar propagates on the light cone. Extending this analysis to an odd-dimensional bulk is more challenging [9] because the scalar propagator is nonvanishing throughout the interior of the light cone, similar to that for the relevant operators studied here.

Our discussion was also limited to $(d/2) \leq \Delta < d$, while unitarity bounds also allow for $(d/2) - 1 \leq \Delta < (d/2)$. In the latter range, we must consider the so-called "alternate quantization" of the bulk scalar [26]. In fact, the asymptotic expansion of the scalar takes precisely the same form as in Eq. (8). However, in this regime, p_0 ($p_{2\Delta-d}$) is the coefficient of the (non-)normalizable mode. Our analysis applies equally well for this range of Δ, and so one still finds $p_{2\Delta-d} \simeq \delta p \delta t^{d-2\Delta}$. That is, the response becomes vanishingly small as $\delta t \to 0$ with δp kept fixed. Hence, to produce a finite $\langle \mathcal{O}_\Delta \rangle$ or finite $\Delta\mathcal{E}$, we would need to scale δp with an inverse power of δt.

When Δ is an integer for even d or a half-integer for odd d, the scaling of the response $\langle \mathcal{O}_\Delta \rangle$ receives additional $\log(\delta t)$ corrections [19]. These logarithmic corrections arise from $\log\rho$ modifications in the asymptotic expansion (8) of the bulk scalar and are easily computed analytically following the present approach [22].

Another exceptional case arises with $\kappa = 2\Delta - d - n$, where n is a positive integer. In this case, Eq. (17) indicates $b_\kappa = 0$. Hence, if the source is given by a series of monomials (14), the scaling of the response will be controlled by the first subleading contribution. With a single monomial, the (subleading) scaling of the response is controlled by nonlinearities in the bulk equations [22], i.e., $p_{2\Delta-d} \simeq \delta t^{-\Delta}(\delta p \delta t^{d-\Delta})^n$, where $n = 2$ if the potential contains a ϕ^3 term and $n = 3$ otherwise.

It is interesting to consider the limit of abrupt quenches with $\delta t = 0$, as this usually sets the starting point in analyses at weak coupling, e.g., Refs. [2,6]. Our holographic result $\Delta\mathcal{E} \simeq \delta p^2/\delta t^{2\Delta-d}$ indicates that the energy density diverges for an abrupt quench with $\Delta > (d/2)$ (a logarithmic divergence appears for $\Delta = (d/2)$ [19,22]). Hence, it would be interesting to carefully compare these holographic results with those for previous weak coupling calculations [27]. Let us note here that many of these studies, e.g., Refs. [2,6], consider the regime $\Delta < d/2$ where $\Delta\mathcal{E}$ does not diverge in our holographic framework. However, singular behavior was observed for abrupt quenches where our holographic model also produces divergences [28]. Of course, the preceding considerations assume a standard protocol where δp is held fixed in the limit $\delta t \to 0$. Instead, if we scale the source to zero as $\delta p \propto \delta t^{\Delta-d/2}$, $\Delta\mathcal{E}$ will remain finite. However, such a limit still produces a divergent response since $p_{2\Delta-d} \sim \delta t^{d-2\Delta}\delta p \propto \delta t^{d/2-\Delta}$. An alternate choice would be to scale $\delta p \propto \delta t^{2\Delta-d}$, which would leave $\langle \mathcal{O}_\Delta \rangle$ finite while $\Delta\mathcal{E} \to 0$.

An important question to ask is to what extent our results are relevant for everyday physical systems. Gauge theories with a dual gravitation description are necessarily strongly coupled and have an ultraviolet fixed point with large central charge. The framework of the gauge-string duality allows for the study of both the finite 't Hooft coupling corrections (the higher-derivative corrections in the gravitational dual) and nonplanar (quantum string-loop) corrections. We expect that our gravitational analyses are robust with respect to the former, as the relevant near-boundary spacetime region is weakly curved. Whether finite central charge corrections are important or not is an open question.

The universal behavior uncovered is of relevance to fast quenches of relevant couplings in the vicinity of an ultraviolet fixed point—as such, it is challenging to observe it in experimental settings. We expect that fixed points in condensed matter systems described by relativistic conformal field theories may exhibit the same universal behavior.

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