Gamma-Ray-Burst Beaming and Gravitational-Wave Observations

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Using the observed rate of short-duration gamma-ray bursts (GRBs) it is possible to make predictions for the detectable rate of compact binary coalescences in gravitational-wave detectors. We show that the nondetection of mergers in the existing LIGO/Virgo data constrains the beaming angles and progenitor masses of gamma-ray bursts, although these limits are fully consistent with existing expectations. We make predictions for the rate of events in future networks of gravitational-wave observatories, finding that the first detection of a neutron-star–neutron-star binary coalescence associated with the progenitors of short GRBs is likely to happen within the first 16 months of observation, even in the case of only two observatories (e.g., LIGO-Hanford and LIGO-Livingston) operating at intermediate sensitivities (e.g., advanced LIGO design sensitivity, but without signal recycling mirrors), and assuming a conservative distribution of beaming angles (e.g., all GRBs beamed within $\theta_j = 30°$). Less conservative assumptions reduce the waiting time until first detection to a period of weeks to months, with an event detection rate of $\gtrsim 10/\mathrm{yr}$. Alternatively, the compact binary coalescence model of short GRBs can be ruled out if a binary is not seen within the first two years of operation of a LIGO-Hanford, LIGO-Livingston, and Virgo network at advanced design sensitivity. We also demonstrate that the gravitational wave detection rate of GRB triggered sources (i.e., those seen first in gamma rays) is lower than the rate of untriggered events (i.e., those seen only in gravitational waves) if $\theta_j \lesssim 30°$, independent of the noise curve, network configuration, and observed GRB rate. The first detection in gravitational waves of a binary GRB progenitor is therefore unlikely to be associated with the observation of a GRB.

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Introduction.—The LIGO and Virgo collaborations have recently released results, investigating the gravitational wave (GW) sky at unprecedented levels of sensitivity [1]. They did not identify any gravitational wave sources, and thereby established new upper limits on the inspiral and merger rates of compact binary systems composed of neutron stars or black holes in the nearby ($z < 0.1$) Universe [2].

Concurrently there has been an active program of observing gamma-ray bursts (GRBs), focusing on rapid follow-up to determine afterglows and identify host galaxies [3–6]. There is growing evidence that most, if not all, short-hard gamma-ray bursts are associated with the mergers of either two neutron stars, or a neutron star with a black hole [7–9]. These studies have also provided redshifts for a subsample of short GRBs, thereby providing preliminary estimates for the rate densities of these events [10,11]. There is tremendous interest in combined gravitational wave and electromagnetic multimessenger observations of these GRBs [12–14], as this would help confirm the first detections of GWs, elucidate the properties of GRBs, and potentially provide interesting measurements of the Hubble constant and the dark energy equation of state [15–19].

One of the most important properties of GRBs is the beaming of the gamma rays during the burst, which relates the observed and total energies of the explosions, and is a crucial factor in estimating the intrinsic GRB event rate. Recent observations suggest beaming opening angles of 1°–30° [3,20,20–22], with numerical studies finding consistent values [23–26].

In this Letter we estimate the limits that arise on the beaming of short-duration GRBs based on the nondetection of GWs from associated binary systems in existing LIGO/Virgo data. We also make projections for the detection rate of binary systems, as a function of mass and beaming angle, for future networks of GW observatories. We emphasize that this rate is *observationally* determined, although it is broadly consistent with the rates arising from population synthesis [27–31]. We also compare the GW detection rates of untriggered and triggered short GRBs.

In what follows we assume that all short GRBs are associated with low-mass compact binary coalescence. While it is conceivable that not all short GRBs are the result of binary coalescences, it is perhaps even more likely that not all binary coalescences result in GRBs. We thus expect that our predictions for the event rates are conservative.

LIGO S6/Virgo VSR2.—From July 2009 to October 2010 the LIGO (Hanford [H] and Livingston [L]) and Virgo [V] observatories conducted a search (S6/VSR2–3) for compact binary coalescences [1,2,32]. They did not detect any gravitational-wave events, and thereby established upper limits on the event rates of coalescences in the local Universe [2].

We follow the approach of [17], taking the representative sensitivities presented in Fig. 1 of [2], and calculating the corresponding horizon distances. As was done in the LIGO compact binary coalescence searches [1], we ignore data below a frequency cutoff of $f_{\rm low} = 40$ Hz for the LIGO detectors, and $f_{\rm low} = 50$ Hz for Virgo, with upper limits set by the frequency of the innermost stable circular orbit. We use nonspinning waveforms based on the stationary phase approximation [17]; comparable results would be found with the inclusion of spin [33]. We utilize the approach of [34] to combine the antenna patterns of different interferometers, taking into account the differing horizon distances (which are functions of the masses) as well as power patterns.

