Dynamic Circulation in a Complex Plasma

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The dynamic circulation of dust particles in a cylindrical complex plasma is studied experimentally. A levitated cloud of charged dust particles rotates around an axis in an ion flow induced by the coupling of applied magnetic field with the electric field due to plasma density gradient. The vertical and horizontal cross sections of the cloud reveal the dynamic circulation with helical trajectories of dust particles with meridional ascending motion near the axis. The dust particles in the center bottom in the cloud remain near a stagnation point and act like tea leaves in a teacup as described by Einstein in 1926.

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The teacup phenomenon is an example of an everyday phenomenon that offers a profound physics description for understanding mechanisms of dynamic motion of Earth's atmosphere such as tornados and hurricanes. The circulation known as a storm in a teacup was studied as early as 1926 by Einstein [1] and later in a context of rotating fluid flow [2,3], and is now applied to a new technology in industrial and medical applications [4].

A complex plasma, the research of which was prompted by the formation of plasma crystals in the laboratory [5] and the observation of spokes in the Saturnian Ring [6], has been studied extensively as a complex system of a plasma [7–12]. Dust particles charged and levitated in a plasma are visible individually through the illumination of laser light and give us the opportunity to study fundamental physics of dynamics like vortex formation [13–15], configuration of minimum energy [16,17], bubble formation [18], and bow shock formation [19]. Recent interest in a complex plasma was advanced in the presence of a magnetic field [20] which produces rotation of a group of particles [21–23] together with a spinning motion of individual dust particles [24–26]. In this Letter, we report on the dynamic circular motion of dust particles in a cylindrical glass tube in the presence of a strong magnetic field. The observation of particle motion in dynamic circulation helps to understand the simple but profound nature of the ubiquitous vortex commonly encountered in nature.

A complex plasma experiment was performed in a cylindrical glass tube as shown in Fig. 1. The experimental area of the device was 50 mm in diameter and 100 mm in length. The cylindrical coordinates (r, θ, z) are with the origin at the inner bottom of the tube and the gravity is in the negative z direction. The base vacuum pressure was as low as 0.1 Pa and argon gas pressure was $p = 5$–25 Pa. The plasma was produced by an rf discharge of 20 W (13.56 MHz). The rf electrodes were located outside of the glass tube near $z = 0$ (powered electrode) and $z = 100$ mm (grounded electrode) to effectively avoid the thermophoretic force acting on dust particles. The plasma was characterized by electron density $n_e \sim 10^{14}$ m^{-3}, electron temperature $T_e \sim 3$ eV measured by a double probe, and the ion temperature was estimated to be $T_i \sim 0.03$ eV. The shielding length was given by the ion Debye length $\lambda_D \approx \lambda_{Di} \sim 0.13$ mm. The magnetic field at a distance z (in mm), $B(h, z) \approx 0.29\{50/(z + h + 50)\}^3$ T, was applied by a cylindrical permanent magnet of 50 mm in diameter placed a distance h (in mm) below the glass tube, and the magnetic field strength was controlled by adjusting the distance by a jack. Dust particles were supplied from a dust reservoir on the top of the glass tube which was located at $z \simeq 600$ mm. The dust particles were acrylic acid resin spheres with radius $a = 1.5$ μm and mass $m_d = 1.7 \times 10^{-14}$ kg and charged in a plasma to $Q = Z_d e \sim -10^4 e$, where e is the elementary electric charge [27]. The levitated dust particles were irradiated with a thin fan green laser light sheet from the radial directions. The laser sheet can be rotated around the laser axis to observe dynamic motion of individual dust particles in

