Magnetic-Field-Tuned Aharonov-Bohm Oscillations and Evidence for Non-Abelian Anyons at $\nu = 5/2$

R. L. Willett,[1] C. Nayak,[2,3] K. Shtengel,[4,2] L. N. Pfeiffer,[5] and K. W. West[5]

[1]*Bell Laboratories, Alcatel-Lucent, Murray Hill, New Jersey 07974, USA*
[2]*Microsoft Research, Station Q, Elings Hall, University of California, Santa Barbara, California 93106, USA*
[3]*Department of Physics, University of California, Santa Barbara, California 93106, USA*
[4]*Department of Physics and Astronomy, University of California, Riverside, California 92521, USA*
[5]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

(Received 25 January 2013; published 28 October 2013)

We show that the resistance of the $\nu = 5/2$ quantum Hall state, confined to an interferometer, oscillates with the magnetic field consistent with an Ising-type non-Abelian state. In three quantum Hall interferometers of different sizes, resistance oscillations at $\nu = 7/3$ and integer filling factors have the magnetic field period expected if the number of quasiparticles contained within the interferometer changes so as to keep the area and the total charge within the interferometer constant. Under these conditions, an Abelian state such as the (3, 3, 1) state would show oscillations with the same period as at an integer quantum Hall state. However, in an Ising-type non-Abelian state there would be a rapid oscillation associated with the "even-odd effect" and a slower one associated with the accumulated Abelian phase due to both the Aharonov-Bohm effect and the Abelian part of the quasiparticle braiding statistics. Our measurements at $\nu = 5/2$ are consistent with the latter.

DOI: 10.1103/PhysRevLett.111.186401 PACS numbers: 71.10.Pm, 73.43.Fj, 05.30.Pr

Introduction.—The origin of the fractional quantum Hall effect [1] at filling factor $\nu = 5/2$ [2–4] has been a long-standing open issue, which is important because it has been conjectured that this state of matter supports non-Abelian anyons [5–9]. Two point-contact Fabry-Pérot interferometers have been proposed to observe the Aharonov-Bohm (AB) effect and the anyonic braiding statistics of quasiparticles [10]. In a non-Abelian state, not only the phase but the also the amplitude of the observed oscillations is indicative of the braiding statistics [11–14]. Specifically, if the $\nu = 5/2$ state is indeed non-Abelian, the quasiparticle parity within the interferometer dictates whether the resistance of an interferometer oscillates with enclosed area (controlled by a side gate) with a period associated with charge $e/4$ quasiparticles. Such oscillations should only be seen when the parity is even—the "even-odd effect" [13,14]. Previous experiments [15,16] are broadly consistent with these predictions [17–20].

In this Letter, we examine the magnetic field dependence of the resistance of a series of interferometers with a large range of active areas. We formulate a model based on the assumption that the total charge in the interferometer and the enclosed area both remain constant as the B field is varied. We test it at $\nu = 7/3$ and integer filling factors and show that it is consistent with the experimental data—in the $\nu = 7/3$ case, it predicts a resistance oscillation with the somewhat surprising flux period $\Phi_0/2$. We thereby determine the effective area of the interference loop in each device (and each preparation of each device, which we describe in the next paragraph). The model also predicts that the resistance in the $\nu = 5/2$ state will oscillate as the product of two oscillations, one with flux period $\Phi_0/5$ and the other with flux period Φ_0, as we explain and compare to our experimental data below.

Interferometers.—The interferometers used in this Letter are fabricated from high-mobility (28×10^6 cm^2/V $\cdot$ s), high-density (4.2×10^{11} cm^{-2}) GaAs/AlGaAs quantum well heterostructures. A 40 nm SiN layer is applied to the heterostructure. The size and shape of the 2D electron channel, which is 200 nm below the surface, is controlled by 100 nm thick Al top gates that are deposited on the SiN layer (see Fig. 1). Prior to charging the top gates, the samples can be briefly illuminated to enhance mobility and to provide different sample preparations since the distribution of localized charges changes with illumination and between cool downs to $\sim$20 mK [15,16,21], where the data were taken. A further description of the interferometer operation and discussion of length scales is presented in the Supplemental Material [22] (see also Refs. [23–25]). Two interfering edge currents result from backscattering across constrictions defined by gate sets 1 and 3. Longitudinal resistance R_L is measured by the voltage drop from contact a to d, with current driven from b to c (see Fig. 1), using standard lock-in techniques. The two standard top gate designs shown in the electron micrographs in Fig. 1 are labeled with device dimension parameters x and y adjusted to produce three separate samples with ratios of lithographic areas of roughly 3:2:1.