Since the LIGO/Virgo network has not detected a gravitational wave source, it provides a 90% upper limit to the rate density: $\mathcal{R} = 2.3/(\sum_i \bar{V}_i \times \Delta t_i)$, where the sum is over the different detector network configurations, $\bar{V}_i$ is the mean detectable volume, Δt_i is the amount of observational time, and the factor of 2.3 is in accordance with a Poisson process [see the discussion below Eq. (1)]. Although to date the LIGO/Virgo collaboration has performed coincident searches (wherein individual observatories separately and independently detect a source), in the future they are expected to utilize a coherent network analysis (wherein data from multiple observatories are combined and analyzed as an ensemble, increasing the overall sensitivity of the network). We follow a coherent approach, and calculate the coherent network signal-to-noise ratio (SNR) threshold, ρ, which we would need to apply, as a function of mass, to match the rate limits which come out of the full analysis presented in [2], finding a range $\rho = 10.7$–12.2, with the specific value depending on the mass and mass ratio of the binaries. In the next section we utilize this same SNR threshold to make predictions for future advanced networks. We note that [2] assumes a uniform distribution of component masses for their binaries. We also consider the case where the neutron star is restricted to have $m_1 = 1.4\,M_\odot$, and the mass of the companion is given by $m_2 = M_{\rm total} - m_1$. Because this entails higher mass ratios for higher mass binaries, it decreases the overall gravitational-wave strength of the sources in comparison to the uniform distributions, and therefore decreases the detectable volume. At $M_{\rm total} \sim 3\,M_\odot$ we find a rate limit of neutron star–black hole binaries of 4.5×10^{-4} Mpc^{-3} yr^{-1}, comparable to the uniform rate in [2], with the rate rising to 6.1×10^{-5} Mpc^{-3} yr^{-1} at $M_{\rm total} = 15\,M_\odot$ and 6.6×10^{-5} Mpc^{-3} yr^{-1} at $M_{\rm total} = 25\,M_\odot$.

We are interested in relating the observed event rates to the beaming angles of GRBs. We define the beaming angle, θ_j, to be the half-opening angle of one of the two polar jets of a gamma-ray burst. Given the paucity of data on the beaming of short GRBs, it is premature to assume knowledge of the distribution of beaming angles. We therefore assume that all short GRBs have a fixed beaming angle, θ_j, defined by $1/(1-\cos\theta_j) \equiv \int P(\theta)/(1-\cos\theta)d\theta$, where $P(\theta)$ is the true distribution of beaming angles. The implied rate density of these coalescences is given by $\mathcal{R} = \mathcal{R}_{\rm GRB}/(1-\cos\theta_j)$. In what follows we assume a conservative observed rate of short GRBs of $\mathcal{R}_{\rm GRB} = 10$ yr^{-1} Gpc^{-3}, based primarily on BATSE and *Swift* observations [10,11,22]; the GW detection rate scales linearly with this intrinsic rate. Given the lack of detected binaries in the existing LIGO/Virgo data, we find that the minimum value of the beaming angle ranges from $\theta_j > 0.8°$ for binaries with $M_{\rm total} = 3\,M_\odot$, rising to 3.5° for $15\,M_\odot$ and 4.5° for $25\,M_\odot$, assuming a uniform distribution of mass ratios and including the prior from the S5 results [35]. If we require one member of the binary to have $M_1 = 1.4\,M_\odot$, these numbers become 0.3°/1.0°/0.9° for $M_{\rm total} = 3/15/25\,M_\odot$, without the S5 prior. These limits are completely consistent with observations and expectations. However, given that GRBs have recently been observed with $\theta_j \sim 5°$, it is apparent that even the current LIGO/Virgo data is on the verge of providing interesting astrophysical constraints. This suggests that the next generation of detectors should provide quick detections, and we explore this prediction in the next section.