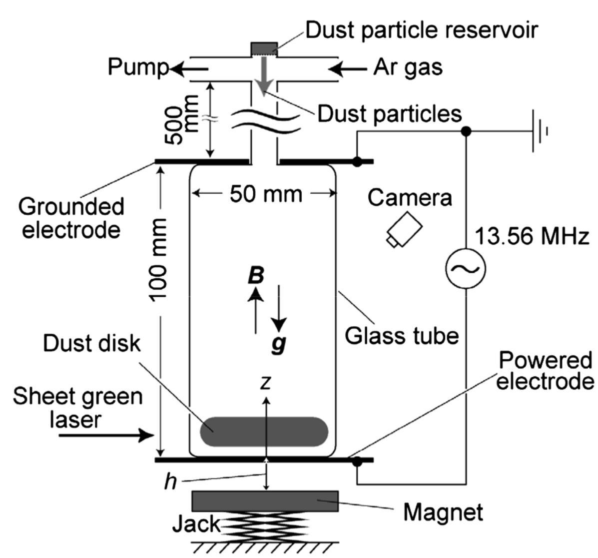

FIG. 1. Schematic drawing of the experimental device.

the vertical direction as well as in the horizontal direction. The scattered laser light from the particles was observed and recorded with a camera placed outside of the device.

Because of the cylindrical symmetry, the location and the dynamic motion of dust particles were well observed by taking pictures in a meridional (vertical) plane as shown in Fig. 2. When $B(h, 0) \leq 0.006$ T with $h \gtrsim 137$ mm, the dust particles levitate a few mm above the glass bottom forming a thin disk of radius 20 mm with a dense group of particles at the rim of the disk near the outer wall as shown in Fig. 2(a). The levitated position is at the transition of the sheath plasma where the downward gravitational force balances with the upward sheath electric force on dust particles. The vertical electric field is given by $E_z = m_d g/|Z_d|e \approx 1$ V/cm. The presence of a dense region of dust particles near the outer wall is characterized by the structure of the sheath near the bottom edge of the cylindrical tube and serves as a source of dust particles in a stronger magnetic field.

When B is increased, the stored dust particles near the wall moved inward to the center and formed a disk of dust particles uniformly distributed above the glass bottom as shown in Fig. 2(b). For $B(17, 0) = 0.12$ T with $p = 20$ Pa, electrons and ions were weakly magnetized and dust particles were rotating around the axis of the tube. The radial electric field is induced by the ambipolar diffusion and the vertical applied B field produces $\boldsymbol{E} \times \boldsymbol{B}$ drift motion of plasma particles, resulting in a solid-body-like azimuthal motion of dust particles with angular velocity 10–20 mm/s. The disk rotation becomes opposite in direction when the magnetic field is reversed, confirming the mechanism of $\boldsymbol{E} \times \boldsymbol{B}$ drift induced rotation. The minimum B field to characterize the magnetized plasma was given by a relation $|\omega_{ce}|\tau_{en} \approx 1$, where $\omega_{ce} = -eB/m_e$ is the electron cyclotron frequency and τ_{en} is the electron-neutral collision time, or $B \simeq 10^{-5} p T_e^{1/2} T_n$ T with p a neutral pressure in Pa, T_e (T_n) electron (neutral) temperature in eV. With $p = 20$ Pa, $T_e \approx 3$ eV, and $T_n \approx 0.03$ eV the critical B field was about 0.01 T.

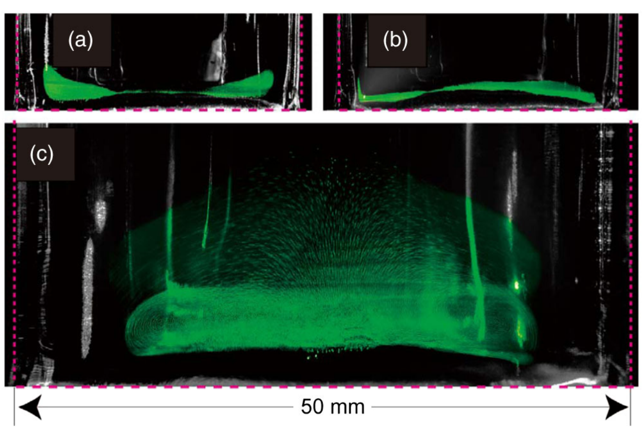

FIG. 2 (color online). Formation of the dust disk at $p = 20$ Pa for various B fields. (a) $B(137, 0) = 60$ G, (b) $B(17, 0) = 1.2$ kG, and (c) $B(12, 0) = 1.5$ kG. The pictures are color processed only for the area where the dust particles exist to keep the color of the laser light. The broken lines show the inner bottom of the tube, while the dotted lines show the wall of the glass tube.