Previous results.—Oscillations in R_L have previously been observed as a function of side gate voltage V_s, which changes the area of the interferometer [15,16]. Interpreted as due to the Aharonov-Bohm effect, the expected period

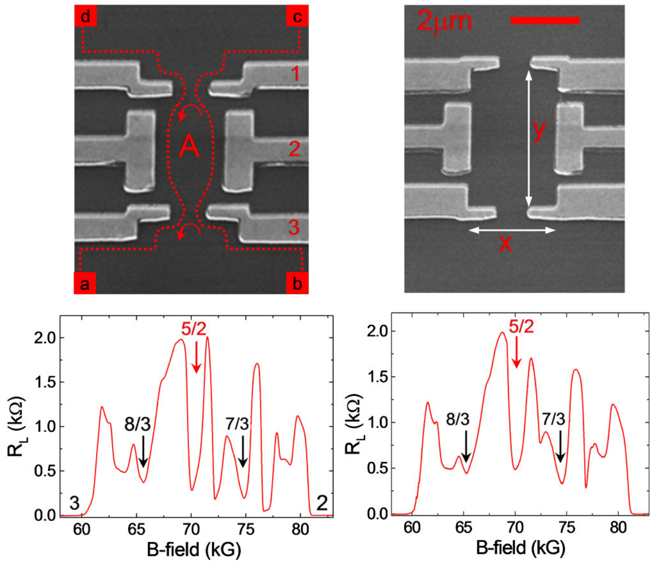

FIG. 1 (color online). Top: electron micrographs of two of the three interferometers used in the measurements The contacts are indicated schematically by a–d. Current injected at contact b can be backscattered at the two quantum point contacts shown, thereby defining an interference loop of area A. Bottom: the longitudinal resistance R_L for the two samples shows minima corresponding to fractional quantum Hall states at $\nu = 7/3, 5/2$, and $8/3$.

of oscillation is $\Delta V_s \propto \Delta A = (e/e^*)\Phi_0/B$ ($\Phi_0 = hc/e$ and e^* is the charge of the interfering quasiparticles), from which the quasiparticle charge e^* could be obtained if the proportionality constant between ΔV_s and ΔA were known. Assuming its independence of the B field, this constant could be determined from the period of R_L oscillations at integer filling factors, where $e^* = e$, or at $\nu = 7/3$, where $e^* = e/3$ is expected. Both filling fractions give similar proportionality constants between ΔV_s and ΔA, consistent with approximate B field independence. At $\nu = 5/2$, R_L oscillations in some intervals of V_s appear consistent with AB oscillations corresponding to $e/4$ charges while in other intervals they seem consistent with $e/2$ charges [15,16]. These results have been interpreted as a manifestation of the even-odd effect [13,14]: charge $e/4$ oscillations should be observed only when there is an even number of charge $e/4$ quasiparticles in the interferometer; charge $e/2$ oscillations should always be observed. When there is an odd number of charge $e/4$ quasiparticles in the interferometer, $e/2$ oscillations should be the only type visible [18]. In Refs. [15,16] only $e/2$ oscillations are visible in certain side-gate voltage intervals (when, according to this interpretation, an odd number of $e/4$ quasiparticles is in the interferometer), but it is not clear whether both $e/4$ and $e/2$ oscillations—or only $e/4$—are present in the other intervals.

By contrast, in this Letter we focus on R_L measurements during magnetic field sweeps. At integer filling, a B-field sweep produces AB oscillations of R_L with period $\Delta B \cdot A = \Phi_0 \approx 41\ \mathrm{G}\,\mu\mathrm{m}^2$, where A is the current-encircled area of the interferometer; we thereby determine the active area for each of the different devices and sample preparations.