Advanced LIGO/Virgo.—We now calculate the expected detection rate of short GRB progenitors in the advanced LIGO and Virgo detectors, as well as with additional detectors in Japan (KAGRA)[36] and India (LIGO-India) [37]. The advanced LIGO detectors are expected to begin operation in ∼2015, and it is hoped that the LIGO and Virgo observatories will achieve their target advanced detector sensitivities by ∼2017, with the Japanese [J] and Indian [I] detectors operating at comparable sensitivities by ∼2020. We assume an identical noise curve for each of these instruments, given by the representative advanced LIGO noise curves in LIGO document T0900288-v3 [38], with $f_{\rm low} = 10$ Hz. We take the target design sensitivity to be given by the ZERO_DET_high_P.txt curve, corresponding to zero detuning of the signal recycling mirror, and high laser power. We also consider an early, less sensitive incarnation of the detectors resulting from the absence of signal recycling mirrors, given by the NO_SRM.txt curve.

We calculate the mean detectable volume, $\bar{V}$, of a variety of ground-based networks. Our results are presented in Table I, where in all cases we have assumed a network SNR threshold of $\rho = 10$, and our sources are taken to be equal-mass binaries with $m_1 = m_2 = 1.4\,M_\odot$. The mean detectable volume of a network is expected to scale as $\mathcal{M}^{5/2}$, with $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$ the chirp mass, although this relation is imperfect since the scaling also depends on the shape of the noise curve. For example, sufficiently massive binaries merge below the seismic noise floor of the detectors, and are undetectable. We note that the

TABLE I. Mean detectable volume and wait times for the detection of binary coalescence associated with short GRBs in future GW detector networks. The network SNR is taken to be 10, and the volume is calculated for a $1.4\,M_\odot$–$1.4\,M_\odot$ binary. $T_{\rm first}$ is the waiting time until first detection (90% of cases), scaled to the value for the "no SRM" HL network (1.8 months for $\theta_j = 10°$; 16 months for $\theta_j = 30°$). The last three columns list the predicted event rate (per year) for three different values of the beaming angle.

Network	$\bar{V}$ (Gpc^3)	$T_{\rm first}$	$\theta_j = 10°$	$\theta_j = 30°$	$\theta_j = 90°$
HL (no SRM)	0.023	1.00	15	1.7	0.23
HLV (no SRM)	0.039	0.59	26	2.9	0.39
HLV	0.076	0.31	50	5.6	0.76
HLVJ	0.11	0.21	74	8.4	1.1
HLVI	0.11	0.20	75	8.5	1.1
HLVJI	0.16	0.15	103	12	1.6

addition of the signal recycling mirror increases the detectable volume by a factor of 2, resulting primarily from the increased sensitivity in the ~200 Hz range.

The rate of binary coalescences is a function of the rate of observed GRBs (in gamma rays), $\mathcal{R}_{\rm GRB}$, the detectable volume, $\bar{V}$, and the beaming of the GRBs, θ_j, and is given by

$$\lambda = \bar{V}\mathcal{R}_{\rm GRB}/(1 - \cos\theta_j). \quad (1)$$

As mentioned above, we assume $\mathcal{R}_{\rm GRB} = 10\ {\rm yr}^{-1}\,{\rm Gpc}^{-3}$. The predicted event rates as a function of beaming angle are presented in Table I. In Fig. 1 we plot these results for the HL and HLV networks; the plot can be straightforwardly rescaled to different values of $M_{\rm total}$ and $\mathcal{R}_{\rm GRB}$.

How long will a given network have to wait before seeing its first event? This is described by a Poisson process, with the probability of waiting a time τ before detecting the first event given by $e^{-\tau\lambda}$, for an event rate λ. We define $t_{\rm first}$ as the waiting time by which, in 90% of cases, the first event will have been observed: $t_{\rm first} = -\ln(0.1)/\lambda = 2.3/\lambda$. To convert to the 50% and 99% values, one multiplies the 90% waiting times by 0.3 and 2, respectively. The waiting times are presented in Table I, scaled to the HL (no SRM) value (1.8 months for $\theta_j = 10°$). The HLVJI network, with all detectors at the advanced zero-detuning high laser power sensitivity, has a waiting time that is a factor of 0.15 that of the HL (no SRM) curve; this works out to 8 days for $\theta_j = 10°$ and 72 days for $\theta_j = 30°$. If the GRB progenitors are larger mass systems, the waiting time is correspondingly shortened. It is to be noted that our results are roughly consistent with the completely independent rate estimates from population synthesis and observed binary pulsars [30].