With a further increase of B, the dust disk becomes thicker extending in a positive z direction with a smaller disk radius. The disk was characterized by the thickness of approximately 6 mm with radius 16 mm, while some particles eject out higher reaching as high as $z \approx 21$ mm as shown for the case $B(12, 0) \approx 0.15$ T in Fig. 2(c). Electrons were strongly magnetized with the electron Larmor radius $r_e \approx 0.03$ mm, while ions are moderately magnetized with the ion Larmor radius $r_i \approx 0.7$ mm. A meridional plane reveals a spectacular movement of dust particles in the thick disk as shown in Fig. 3. The dust particles move upward against gravity near the center of the tube from $z \approx 3$ mm. Such ascending motion of dust particles was followed by radial movement toward the outer wall and then downward along the wall. After hitting close to the tube bottom, dust particles move inward along the bottom of the glass tube. While dust particles move in a closed circle in a meridional plane, the dust cloud rotates around the z axis with dust angular velocity $v_\theta \approx 20$ mm/s at $z \approx 5$ mm. Figure 4 shows a schematic illustration of the meridional movement of dust particles for the

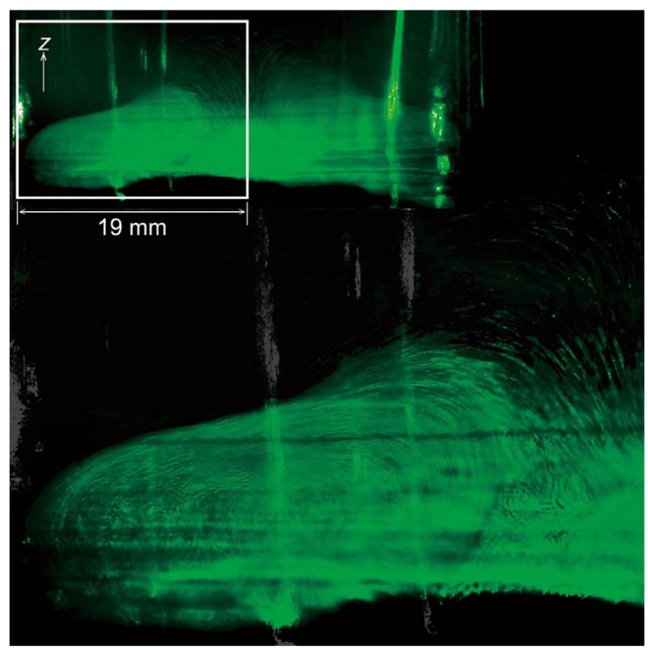

FIG. 3 (color online). Vertical dust motion in the meridional dust flow at $p = 13$ Pa and $B(8, 0) = 1.9$ kG. Dust particles ascend along the z axis at $r \approx 0$, move toward the outer wall, then descend to the bottom and move inward radially, completing the circulation. The upper inset shows a whole picture of the dust flow. Some of the levitating dust particles are gathered and remained in the center without involving the circulation similar to Einstein's tea leaves.