Model.—The key assumption in our interpretation of the experimental data is that the charge contained within the interference loop and the area of the loop remain constant as the magnetic field is varied. It is natural to assume that the charge contained within the loop remains constant if it is primarily determined by the local electrostatic potential. In such a case, as the magnetic field is varied, one of two possibilities will occur. Quasiparticles will be created in the bulk or else the quantum Hall droplet will shrink or expand; in the former case, the area of the interference loop will remain constant. We expect this scenario to hold if there are localized states in the bulk that have very low energy as a result of disorder so that it is energetically favorable to create quasiparticles there, rather than to change the charge density at the edge. When this scenario holds, increasing the flux through the interferometer by Φ causes the number of charge e^* quasiparticles to change by $N_{e^*} = (\nu\Phi/\Phi_0)/(e^*/e)$.

Meanwhile, changing the flux by Φ and the number of charge e^* quasiparticles by N_{e^*} causes a change $\Delta\gamma$ in the phase acquired by a quasiparticle taking one path around the interferometer relative to the phase acquired by a quasiparticle going around the other [26],

$$\begin{aligned}\Delta\gamma &= 2\pi(\Phi/\Phi_0)(e^*/e) - 2\theta_{e^*}N_{e^*} \\ &= (\Phi/\Phi_0)[2\pi(e^*/e) - 2\theta_{e^*}(\nu e/e^*)]. \qquad (1)\end{aligned}$$

The first term on the right-hand side is the electromagnetic AB phase seen by a charge e^* quasiparticle encircling flux Φ. The second term is the statistical phase seen by a charge e^* quasiparticle encircling N_{e^*} such quasiparticles; the phase acquired when a single charge e^* quasiparticle encircles another is $2\theta_{e^*}$, assuming that the particles are Abelian. For non-Abelian particles, more care is required, as we will see below.

In an integer quantum Hall state, $e^* = e$ and $\theta_e = \pi$, so $\Delta\gamma = 2\pi(\Phi/\Phi_0)$ and R_L will oscillate with B-field period $\Delta B_0 \cdot A = \Phi_0 \approx 41\ \mathrm{G}\,\mu\mathrm{m}^2$. Now consider the $\nu = 7/3$ state. If it is in the same universality class as the $\nu = 1/3$ Laughlin state, then $e^* = e/3$ and $2\theta_{e/3} = 2\pi/3$. Then $\Delta\gamma = -4\pi(\Phi/\Phi_0)$. Consequently, R_L will show oscillations with period $\Delta B_1 \cdot A = \Phi_0/2 \approx 20\ \mathrm{G}\,\mu\mathrm{m}^2$, i.e., half that in an integer quantum Hall state.

Now consider the case of $\nu = 5/2$. If the system is in an Ising-type topological phase such as the Moore-Read state [5] or the anti-Pfaffian state [27,28], then when there is an even number of charge $e/4$ quasiparticles in the interference loop, the Ising topological charge will be 1 or ψ, but when there is an odd number in the interference loop, the Ising topological charge will be σ. As a result, if one particular topological charge is energetically favorable for even quasiparticle number—let us suppose, for the sake of concreteness, that it is 1—then the non-Abelian Ising topological charge has a periodicity of two quasiparticles or, taking $N_{e/4} = (\nu e\Phi/\Phi_0)/(e/4)$, a flux period $\Phi = 2\Phi_0(e/4e)/\nu = \Phi_0/5$. Hence, R_L oscillates with magnetic field period $\Delta B_2 \cdot A \approx 8$ G μm^2. However, there is also an Abelian phase (1) which can have a different periodicity. The Abelian phase acquired when a charge $e/4$ quasiparticle encircles $2N$ quasiparticles with Ising charge 1 (or, equivalently, N charge $e/2$ quasiparticles) is $\theta = \pi/4$ and $N = (\nu e\Phi/\Phi_0)/(e/2)$. Hence, Eq. (1) now reads $\Delta\gamma = -2\pi(\Phi/\Phi_0)$. Therefore, there is also a slower oscillation in R_L with magnetic field period $\Delta B_0 \cdot A = \Phi_0 \approx 41$ G μm^2.