Alternatively, the binary origin of short GRBs would be under pressure if no coalescences were observed within a sufficient amount of time. If we take a conservative upper limit of $\theta_j = 45°$, we find that the binary origin can be falsified at the 99% level in 70 months for HL (no SRM), and in 21 months for HLV. However, even if short GRBs are not the result of binary mergers, we nonetheless expect a population of merging systems, and these should be observable by future observatories [30]. The binary origin of short GRBs would also be ruled out by direct observation of a sufficiently nearby GRB without the detection of attendant GWs.

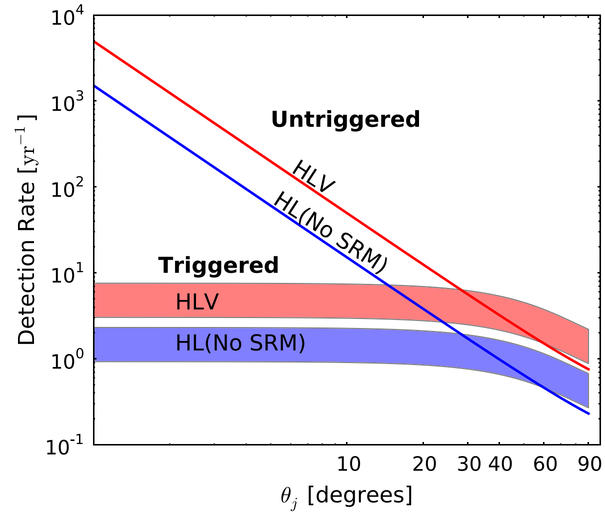

FIG. 1 (color online). Predicted detection rate as a function of the beaming angle, θ_j. Results are shown for both untriggered and triggered GW observations of GRB progenitors, where we have assumed the observed local short GRB rate is $\mathcal{R}_{\rm GRB} = 10\ {\rm yr}^{-1}\,{\rm Gpc}^{-3}$. We plot results for two different GW networks: LIGO Hanford + Livingston (HL) operating without a signal recycling mirror (a potential early sensitivity), and Hanford + Livingston + Virgo (HLV) operating at the design sensitivity. Even in the pessimistic case (HL, no SRM, all short GRBs are beamed with $\theta_j = 30°$), more than 1 detection per year is expected. For more substantial GW networks and reasonable beaming angles, the rates approach ~50 per year, implying a first detection within a month of operation. The bands show the detection rates for GRB triggered GW observations, for a range of values of the reduced SNR threshold (due to the presence of a trigger; see text). The rates of triggered and untriggered GW observations of GRBs match at $\theta_j \sim 30°$ for $\rho \sim 7.5$. For lower values of θ_j untriggered observations occur more frequently than those triggered by GRBs, in which case the first observed binary GRB progenitors would be expected to be seen solely in GWs.

Triggered versus untriggered.—An important question when considering the future GW detection of binary mergers associated with short GRBs is whether triggered or untriggered detections are more likely. We assume that when a binary merges it emits both GWs and gamma rays. If the gamma rays happen to point at the Earth, we will see the event as a gamma-ray burst; we call this a GRB triggered event. This improves the sensitivity of the GW search, as it reduces the need to marginalize over an unknown time and sky position. In addition, because the gamma rays are thought to be beamed, a GRB triggered source is expected to be almost face-on, thereby increasing the strength of the signal in GWs when compared with a source with a random inclination. Alternatively, if the initial detection happens only in GWs, we call this an untriggered event. Given a fixed observed GRB rate, for small beaming angles the rate density of GRB progenitors increases (approaching ∞ as $\theta_j \to 0$), ensuring that for sufficiently small beaming the untriggered rate dominates over triggered GRBs.

In the previous section we calculated the waiting time and event rates for untriggered observations of GRB progenitor systems. We now consider the equivalent calculation in the case of a GW binary with a GRB trigger. If we assume the gamma-ray emission is uniform within θ_j, then the inclination of the binary, as far as the GW emission is concerned, will range uniformly in $\cos(\theta)$ from $\theta = 0°$ (face-on) to $\theta = \theta_j$ (at the edge of the gamma-ray emission). For $\theta_j = 10°$ the detectable volume, and thus the rate, are improved by a factor of 3.4, while for $\theta_j = 30°$ the improvement is 2.8. The improvement in the rate for a perfectly face-on binary is 3.5.