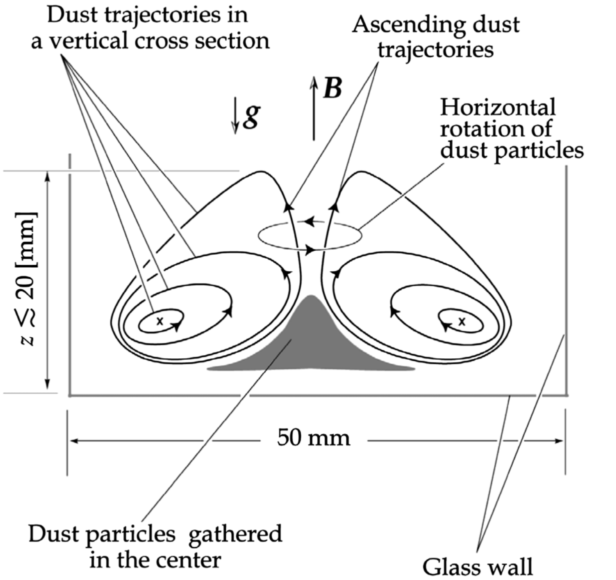

FIG. 4. An illustration of the meridional flow of dust particles. Some levitating dust particles retained at the center near the stagnation point.

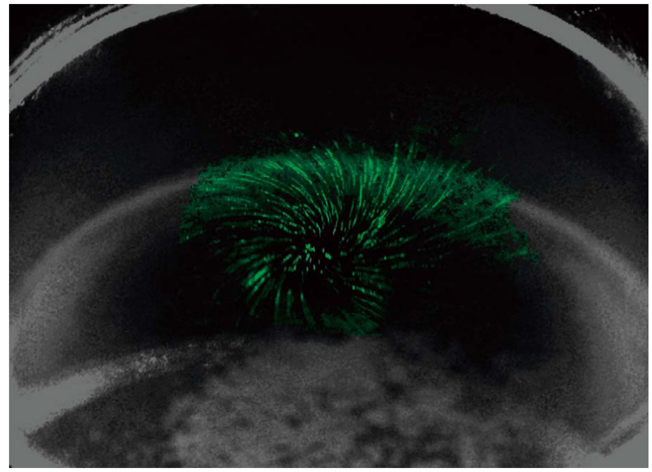

FIG. 5 (color online). Horizontal cross section of the dust disk at $z = 11$ mm. A laser sheet with the thickness about 2 mm illuminates the dust particles in a horizontal plane and the cross section is photographed from the top of the tube. A side wall of the glass tube illuminated by laser light is shown as an eclipse at $z = 11$ mm because of the angle of the picture taken. Dust mass ejection, shown as a firework, is captured in the picture. Dust particles at the center of the tube rise vertically with helical motion from near the bottom ($z \approx 3$ mm) to $z \approx 11$ mm and spread toward the tube wall at and above $z = 11$ mm. Dust particles remaining on the tube bottom without levitation are shown as a white cloud in the lower part of the picture with a circling border of the tube edge, scattered by the stray laser light from the glass wall.

observation shown in Fig. 3. Dust particles are located near the bottom of the tube in a thick disk form and show the rising motion around $r = 0$ against the gravity followed by a circular motion. There is indeed a stagnation point around the center near the bottom of the tube. On the other hand, a group of dust particles shown as a shaded area in Fig. 4 around the central bottom was observed without involving the dynamic rotation.

The horizontal cross sections were photographed from the top of the tube by shining sheet laser with about 2 mm in thickness for $B(12, 0) \approx 0.15$ T and $p = 20$ Pa. At $z = 3$ mm, the dust clouds are in a ring form with a void in the center, indicating the presence of stagnation point around $z = 3$ mm at $r = 0$ with dust particles much less density without involving in the meridional dynamic flow. At $z = 5$ mm, dust particles spread uniformly in a disk around the axis and the dust particles rotate around the z axis like a rigid body rotation. A cross section at $z = 11$ mm taken from the top of the tube is shown in Fig. 5. As shown in Figs. 2(c) and 3, particles rise vertically at around $r = 0$, followed by the radial outward motion. The picture reveals that the particles spurted out from the center ($r = 0$) after rising in helical motion around the z axis. The uprising particles spread like a firework and move toward the tube wall. The vertical cross section shown in Figs. 2(c) and 3, and the horizontal cross sections in Fig. 5 reveal the rotation of the dust cloud around the z axis while dust particles in the boundary layer close to the bottom tube drift in a helical flow with ascending motion at the center like an updraft in the atmosphere.