If, however, the Ising charge is not fixed to 1 for any even number of quasiparticles, but may be randomly either 1 or ψ, then the slower, period Φ_0, oscillation will be afflicted by random π phase shifts that could wash it out. If the system were in an Abelian (3, 3, 1) state, then similar considerations lead to a period Φ_0 oscillation but no rapid period $\Phi_0/5$ oscillation.

Comparison with experiment.—The overall B-sweep trace between filling factors 2 to 3 of R_L across two of the interferometers is shown in the bottom two panels of Fig. 1; they clearly demonstrate fractional quantum Hall states at $\nu = 7/3$, $8/3$, and $5/2$. This overall trace is averaged locally to define a background, which we subtract from the raw R_L measurement to make the oscillations clearer. The ΔB_0 period should change with area according to our model. From the three devices used and the multiple preparations and gate values employed, the measured ΔB_0 periods show active areas ranging from 0.1 to $\sim$0.6 μm^2.

To put our picture to test, we first consider $\nu = 4$ and $7/3$. Oscillations of R_L with B are shown in the upper left inset of Fig. 2. The period ΔB_0 of oscillations is found to be

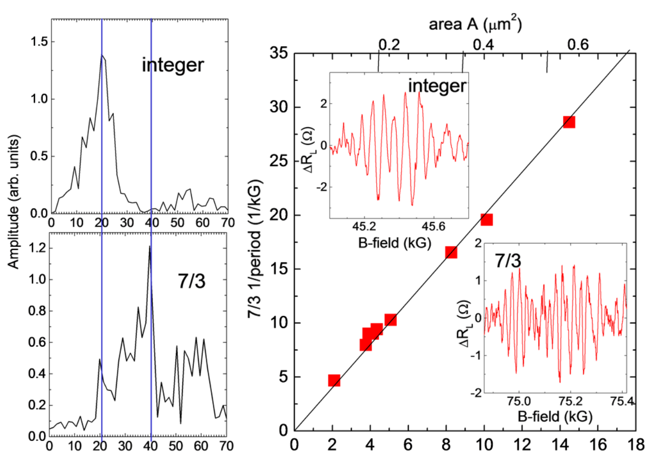

FIG. 2 (color online). Oscillations with magnetic field at the integer state $\nu = 4$ and also at $\nu = 7/3$ (insets), with FFTs of these oscillations shown in the left-hand panels. The ratio between these two oscillation periods is the same in eight device preparations of varying size.

similar near integer filling 2, 3, and 4 for each device, consistent with this being an AB oscillation and not Coulomb effects [29]. From the periodicity ΔB_0, we determine the active area of this preparation.

We now turn to $\nu = 7/3$. Oscillations at 7/3 (and integer filling factors) are shown in Fig. 2 insets; corresponding Fourier transforms in the left panels show peaks. The 7/3 peak frequency is twice the $\nu = 4$ peak frequency (or half the period), consistent with the analysis above. The same R_L measurements comparing $\nu = 7/3$ and integer filling factors were carried out on the three different devices and different preparations of this study, as summarized in the right panel of Fig. 2. The R_L oscillations at $\nu = 7/3$ consistently occur at twice the frequency of their respective integer filling factor oscillations over the full range of device areas studied. We conclude that the assumptions and analysis outlined above are valid.

Figure 3 presents the comparison between interference oscillations of ΔR_L at $\nu = 5/2$, $7/3$, and $\nu = 4$ observed in the same sample or preparation. (The overall B-sweep