We now estimate the reduction in SNR threshold due to the known time and sky position of the source [17]. The false alarm rate is related to the number of templates needed to search the data for the desired waveforms, and we therefore assume $\exp(-\rho^2/2) \propto 1/\#$ of templates [39], where in the untriggered case we took $\rho = 10$. If we take this threshold to have been based on roughly one year of observation, the existence of a GRB trigger now reduces the observational window down to ~ 10 sec, for a reduction in the number of templates by a factor of $\sim 10^6$. If the sky localization in the untriggered case is ~ 5 deg [40], then compared to a full-sky search (41, 253 deg^2), the reduction in the number of templates is a factor of $\sim 10^3$. The total number of templates is down by a factor of 10^9, which for the equivalent false alarm rate would imply that the SNR threshold is reduced to $\rho \sim \sqrt{\{-2\log[\exp(-10^2/2) \times 10^9]\}} = 7.7$.

In Fig. 1 we compare the detection rates for untriggered and triggered GRBs. We find that the rates of triggered and untriggered events are similar if the average GRB beaming angle is $\theta_j \sim 30°$. This result is independent of the network configuration, the individual noise curves, and the assumed GRB rate, and is weakly dependent on the specific value of the threshold improvement due to the reduced number of templates (as shown by the width of the bands in the figure). If the GRBs are beamed at $\theta_j = 10°$ or less, then the detected rate of untriggered GRBs is at least an order of magnitude larger than the rate of triggered sources. These estimates assume that the GRBs are being detected throughout the sky. If this isn't the case (e.g., the Swift BAT only sees 1/9 of the sky), the rates in the triggered case are further suppressed.

It is to be noted that this process can be inverted, and the wait time before first detection (and in between the first few detections) may be used to infer the beaming angle of GRBs. In addition, the relative rates of triggered and untriggered GRBs will help establish the beaming, and will be an important consistency test when compared with explicit determinations of the beaming distribution based on GW measurements of the inclination of GRB sources and jet break measurements in the electromagnetic spectrum.

As discussed above, recent observations have measured short GRBs with $\theta_j \lesssim 10°$, indicating that it is likely that the detection rate of untriggered GRBs will be significantly greater than the rate of triggered ones, and implying that the first detection of a binary system which is a progenitor of a short GRB will not be triggered by a GRB. Although triggered GRBs may be less frequent than untriggered ones, multimessenger observations of these systems hold tremendous scientific potential, and should be aggressively pursued [41].

Discussion.—We have explored the connection between the observed short GRB rate, the beaming angle of short GRBs, and the predicted rate of detectable binary systems associated with progenitors of GRBs in networks of gravitational-wave observatories. We have shown that existing LIGO/Virgo data provides preliminary constraints on the beaming angle and mass distribution of short GRB progenitor systems. For example, we find that short GRB progenitors with mass $M_{\mathrm{total}} > 20\,M_\odot$ (uniformly distributed in component mass) and with beaming angles of $\theta_j < 4°$ are ruled out by existing LIGO/Virgo data. These constraints, while novel, are fully consistent with our current understanding of the short GRB engine and rates.

We have analyzed the observed rate of short GRB progenitors in future networks of GW detectors. We find that, even in the case of only two detectors (HL) operating at conservative sensitivity, in 90% of cases we would expect a first detection of a binary within 16 months if the GRBs are beamed within $\theta_j = 30°$, and within 55 days if $\theta_j = 10°$. The expected event rates are 1.7 yr^{-1} ($\theta_j = 30°$) and 15 yr^{-1} ($\theta_j = 10°$). We find that the HLV network operating at design sensitivity would shorten these times to 4.9 months ($\theta_j = 30°$) and 17 days ($\theta_j = 10°$), with corresponding event rates of 5.6 yr^{-1} and 49 yr^{-1}. Alternatively, the binary coalescence model for short GRB progenitors can be ruled out if a HLV network does not observe a binary within the first two years of observation.

Finally, we have shown that the rate of GRB triggered observations of GW systems associated with GRB progenitors is lower than the rate of untriggered observations if $\theta_j \lesssim 30°$. This result is independent of network, noise curve, and GRB rate, and when coupled with recent observations of small beaming angles for short GRBs, suggests that the first detections of GRB progenitors with advanced GW networks will not involve the observation of gamma rays from GRBs.

We conclude that, assuming short GRBs are the result of the merger of compact objects, and assuming that the resulting gamma rays are beamed, the first detection of gravitational waves from binary coalescence associated with a GRB progenitor will be untriggered, and may occur within weeks to months of operation of an early network of advanced sensitivity ground-based gravitational wave observatories.

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