In a magnetized plasma the radial ambipolar diffusion is suppressed due to the electron magnetization and the current starts to flow in an azimuthal direction. Thus the azimuthal electric field associated with the current density is given by the plasma density gradient as

$$E_\theta = \frac{1}{|\omega_{ce}|\tau_{en}} \frac{\kappa T_e}{e} \frac{1}{n_e} \frac{\partial n_e}{\partial r} \approx 9 \text{ V/m}, \qquad (1)$$

for $|\omega_{ce}|\tau_{en} \approx 17$, $\kappa T_e/e \approx 3$V, and $L_n \equiv (\partial n_e/n_e \partial r)^{-1} \approx 2 \times 10^{-2}$ m. And the electric field will then produce the azimuthal motion of ions with angular velocity $v_{\theta,\text{ion}} = eE_\theta/m_i \nu_{in} \approx 40$ m/s. Those ions circling around the cylindrical tube will move charged dust particles producing the dust flow around the z axis. Our observed maximum dust angular velocity was $v_{\theta,\text{dust}} \approx 0.02$ m/s, near the wall. The rotating magnetohydrodynamics (MHD) fluid involving dust particles may be described by the Navier-Stokes equation with the continuity equation of an incompressible fluid of constant mass density

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla}\right)\boldsymbol{v} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \nu \boldsymbol{\nabla}^2 \boldsymbol{v} + \frac{1}{\rho}\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{f}, \qquad (2)$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0, \qquad (3)$$

where ρ is a mass density of dust fluid, ν is a kinematic viscosity, and $\boldsymbol{f}$ is an external force including gravitational force, drag force, thermophoretic force and others. The gravitational force with gravitational acceleration $\boldsymbol{g}$ is in the negative z direction, while the drag forces by neutral particles and ions are in the azimuthal direction. The drag forces and the thermophoretic force are negligible in our experimental conditions [28]. We consider rotating fluid with constant angular frequency Ω far from the tube bottom ($z = 0$). Equations (2) and (3) can be solved for steady, axisymmetric flow with $\boldsymbol{v} = (v_r, v_\theta, v_z)$ in cylindrical coordinates with boundary conditions $v_\theta = r\Omega$, $v_r = 0$ at $z = \infty$ and $v_r = v_\theta = v_z = 0$ at $z = 0$. We assume $\boldsymbol{J} = (0, J_\theta, 0)$ and $\boldsymbol{B} = (0, 0, B)$. The equilibrium condition requires

$$-r\Omega^2 = -\left(\frac{\partial p}{\partial r} - J_\theta B\right)\Big/\rho, \tag{4}$$

indicating that the centrifugal force on a dust particle is balanced by the pressure gradient and the Lorentz force. By introducing dimensionless parameters, $\bar{v}_r = v_r/r\Omega$, $\bar{v}_\theta = v_\theta/r\Omega$, $\bar{v}_z = v_z/\sqrt{\nu\Omega}$, $\bar{p} = p/\rho\nu\Omega$, and $\bar{g} = g/\sqrt{\nu\Omega^3}$ with $\bar{z} = z/\sqrt{\nu/\Omega}$, Equations (2) and (3) can be expressed by a set of three ordinary differential equations $\bar{v}_r'' = 1 + \bar{v}_r^2 - \bar{v}_\theta^2 + \bar{v}_z\bar{v}_r'$, $\bar{v}_\theta'' = 2\bar{v}_r\bar{v}_\theta + \bar{v}_\theta'\bar{v}_z$, and $\bar{v}_z' = -2\bar{v}_r$ supplemented by the pressure gradient equation $\bar{p}' = \bar{v}_z'' - \bar{v}_z\bar{v}_z' - \bar{g}$. Here the prime shows the derivative with respect to $\bar{z}$. The set of equations is well studied as similarity solutions for the rotating fluid [3]. The solution shows the presence of a stagnation point at $(r, z) = (0, 0)$ and the presence of a thin boundary layer near the bottom of the tube ($0 \leq \bar{z} \lesssim 3$) where the fluid moves inward causing the meridional flow of dust particles directing toward the center of rotation. Our observation shows the boundary layer $3\sqrt{\nu/\Omega} \approx 5$ mm. As Eqs. (2) and (3) show, dust particles drift in the azimuthal direction and the centrifugal force on a particle is given by $F_C = m_d r(\alpha\Omega)^2$ with α, a number less than unity. The centrifugal force is balanced by an inward drag force by neutral particles $F_{\mathrm{Drag}} = C_D \pi a^2 m_n n_n v_r^2/2$, where C_D is a drag coefficient, m_n is a neutral mass, n_n is a neutral density, and $v_r (= \beta r\Omega)$ is a representative radial velocity of dust particles with $\beta < 1$. The balancing equation gives the equilibrium radius as