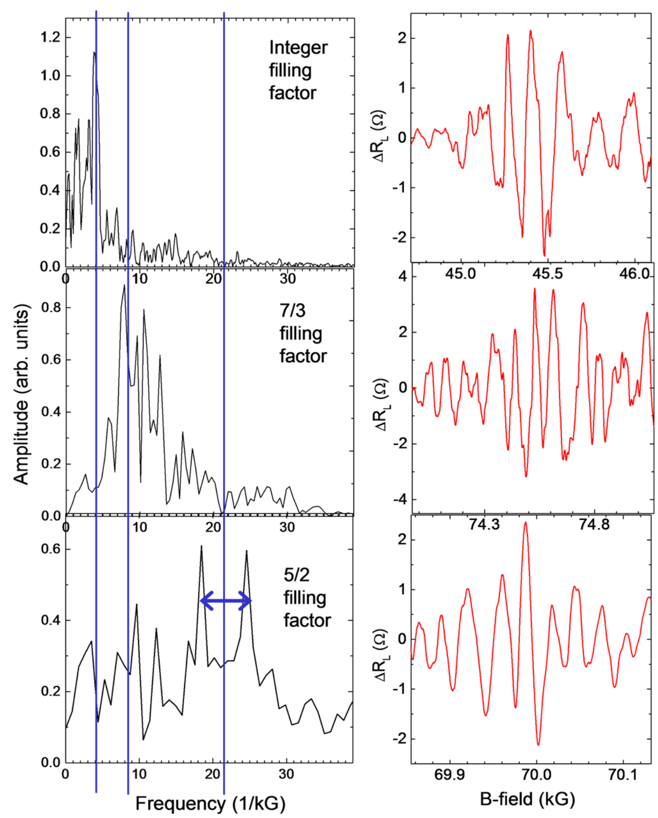

FIG. 3 (color online). Oscillations in R_L as a function of magnetic field and the associated Fourier transforms at $\nu = 4$, $\nu = 7/3$, and $\nu = 5/2$. The vertical blue lines mark $5\times$, $2\times$, and the integer frequency. The oscillations at $\nu = 7/3$ are observed to have twice the frequency of those at $\nu = 4$, which is consistent with the theoretical model explained in the text. The oscillations at $\nu = 5/2$ show beating between a fast oscillation with a period that is 1/5 that at $\nu = 4$ and a slow one with the same period as at $\nu = 4$.

trace of R_L for this interferometer is shown in the bottom right panel of Fig. 1.) Sets of oscillations are shown with their respective Fourier transforms. Once again, the $\nu = 7/3$ oscillations are observed at half the B-field period of those at $\nu = 3$. Importantly, the oscillations at $5/2$ contain a higher frequency component, and the fast Fourier transform (FFT) spectrum demonstrates that the predominant frequency is 5 times that of the integer oscillation frequency. This value is consistent with the expected oscillation frequency for expression or suppression of non-Abelian $e/4$ interference due to the changing number of quasiparticles with varying magnetic field. Moreover, the peak centered around 5 times the integer frequency is split, with the splitting being roughly twice the frequency observed at integer plateau. This corresponds to beats which are further consistent with the above prediction for the interplay between the AB and statistical contributions for a non-Abelian $\nu = 5/2$ state. Other such data sets are presented in the Supplemental Material [22].

The observation of a small oscillation period at $\nu = 5/2$ in a series of samples with different interferometer sizes and different sample preparations is further demonstrated in Figs. 4 and 5. Figure 4 (top panel) shows the B-sweep results for another preparation, focusing on the small period resistive oscillations corresponding to multiple parity changes in the enclosed $e/4$ quasiparticle number near $5/2$. In our model, the five periods of oscillation shown here represent ten parity changes. The B-field range near $5/2$ where this data is taken is marked in the overall R_L trace. The distinct resistive oscillations (black trace; the blue trace is a coarse smoothing of the data; the reproducibility of the traces is demonstrated in the Supplemental Material [22]) near $5/2$ in this preparation have a period $\Delta B_2 \approx 26$ G, versus a period at integer fillings of $\Delta B_0 \approx 125$ G. This is precisely the same five-fold ratio shown in Fig. 3, once again in agreement with our model. Note here that splitting in the $5/2$ peak is not resolved, which may be an indication that the fermion parity in the interferometer is not constant, a possibility discussed above. Or, instead, sweeps through a wider B-field interval may be necessary to observe this slow oscillation in some samples or preparations. Wider sweeps may also reveal oscillations due to transport by charge $e/2$ quasiparticles, which should have a period $\Delta B \cdot A = \Phi_0/2$ (by essentially the same argument as for $7/3$). They are not apparent in the magnetic field sweeps in Figs. 3 and 4, even though they are seen in side-gate voltage sweeps [15,16].

Figure 5 summarizes the principal result of this study. The oscillation period in units of flux (i.e., $\Delta B_2 \cdot A$) at $\nu = 5/2$ measured for different samples or preparations is approximately independent of the device area derived from ΔB_0. The observed values of $\Delta B_2 \cdot A$ are shown to be in reasonable agreement with the expected value of 8 G μm^2.

The interferometers studied in this Letter are small and the interference loops have large aspect ratios. This may contribute to aperiodicities in the AB effect at all filling factors. It may cause bulk $e/4$ quasiparticles to be close to each other and to the edge, which may complicate our simple theoretical picture [18,20,30]. Finally, it leads to large oscillation periods ΔB and, as a result, a limited number of oscillations from which beating effects and Fourier transforms can be extracted. This is the price that must, seemingly, be paid to observe interference effects at $7/3$ and $5/2$ because oscillation amplitudes diminish with increasing interferometer size [29] due to increased dephasing for longer paths. Nevertheless, further ongoing experiments are investigating larger interferometers with smaller aspect ratios.

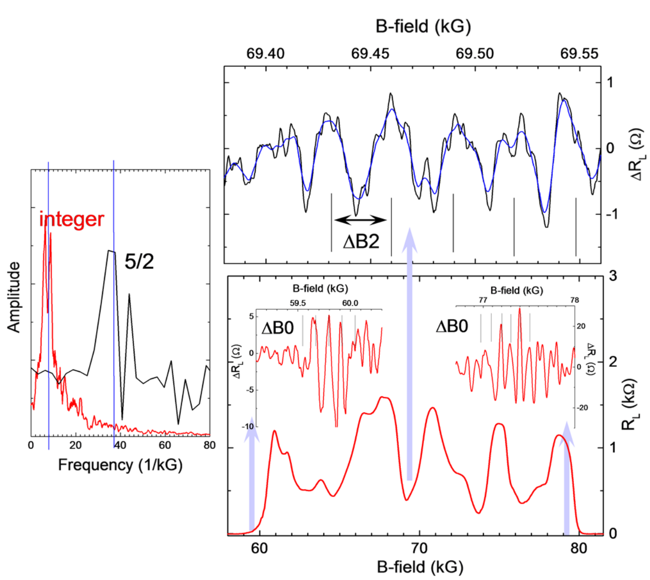

FIG. 4 (color online). Oscillations at $\nu = 2$ (right inset), $\nu = 3$ (left inset), and at $\nu = 5/2$ (top panel). From the Fourier transforms of R_L versus B (left panel), the oscillation period at $\nu = 5/2$ is $1/5$ as large as at $\nu =$ integer. Sometimes it shows beating with the same period as at $\nu = 2$.

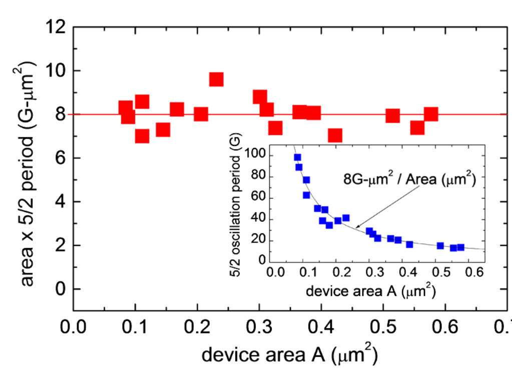

FIG. 5 (color online). The oscillation period in units of flux is independent of the device area and is approximately 8 G μm^2 = $\Phi_0/5$. Equivalently (inset) the oscillation period in units of magnetic field is inversely proportional to the area.

To conclude, this experiment provides the necessary complement to prior measurements [15,16] where AB oscillations were examined as a function of the active interferometer area A controlled by side-gate voltage. By sweeping the B field in these multiple area devices instead, the previous experimental limitation coming from slow gate charging has been avoided. The resistance oscillations observed near filling factor $\nu = 5/2$ in multiple devices show a period consistent with the additional magnetic field needed to add one quasihole to their respective (different) active areas (thereby changing the quasiparticle parity). We stress that the presence of such a period is indicative of a non-Abelian nature of the $\nu = 5/2$ state. While this interpretation is based on several assumptions discussed earlier, using the $\nu = 7/3$ fractional quantum Hall state for control measurements significantly strengthens our case.