$$r = \frac{8}{3C_D}\frac{\rho_d}{\rho_n}\left(\frac{\alpha}{\beta}\right)^2 a, \tag{5}$$

where ρ_d is a dust mass density and $\rho_n = m_n n_n$. Equation (5) with $\alpha/\beta \approx 0.03$ gives an observed equilibrium radius of about 0.02 m.

The mechanism of the meridional dust flow is understood in the following way. Initially dust particles are driven by the ion azimuthal motion caused by the radial plasma density gradient in the presence of a strong vertical magnetic field. While the MHD dust fluid forms a rotation around the axis, the angular velocity of dust particles near the tube bottom is reduced by the friction from the sheath-plasma transition, resulting in the reduction of the centrifugal force. And the equilibrium given by Eq. (3) within the rotating MHD fluid is disturbed near the bottom. As a result, the pressure gradient force together with the Lorentz force which remain the same near the bottom generate a radial inward flow of dust particles toward the axis. Because of the continuity, the radial inward motion will be compensated by an axial upward flow. Although the solution to the three ordinary differential equations shows the upward constant motion with $v_z \approx 1.4\sqrt{\nu\Omega}$ at $z \geq 10\sqrt{\nu/\Omega}$, our observation of dust particles clearly shows the upper limit in the ascending motion as high as $z \approx 20$ mm, which is about $12\sqrt{\nu/\Omega}$. This is caused by the heavy charged dust particles placed in the boundary of the sheath and the plasma. Dust particles near the bottom ascend along the axis in the sheath where the sheath electric force pushes charged dust particles upward while the gravitational force pushes dust particles downward. The presence of the strong sheath electric field is responsible for the spectacular dust ejection observed to be like fireworks. When rising dust particles move in a plasma region outside of the sheath, the dust particles feel only gravitational force, which eventually limits the height of dust particles. Thus the ascending motion of dust particles at the axis is followed by the outward movement toward the wall and then dust particles descend near the tube wall as shown in Fig. 4.

A circulation with an inward flow at the bottom has been known as a teacup phenomenon [2], also known as Einstein's tea leaves [1]. We note that Einstein in 1926 explained that tea leaves gather in the center of the teacup when the tea is stirred as a result of a secondary, rim-to-center circulation caused by the fluid rubbing against the bottom of the cup. It is indeed observed in our complex plasma experiment that there were some levitated dust particles staying close to the bottom near the center as shown in Fig. 2 together with the illustration in Fig. 3. Because of the nature of less dense dust particles remaining near the stagnation point, the top cross-sectional view at $z = 3$ mm appeared as a void structure.

In summary, we have observed the dynamic circulation of charged dust particles, similar to the teacup phenomenon, in a cylindrical complex plasma by the application of a strong magnetic field of more than 0.15 T. The azimuthal electric field produced by the density gradient in a magnetized plasma pushes ions which cause a drifting motion of dust particles in a cylindrical plasma. The meridional dust flow in the sheath near the bottom is directed toward the center of the rotation. Ascending dust particles near the center along the axis follow the secondary circulation. The retained dust particles without rotation were observed in the sheath at the center near the bottom, like the tea leaves collecting in the center of the teacup as described by Einstein in 1926.

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