R. L. W., C. N. and K. S. acknowledge the hospitality of KITP supported in part by the NSF under Grant No. PHY11-25915. C. N. and K. S. are supported in part by the DARPA-QuEST program. K. S. is supported in part by the NSF under Grant No. DMR-0748925. The work at Princeton was partially funded by the Gordon and Betty Moore Foundation through Grant No. GBMF2719 and by the National Science Foundation, Grant No. MRSEC-DMR-0819860, at the Princeton Center for Complex Materials.

[1] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[2] R. Willett, J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. **59**, 1776 (1987).
[3] W. Pan, J.-S. Xia, V. Shvarts, D. E. Adams, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **83**, 3530 (1999).
[4] J. P. Eisenstein, K. B. Cooper, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **88**, 076801 (2002).
[5] G. Moore and N. Read, Nucl. Phys. **B360**, 362 (1991).
[6] M. Greiter, X. G. Wen, and F. Wilczek, Nucl. Phys. **B374**, 567 (1992).
[7] C. Nayak and F. Wilczek, Nucl. Phys. **B479**, 529 (1996).
[8] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[9] P. Bonderson, V. Gurarie, and C. Nayak, Phys. Rev. B **83**, 075303 (2011).
[10] C. de C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen, Phys. Rev. B **55**, 2331 (1997).
[11] E. Fradkin, C. Nayak, A. Tsvelik, and F. Wilczek, Nucl. Phys. **B516**, 704 (1998).
[12] S. Das Sarma, M. Freedman, and C. Nayak, Phys. Rev. Lett. **94**, 166802 (2005).
[13] A. Stern and B. I. Halperin, Phys. Rev. Lett. **96**, 016802 (2006).
[14] P. Bonderson, A. Kitaev, and K. Shtengel, Phys. Rev. Lett. **96**, 016803 (2006).
[15] R. L. Willett, L. N. Pfeiffer, and K. W. West, Proc. Natl. Acad. Sci. U.S.A. **106**, 8853 (2009).
[16] R. L. Willett, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **82**, 205301 (2010).
[17] X. Wan, Z.-X. Hu, E. H. Rezayi, and K. Yang, Phys. Rev. B **77**, 165316 (2008).
[18] W. Bishara, P. Bonderson, C. Nayak, K. Shtengel, and J. K. Slingerland, Phys. Rev. B **80**, 155303 (2009).
[19] W. Bishara and C. Nayak, Phys. Rev. B **80**, 155304 (2009).
[20] B. Rosenow, B. I. Halperin, S. H. Simon, and A. Stern, Phys. Rev. B **80**, 155305 (2009).
[21] R. L. Willett, M. J. Manfra, L. N. Pfeiffer, and K. W. West, Appl. Phys. Lett. **91**, 052105 (2007).
[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.111.186401 for further details of the theoretical model used to analyze the data and additional data sets.
[23] Y. Zhang, D. T. McClure, E. M. Levenson-Falk, C. M. Marcus, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **79**, 241304 (2009).
[24] N. Ofek, A. Bid, M. Heiblum, A. Stern, V. Umansky, and D. Mahalu, Proc. Natl. Acad. Sci. U.S.A. **107**, 5276 (2010).
[25] D. T. McClure, Ph.D. thesis, Harvard University, 2012, http://qdev.nbi.ku.dk/student_theses/pdf_files/McClure_Thesis.pdf_copy/.
[26] The relative minus sign can be understood using the argument in Ref. [10], where it is explained why the AB and statistical phases should cancel under certain conditions.
[27] M. Levin, B. I. Halperin, and B. Rosenow, Phys. Rev. Lett. **99**, 236806 (2007).
[28] S.-S. Lee, S. Ryu, C. Nayak, and M. P. A. Fisher, Phys. Rev. Lett. **99**, 236807 (2007).
[29] R. L. Willett, L. N. Pfeiffer, K. W. West, and M. J. Manfra, arXiv:1301.2594.
[30] B. Rosenow and S. H. Simon, Phys. Rev. B **85**, 201302 (2012).